Logical Operators

p		A proposition taking on exactly one value from {True, False}
~p	Not p	If p is True, then ~p is False. If p is False, then ~p is True.
$p \vee q$	p OR q	False if both p,q are False, otherwise True.
$p \wedge q$	p AND q	True if both p,q are True, otherwise False.
$p \rightarrow q$	IF p THEN q	False if p is True and q false, otherwise True.

Order of Operations

1. Parentheses
2. Negation (NOT)
3. Conjunction (AND)
4. Disjunction (OR)
5. Implication (IF/THEN)

Evaluate from left to right.

Replacement Rules

$p \vee p$	p	Tautology
$p \wedge p$	p	Tautology
False$\wedge p$	False	Constant Evaluation
True$\wedge p$	p	Constant Evaluation
False$\vee p$	p	Constant Evaluation
True$\vee p$	True	Constant Evaluation
$p \wedge \sim p$	False	Non-Contradiction
$p \vee \sim p$	True	Excluded Middle
$p \rightarrow p$	True	Identity
False$\rightarrow p$	True	False implies anything

Replacement Rules

p∨q	q∨p	Commutativity
p∧q	q∧p	Commutativity
(p∨q)∨r	p∨(q∨r)	Associativity
(p∧q)∧r	p∧(q∧r)	Associativity
p(q∨r)	pq∨pr	Distribution
p∨(qr)	(p∨q)(p∨r)	Distribution
~~p	p	Double Negation
p→q	~p∨q	Material Implication
~(p∨q)	~p~q	De Morgan’s
~(pq)	~p∨~q	De Morgan’s

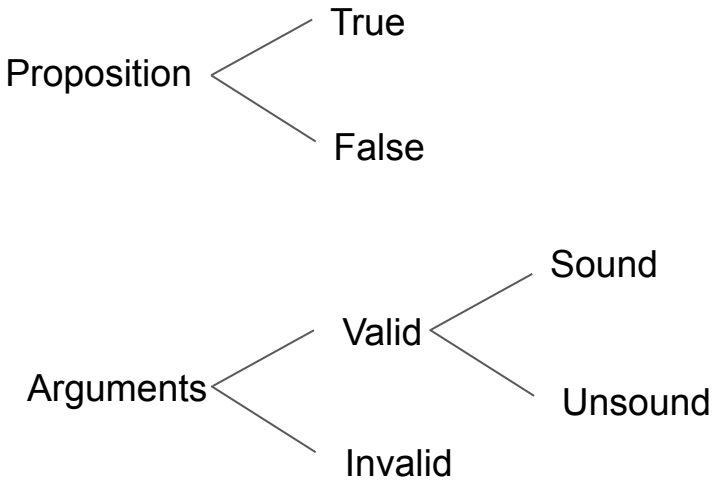
Syllogisms

$p \rightarrow q$ p $\therefore q$	Modus Ponens
$p \rightarrow q$ $\sim q$ $\therefore \sim p$	Modus Tollens
$p \vee q$ $\sim p$ $\therefore q$	Disjunctive Syllogism
$p \rightarrow q$ $q \rightarrow r$ $\therefore p \rightarrow r$	Hypothetical Syllogism

Syllogisms

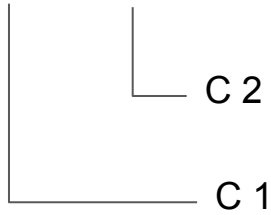
$(p \rightarrow q)(r \rightarrow s)$ $p \vee r$ $\therefore q \vee s$	Constructive Dilemma
$p \rightarrow q$ $\therefore p \rightarrow pq$	Absorption
pq $\therefore p$	Simplification
p q $\therefore pq$	Conjunction
p $\therefore p \vee q$	Addition

Terminology



Major Premise : defines a relationship between two distinct argument components.

If P then Q



Relationship:

If...then
(Implication)

Minor Premise: provides information about one of the components from the major premise.

P [is true]

Contradictories

Proposition	Contradictory
$\forall (x)\phi x$	$\exists (x)\sim\phi x$
$\exists (x)\phi x$	$\forall (x)\sim\phi x$

Quantifier Rules of Inference

$\forall (x)\phi x$ $\therefore \phi y$	Universal Instantiation
ϕy $\therefore \forall (x)\phi x$	Universal Generalization
$\exists (x)\phi x$ $\therefore \phi y$	Existential Instantiation
ϕy $\therefore \exists (x)\phi x$	Existential Generalization