A GRAND TOUR OF PHYSICS

QUANTUM MECHANICS

LECTURE 3

\[ \Delta p \Delta x \geq \frac{1}{2} \hbar \]

\[ \Delta E \Delta t \geq \frac{1}{2} \hbar \]

SCHRÖDINGER'S WAVE EQUATION

\[ \frac{\partial}{\partial t} \Psi = -i\hbar \nabla^2 \Psi = E \Psi \]

\( \Psi \) = WAVE FUNCTION

HYDROGEN ATOM

APR. 5, 2019
1:30 – 3:30
TNCC
ROOM 328.

DR. GEORGE DERISE
PROFESSOR EMERITUS, MATHEMATICS
THOMAS NELSON COMMUNITY COLLEGE
SPRING 2019
QUANTUM MECHANICS’ BIRTHDAY – OCT. 7, 1900
“In an act of desperation....”

MAX PLANCK QUANTUM HYPOTHESIS 1900
Maxwell’s theory of electromagnetism cannot explain the photoelectric effect. The assumption of light as a particle explains the photoelectric effect.

LIGHT IS A PARTICLE (A PHOTON)
WITH ENERGY COMING IN DISCRETE BUNDLES-QUANTA!

The energy formula is the same as that used by Planck in black body radiation.
\[
\begin{align*}
\frac{9}{5} & \quad \frac{16}{12} & \quad \frac{25}{21} & \quad \frac{36}{32} & \quad \frac{49}{45} & \quad \frac{64}{60} & \quad \ldots \\
\frac{3^2}{3^2 - 4} & \quad \frac{4^2}{4^2 - 4} & \quad \frac{5^2}{5^2 - 4} & \quad \frac{6^2}{6^2 - 4} & \quad \frac{7^2}{7^2 - 4} & \quad \frac{8^2}{8^2 - 4} & \quad \ldots & \quad \frac{m^2}{m^2 - 4} \\
1.8 & \quad 1.33 & \quad 1.19 & \quad 1.125 & \quad 1.089 & \quad 1.067 & \quad \ldots & \quad 1 \\
656.3 & \quad 486.1 & \quad 434.0 & \quad 397.0 & \quad \ldots & \quad \ldots & \quad \ldots & \quad 364.6
\end{align*}
\]
H ATOM - BOHR - RUTHERFORD MODEL

QUANTIZED ENERGY STATES

1. The electron moves in a circular orbit around the nucleus under the influence of Coulomb’s law.

2. The electron can only move in very particular orbits because of a quantum condition. \( l = n\hbar \); This is unlike classical mechanics.

3. Even though the electron is accelerating constantly (circular orbits) EM energy is not emitted.

4. Electrons can only gain and lose energy by jumping from one allowed orbit to another, absorbing or emitting electromagnetic radiation with a frequency \( \nu \) determined by the energy difference of the levels according to the Planck relation.
BOHR’S HYDROGEN ATOM: DISCRETE QUANTUM ORBITS AND ENERGIES

\[ E_n = n^2 E_1 \]

\[ E_1 = \frac{\hbar^2 \pi^2}{2mL^2} \]

<table>
<thead>
<tr>
<th>ORBIT</th>
<th>QUANTUM NUMBER</th>
<th>DISTANCE to NUCLEUS</th>
<th>ENERGY LEVELS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>1</td>
<td>1a_0</td>
<td>-E_1/1</td>
</tr>
<tr>
<td>2nd</td>
<td>2</td>
<td>4a_0</td>
<td>-E_1/4</td>
</tr>
<tr>
<td>3rd</td>
<td>3</td>
<td>9a_0</td>
<td>-E_1/9</td>
</tr>
<tr>
<td>4th</td>
<td>4</td>
<td>16a_0</td>
<td>-E_1/16</td>
</tr>
<tr>
<td>5th</td>
<td>5</td>
<td>25a_0</td>
<td>-E_1/25</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>nth</td>
<td>n</td>
<td>n^2 a_0</td>
<td>-E_1/n^2</td>
</tr>
</tbody>
</table>
If a light wave consists of photons (particles) then, perhaps particles (e.g. electrons) are transported by waves. (1924 PhD thesis)

Confirmed by the experiment of Davisson and Germer 1927. Electrons scattered by the surface of a crystal displays a diffraction pattern.
\[ \sqrt{2X(0)} = \sqrt{\frac{\hbar}{2\pi}} \begin{bmatrix} 0 & \sqrt{1} & 0 & 0 & 0 & \cdots \\ \sqrt{1} & 0 & \sqrt{2} & 0 & 0 & \cdots \\ 0 & \sqrt{2} & 0 & \sqrt{3} & 0 & \cdots \\ 0 & 0 & \sqrt{3} & 0 & \sqrt{4} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}, \]

