A GRAND TOUR OF PHYSICS
ELECTRICITY & MAGNETISM
SPECIAL RELATIVITY

LECTURE 2

MAXWELL’S THEORY OF ELECTROMAGNETISM

\[ \nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0} \]

\[ \nabla \cdot \mathbf{B} = 0 \]

\[ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \]

\[ \nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \]

EINSTEIN’S THEORY OF SPECIAL RELATIVITY

\[ x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}} \]

\[ t' = \frac{t - \left(\frac{v}{c^2}\right)x}{\sqrt{1 - \frac{v^2}{c^2}}} \]
ELECTRICITY AND MAGNETISM

COULOMB’S LAW OF ELECTRICITY

\[ F = k \frac{q_1 q_2}{r^2} \]

NEWTON’S LAW OF GRAVITATION

\[ \vec{F} = -\frac{G m_1 m_2}{r^2} \hat{r} \]

BUT...
The electrostatic force: + and - charges
The gravitational force: mass always +

ALSO:
The relative magnitude of the two forces.
The electrostatic repulsion between two electrons is approximately \(10^{42}\) times larger than the corresponding gravitational attraction.

BUT BOTH HAVE THE \(1/r^2\) Singularity.
The proton (p) has exactly the same charge (+) as the electron e⁻ ( - ) but
the mass of a proton is about 1836.153 times the mass of an electron. *(Why?)*

**CHARGE IS QUANTIZED:**
The charge of a system is an integral multiple of the electric charge of an electron.
Charge comes in discrete units.
Q = ±Ne
Why is charge quantized *(?)*
**LAW OF CHARGE CONSERVATION:**

The total electric charge in an isolated system never changes.

The change in the amount of electric charge in any volume of space is exactly equal to the amount of charge flowing into the volume minus the amount of charge flowing out of the volume.

**CONSERVATION OF CHARGE - Examples**

**CHEMICAL REACTION:** $\text{Na}^+ + \text{Cl}^- = \text{NaCl}$

charge: $(+e) + (-e) = 0$

**RADIOACTIVE DECAY:** $n \rightarrow p + e^- + \nu$

charge: $(0) = (+e) + (-e) + 0$
SCALAR FIELD
FOR EVERY POINT IN SPACE
THERE IS A NUMBER.

VECTOR FIELD
FOR EVERY POINT IN SPACE
THERE IS A VECTOR.
ELECTRIC FIELD LINES START ON POSITIVE CHARGES AND END ON NEGATIVE CHARGES.

THE DENSITY OF ELECTRIC FIELD LINES INDICATES THE STRENGTH OF THE E FIELD.

THE FIELD IS STRONGER WHERE THE LINES GET CLOSER TOGETHER.

FIELD-LINES NEVER CROSS.

THE DIRECTION OF THE ELECTRIC FORCE AT ANY POINT IS TANGENT TO THE FIELD LINE.
**POTENTIAL DIFFERENCE (VOLTAGE)**

AN ELECTRIC POTENTIAL IS THE AMOUNT OF ENERGY NEEDED TO MOVE A UNIT OF A POSITIVE CHARGE FROM ONE POINT TO ANOTHER POINT INSIDE THE FIELD.

THE POTENTIAL DIFFERENCE IS THE ENERGY CHANGE DIVIDED BY THE CHARGE. $V = E/Q$

EQUIPOTENTIAL LINES ARE ALL THE POINTS IN SPACE WHERE THE POTENTIAL IS THE SAME.

![Earth's Gravitational Field](earth_gravity_field.png)
GAUSS’ LAW
ELECTRICITY

\[ \text{div } E = \rho \]

\[ \Delta \Phi = E \Delta A \]

\[ \Phi_{\text{electric}} = \frac{Q}{\varepsilon_0} \]

http://hyperphysics.phy-astr.gsu.edu/hbase/electric/gaulaw.html
MAGNETIC FIELDS

http://hyperphysics.phy-astr.gsu.edu/
Dirac

The existence of one single magnetic monopole would explain quantization of electric charge everywhere in the Universe.
FARADAY’S LAW

A CHANGING MAGNETIC FIELD
PRODUCES AN ELECTRIC FIELD

\[
\frac{\partial B}{\partial t} = - \text{curl } E
\]

MICHAEL FARADAY
JAMES CLERK MAXWELL
AMPERE’S LAW
AN ELECTRIC CURRENT PRODUCES A MAGNETIC FIELD

An electric current produces a circular magnetic field as it flows through a wire. (1820)

\[ J = I/A \]
\[ J = \text{current density in amperes/m}^2 \]
\[ I = \text{current through a conductor, in amperes} \]
\[ A = \text{cross-sectional area of the conductor, m}^2 \]
BEFORE MAXWELL

\[ \nabla \cdot \vec{B} = 0 \]

\[ \nabla \cdot \vec{D} = \rho_v \]

\[ \nabla \times E = -\frac{\partial B}{\partial t} \]

\[ \nabla \times H = J \]

THE EQUATIONS ARE INCONSISTENT

LAW OF CONSERVATION OF CHARGE FAILS
AFTER MAXWELL

\( \nabla \cdot \vec{B} = 0 \)
\( \nabla \cdot \vec{D} = \rho_v \)
\( \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \)
\( \nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \)

and then there was light.

THE EQUATIONS ARE CONSISTENT

LAW OF CONSERVATION OF CHARGE HOLDS

WAVE EQUATION - LIGHT IS A WAVE
MAXWELL’S EQUATIONS IMPLY
THE PROPAGATION OF ELECTROMAGNETIC WAVES
AT THE SPEED OF LIGHT

THE ELECTRIC AND MAGNETIC FIELDS ARE IN PHASE.
PROPERTIES OF WAVES

- Wavelength ($\lambda$)
- Period (period)
- Amplitude

Signal with lower frequency:
- Frequency = 1 cycle per second
- Period ($T$) = 1 s

Signal with higher frequency:
- Frequency = 2 cycles per second
- Period ($T$) = 0.5 s

Phase shift
ELECTROMAGNETIC SPECTRUM

Penetrates Earth’s Atmosphere?

Radiation Type

- Radio: $10^3$ m
- Microwave: $10^{-2}$ m
- Infrared: $10^{-5}$ m
- Visible: $0.5 \times 10^{-6}$ m
- Ultraviolet: $10^{-8}$ m
- X-ray: $10^{-10}$ m
- Gamma ray: $10^{-12}$ m

Approximate Scale of Wavelength

- Buildings
- Humans
- Butterflies
- Needle Point
- Protozoans
- Molecules
- Atoms
- Atomic Nuclei

Frequency (Hz)

- $10^4$
- $10^8$
- $10^{12}$
- $10^{15}$
- $10^{16}$
- $10^{18}$
- $10^{20}$

Temperature of objects at which this radiation is the most intense wavelength emitted

- 1 K: -272 °C
- 100 K: -173 °C
- 10,000 K: 9,727 °C
- 10,000,000 K: ~10,000,000 °C
\[ \text{div } E = \rho \]
\[ \text{div } B = 0 \]
\[ \text{curl } E + \frac{\partial B}{\partial t} = 0 \]
\[ \text{curl } B - \frac{\partial E}{\partial t} = j \]

\text{ELECTRIC-MAGNETIC DUALITY}

\[ \begin{align*}
\text{div } E &= 0 \\
\text{div } B &= 0 \\
\text{curl } B - \frac{\partial E}{\partial t} &= 0 \\
\text{curl } E + \frac{\partial B}{\partial t} &= 0
\end{align*} \]

\[ B \rightarrow E \quad E \rightarrow -B \]
"It is just as foolish to complain that people are selfish and treacherous as it is to complain that the magnetic field does not increase unless the electric field has a curl."

\[ \text{curl} E + \frac{\partial B}{\partial t} = 0 \]

JOHN VON NEUMANN
Conservation of Charge now holds; Maxwell’s Equations without Displacement Current were inconsistent.

1887: Heinrich Hertz conducted a series of experiments that not only confirmed the existence of electromagnetic waves, but also verified that they travel at the speed of light, denoted by c.

WAVE EQUATION!!
Light (in fact all Electromagnetic radiation) propagates as a wave.

MAXWELL’S EQUATIONS

\[
\begin{align*}
\text{div } E &= \rho \\
\text{div } B &= 0 \\
\text{curl } E + \frac{\partial B}{\partial t} &= 0 \\
\text{curl } B - \frac{\partial E}{\partial t} &= j
\end{align*}
\]

Maxwell’s Theoretical ‘idea’ - Displacement Current
INERTIAL FRAME OF REFERENCE

A frame of reference in which a body with zero net force acting upon it is not accelerating;

\[ F = ma \]
\[ 0 = ma, \quad m \neq 0 \]
\[ a = 0, \]
acceleration is 0
velocity is constant

GALILEON’S SHIP;
GALILEO IN A GONDOLA

All inertial frames are in a state of constant, rectilinear motion with respect to one another. Measurements in one inertial frame can be converted to measurements in another by a simple transformation—(the Galilean Transformation in Newtonian physics).
INERTIAL FRAME OF REFERENCE (GALILEAN REFERENCE FRAME):
...one that is not undergoing acceleration.
All inertial frames are in a state of constant, rectilinear motion (constant velocity) with respect to one another.