HEISENBERG IN HELIGOLAND

MATRIX MECHANICS

\[ AB \neq BA \]
\[
\begin{pmatrix}
A & B \\
C & D
\end{pmatrix} \times \begin{pmatrix}
E & F \\
G & H
\end{pmatrix} = \begin{pmatrix}
AE+BG & AF+BH \\
CE+DG & CF+DH
\end{pmatrix}
\]

\[
\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \times \begin{pmatrix} 2 & 0 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 4 & 4 \\ 10 & 8 \end{pmatrix}
\]

\[
\begin{pmatrix} 2 & 0 \\ 1 & 2 \end{pmatrix} \times \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 2 & 4 \\ 7 & 10 \end{pmatrix}
\]

\[AB \neq BA\]

MATRIX ALGEBRA- A NON COMMUTATIVE ALGEBRA
HEISENBERG’S UNCERTAINTY PRINCIPLE

\[ \Delta p \Delta x \geq \frac{1}{2} \hbar \]

\[ \Delta E \Delta t \geq \frac{1}{2} \hbar \]

THE MORE PRECISELY THE POSITION OF SOME PARTICLE IS DETERMINED, THE LESS PRECISELY ITS MOMENTUM CAN BE KNOWN, AND VICE VERSA.
ONE ESSENTIAL DIFFERENCE BETWEEN CLASSICAL AND QUANTUM MECHANICS

CLASSICAL MECHANICS:

MINIMIZATION OF THE ‘ACTION’
GIVES AN EQUATION OF MOTION

DETERMINISTIC

QUANTUM MECHANICS:

SUMS CONTRIBUTIONS OF
PROBABILITY FUNCTIONS
FROM ALL POSSIBLE PATHS

PROBABILISTIC

ANOTHER ESSENTIAL DIFFERENCE BETWEEN CLASSICAL AND QUANTUM MECHANICS

PHASE PLANE DIAGRAM
QUANTUM MECHANICS

SCHRÖDINGER’S WAVE EQUATION

\[ i\hbar \frac{\partial}{\partial t} \Psi = E\Psi \]

\( \Psi = \text{WAVE FUNCTION} \)

HYDROGEN ATOM

ERWIN SCHRÖDINGER
transistors, personal computers, lasers, Blu-ray players, CDs, DVDs, MRI machines, PET scans, atomic clocks, GPSs, ...

understanding of molecular structure...

“If quantum mechanics suddenly went on strike every single machine that we have in the U.S., almost, would stop functioning.”
Max Tegmark MIT explains in “Quantum Leap”
“I have prepared a two-minute course in quantum mechanics for you.”
Steven Weinberg

Fig. 1 A coin as an example of a simple quantum mechanical system. Probability of heads = $H^2$, probability of tails = $T^2$, so $H^2 + T^2 = 1$. The length of the state vector is $\sqrt{H^2 + T^2} = 1$. 
a and b are called PROBABILITY AMPLITUDES

\[ a^2 + b^2 = 1 \]

DYNAMICS: How the vector rotates with time.

\[
v = a \begin{pmatrix} 1 \\ 0 \end{pmatrix} + b \begin{pmatrix} 0 \\ 1 \end{pmatrix}
\]

PROBABILITY OF GETTING HEADS \( P(H) = a^2 \)

PROBABILITY OF GETTING TAILS \( P(T) = b^2 \)

INTERMEDIATE POSITION, A "SUPERPOSITION OF STATES"

BY LOOKING AT THE COIN (PERFORM AN EXPERIMENT OR MEASUREMENT) YOU FORCE IT INTO ONE OF THE "PURE STATES" H OR T

\[
\begin{align*}
9/25 + 16/25 &= 1 \\
.36 + .64 &= 1
\end{align*}
\]
A "Schrödinger Cat" Superposition State of an Atom

C. Monroe, D.M. Meekhof, B.E. King, and D.J. Wineland

Time and Frequency Division, MS 847, National Institute of Standards and Technology

Boulder, CO 80303

A "Schrödinger cat"-like state of matter has been generated at the single atom level. A
LIGHT IS A WAVE
YOUNG’S INTERFERENCE EXPERIMENT

THOMAS YOUNG (1807): LIGHT HAS THE PROPERTIES OF A WAVE

Light (waves) passing through two slits (double-slit) add together or cancel each other.

INTERFERENCE FRINGES APPEAR.
This phenomenon cannot be explained unless light is considered as a wave.

http://phototerrace.net/en/photon/duality/
YOUNG’S EXPERIMENT MODIFIED- ONE SLIT OPENED

“Each electron goes through either hole 1 or 2.”

So, all electrons that arrive on the backdrop came from either hole 1 or hole 2.

The result $P_{12}$ obtained when both holes are open is not the sum of $P_1$ and $P_2$ the probabilities for each hole alone.

This is just like the interference that sound waves or light waves would exhibit.

Actually the mathematics used to get $P_{12}$ is that that implies waves.

But electrons are particles.

The math does not imply “Each electron goes through either hole 1 or 2.”

That statement is not true.

So, the electron is both a wave and a particle!
\[ I = I_1 + I_2 = H^2 + J^2 \]

\[ I = (H + J)^2 = H^2 + 2HJ + J^2 \]

- \( I_1 \) = intensity of wave, proportional to \( H^2 \)
- \( I_2 \) = intensity of wave, proportional to \( J^2 \)
“Light is not only a wave but also a particle”

Young’s Interference Experiment was carried out using technology to detect individual light particles to see if interference fringes appear even if the light is drastically weakened to the level having only one particle.

Results from the experiment confirmed that one photon exhibited an interference fringe.

When light weakened to an extreme brightness limit and projected on a screen is detected, it behaves like a particle. LEFT

When the recorded particle count increases an interference fringe appears. RIGHT

http://photonterrace.net/en/photon/duality/
WAVE PARTICLE DUALITY
NIELS BOHR’S “COPENHAGEN INTERPRETATION”

• The uncertainty principle.

• The statistical interpretation of Max Born interprets the Schrödinger wave function as yielding the probability of an outcome in any given state.

• Bohr’s complementarity concept: the idea of wave-particle duality.

• Collapse of the wave function:
The Schrödinger equation has the state vector evolving deterministically.

The solution represents the probability of observing a particular outcome when an experiment is performed.

The act of measurement affects the system, causing the set of probabilities to reduce to only one of the possible values immediately after the measurement (collapse of the wave function).

• Positivism: An emphasis on discussing solely the observable outcomes of the experiments rather than on the "meaning" or underlying "reality".
WAVE PARTICLE DUALITY

Waves and Particles:

Waves:
- Spread in space and time
- Can be superposed – show interference effects
- Pass through each other

Particles:
- Localized in space and time
- Cannot pass through each other - they bounce or shatter

<table>
<thead>
<tr>
<th>Phenomenon</th>
<th>Can be explained in terms of waves.</th>
<th>Can be explained in terms of particles.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reflection</td>
<td>![Wave Symbol] ![Particle Symbol]</td>
<td>![Particle Symbol] ![Wave Symbol]</td>
</tr>
<tr>
<td>Refraction</td>
<td>![Wave Symbol] ![Particle Symbol]</td>
<td>![Particle Symbol] ![Wave Symbol]</td>
</tr>
<tr>
<td>Interference</td>
<td>![Wave Symbol] ![Particle Symbol]</td>
<td>![Particle Symbol] ![Wave Symbol] ![X Symbol]</td>
</tr>
<tr>
<td>Diffraction</td>
<td>![Wave Symbol] ![Particle Symbol]</td>
<td>![Particle Symbol] ![Wave Symbol] ![X Symbol]</td>
</tr>
<tr>
<td>Polarization</td>
<td>![Wave Symbol] ![Particle Symbol]</td>
<td>![Particle Symbol] ![Wave Symbol] ![X Symbol]</td>
</tr>
<tr>
<td>Photoelectric effect</td>
<td>![Wave Symbol] ![Particle Symbol]</td>
<td>![Particle Symbol] ![Wave Symbol] ![X Symbol]</td>
</tr>
</tbody>
</table>

https://www.physics.uci.edu/~silverma/quantum.ppt

http://semesters.in/wave-particle-duality/
DR. QUANTUM DOUBLE SLIT EXPERIMENT
https://www.youtube.com/watch?v=DfPeprQ7oGc
EINSTEIN: "I am convinced that He (God) does not play dice."
BOHR: “Einstein, stop telling God what to do”
OPINION OF THE PRACTICAL QUANTUM MECHANIC OF THE COPENHAGEN INTERPRETATION -

I DON’T CARE
QM WORKS!!

Andrew Zimmerman Jones. www.thoughtco.com