For any inertial frame of reference the laws of physics are the same, regardless of the velocity of that frame of reference in terms of any other frame of reference.

If zero net force acting upon a body, it is not accelerating; that is, such a body is at rest or it is moving at a constant speed in a straight line. (Newton’s First Law)

Newton's first law of motion is often stated as... An object at rest stays at rest and an object in motion stays in motion with the same speed and in the same direction unless acted upon by an unbalanced force.
Light was sent off in two directions and reflected from places the same distance from the starting point and then the travel times were compared.

There was a difference in the travel times. This difference is detecting light’s movement through the two different paths.
Which would take longer, for a man to swim from A to B and back, or to swim across the river, from A to C and back?

AB = AC, v is the speed of the current.

But if A to C he must shoot for a point to the right of C to compensate for the current. ... Simple algebra shows that

IT TAKES LONGER TO SWIM UPSTREAM AND BACK THAN TO SWIM THE SAME DISTANCE ACROSS STREAM AND BACK.

LIGHT IS A WAVE.
TO TRAVEL THROUGH SPACE THERE MUST BE A MEDIUM (ETHER).
MICHELSON AND MORLEY EXPERIMENT (1887).
1. **Universality of the speed of light:**
The speed of light in a vacuum is a universal constant-independent of the motion of the source or the observer. The speed of light is the same in **ALL** inertial reference frames.

2. **Principle of Covariance:**
All inertial frames (those moving at constant velocity with respect to one another) are equivalent for the observation and formulation of physical laws.

Physical laws must have the same mathematical form when observed in any inertial reference frame.
Path in ordinary space

Path in spacetime

Your world line

$\mathbb{R}^{3,1}$

Spacetime
LIGHT CONE OF SPECIAL RELATIVITY
Minkowski Space-time Diagram

Light Cone

Future light-cone
Past light-cone
Space-like separation
Speed of light
Event "Now"
Time
World-line

x
y

LIGHT CONE
WORLD-LINE OF EARTH IN ONE YEAR
A, B, and C all start at the same place O at the same time $t=0$.
A is at “rest”. B is moving away from A at a velocity 1 unit/5 sec.
C is moving away from A at a velocity of .6 units/sec.
C is moving away from A faster than B is.
$x' = x + 3$
$y' = y - 2$

$(0,0) \rightarrow (3,-2)$
$(-2,4) \rightarrow (1,2)$
$(3,3) \rightarrow (6,1)$

EXAMPLE OF A GEOMETRIC TRANSFORMATION
\[ x = \frac{4}{5}x' + \frac{3}{5}y' \]
\[ y = -\frac{3}{5}x' + \frac{4}{5}y' \]

or, in the example of the vector OA,
\[ 7 = \frac{4}{5} 2 + \frac{3}{5} 9 \]
\[ 6 = -\frac{3}{5} 2 + \frac{4}{5} 9 \]
The Galilean transformation is a good approximation only at relative speeds much smaller than the speed of light. For speeds close to $c$ we need new transformations.

Lorentz Transformations reflect the fact that observers moving at different velocities may measure different distances, elapsed times, and even different orderings of events, but always such that the speed of light is the same in all inertial reference frames.

The Lorentz transformations preserve the spacetime interval between any two events. This property is the defining property of a Lorentz transformation.

They are the transformations under which Maxwell's equations are invariant when transformed. Newton's equations are not invariant under a Lorentz transformation.

\[ ds^2 = -dt^2 + dx^2. \]
LORENTZ TRANSFORMATIONS

\[
x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}}
\]

\[
y' = y
\]

\[
z' = z
\]

\[
t' = \frac{t - (v/c^2)x}{\sqrt{1 - \frac{v^2}{c^2}}}
\]

\[(x, y, z, t) \leftrightarrow (x', y', z', t')\]
SPECIAL RELATIVITY – ‘SIMULTANEITY’

\[ t_a = t_b \]

\[ \text{E}_a \quad \text{E}_b \]
Reference Frame $x',t'$ moves at $.8c$.

$x,t$ is “at rest”

A 5 unit length bar in $x,t$ contracts to a 3 unit length bar in $x',t'$ Frame.
\[ \sqrt{1 - \frac{v^2}{c^2}} \]

Let \( v = 0.6c \)

\( v^2 = 0.36c^2 \)

\[ \frac{v^2}{c^2} = 0.36 \]

\[ 1 - \frac{v^2}{c^2} = 0.64 \]

\[ \sqrt{1 - \frac{v^2}{c^2}} = 0.8 \]

\[ L = L_0 \sqrt{1 - \frac{v^2}{c^2}} \]

\[ L = 0.8L_0 \]

\[ T = \frac{T_0}{\sqrt{1 - \frac{v^2}{c^2}}} \]

\[ T = 1.25T_0 \]

\[ m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \]

\[ m = 1.25m_0 \]
\[ L = L_0 \sqrt{1 - \frac{v^2}{c^2}} \]

\[ T = \frac{T_0}{\sqrt{1 - \frac{v^2}{c^2}}} \]

\[ m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \]

<table>
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<tr>
<th>( v )</th>
<th>( \gamma = \frac{1}{\sqrt{1 - v^2/c^2}} )</th>
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<td>0.01 c</td>
<td>1.00005</td>
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<td>1.005</td>
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<tr>
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<td>7.07</td>
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<tr>
<td>1.00c</td>
<td>( \infty )</td>
</tr>
<tr>
<td>Larger than c</td>
<td>Imaginary number</td>
</tr>
</tbody>
</table>
Lorentzcontractie

\[ L = L_0 \sqrt{1 - \frac{v^2}{c^2}} \]

Voorwerpen vervormen afhankelijk van
de snelheid van de waarnemer

Hendrik Lorentz (1853 - 1928)
Albert Einstein (1879 - 1955)
SPECIAL RELATIVITY

TIME DILATION

MASS INCREASE
SPECIAL RELATIVITY

SPACETIME DISTANCE BETWEEN TWO EVENTS

$$ds^2 = -dt^2 + dx^2.$$
TWIN PARADOX

Twin 1 who stays at home is 10 years older.
Twin 2 ages only 6 years; he comes back four years younger.
METRIC FOR SPECIAL RELATIVITY
METRIC FOR ‘FLAT MINKOWSKI SPACETIME’

\[(ds)^2 = -c^2 (dt)^2 + (dx)^2 + (dy)^2 + (dz)^2\]

\[\eta_{\mu\nu} = \begin{bmatrix}
-c^2 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}\]
EINSTEIN’S ENERGY-MOMENTUM EQUATION

\[ E^2 = (pc)^2 + (mc^2)^2 \]

CLASSICAL MECHANICS

\[ E_k = \frac{1}{2}mv^2 \rightarrow \text{kinetic energy} \]
\[ p = mv \rightarrow \text{linear momentum} \]

RELATIVISTIC MECHANICS

\[ E_k = (\gamma - 1)mc^2 \rightarrow \text{kinetic energy} \]
\[ p = \gamma mv \rightarrow \text{linear momentum} \]
where \( \gamma = \frac{1}{\sqrt{1-(v/c)^2}} \rightarrow \text{Lorentz factor} \)

\[ E = mc^2 \]
\[ ds^2 = dx^2 + dy^2 \]

**PYTHAGOREAN THEOREM**

**EUCLIDEAN GEOMETRY**

\[ ds^2 = dt^2 - dx^2 \]

**MINKOWSKI METRIC**

**GEOMETRY OF MINKOWSKI SPACETIME**
EXPERIMENTAL SUPPORT FOR SPECIAL RELATIVITY

TIME DILATION - THE MUON EXPERIMENT:
Muons are created in cosmic ray showers at a typical height of 10 – 60 km.
Their mean lifetime is about 2 microseconds and travel at about 0.99c.
Measurements are made at the top of a mountain and at sea level.
Without time dilation it would take about 100 half-lives to reach the ground.
None should reach the ground.
But a significant number of muons do reach the ground.
For muons travelling at 0.99c, the time dilation factor is about 7. (see previous chart)
Their half-life observed in our ground frame of reference is longer by a factor of 7.
Hafele and Keating Experiment
During October, 1971, four cesium atomic beam clocks were flown on commercial jet flights around the world twice, once eastward and once westward, to test Einstein's theory of relativity with macroscopic clocks. These results provide an unambiguous empirical resolution of the famous clock "paradox" with macroscopic clocks.

\[ E = mc^2 \]

The inter-convertibility of energy and mass has, of course, been observed.
EINSTEIN’S SPECIAL THEORY OF RELATIVITY

Summary

• All inertial reference frames obey the same laws of physics.

• The speed of light in empty space, $c$, does not change when the reference frame changes.

• No object can move faster than the speed of light

• At everyday velocities, the consequences of special relativity are too small to be noticed or measured.

• At relativistic velocities, an observer will see that clocks, lengths in the direction of travel, momentum and energy will be different than those at rest with respect to the observer.

• Even at rest, objects with mass have energy

$$E = mc^2$$