The determination of the neutrino flux in the NuMI beamline poses a challenge for short and long baseline neutrino experiments. Despite much work in recent years to more accurately determine neutrino flux, current simulations do not agree with data. The geometry and physics of the NuMI beamline are simulated by a Geant4 based Monte Carlo simulation called g4numi. The NuMI beam is formed by separating a high energy proton beam from the accelerator at Fermilab and colliding it with a target to produce short-lived particles such as pions and kaons. These particles are focused by two magnetic horns in series and then allowed to decay into neutrinos through a long decay pipe. The remaining particles are removed by absorbers and layers of rock. My work will focus on modifying the simulation, specifically the geometry of the target and magnetic horns through which the protons collide and its products are focused, in order to have the simulation better agree with data. By making these changes to the simulation I hope to lessen the discrepancy in flux measurements between simulation and data. This work is extremely vital to experiments such as MINERνA which requires an accurate measure of neutrino flux in order to determine neutrino-nucleus cross sections within the detector. An accurate determination of neutrino flux is also crucial for neutrino oscillation experiments such as MINOS, NOνA and in the future DUNE.
I. A BRIEF BACKGROUND ON NEUTRINOS

Particle physics is the study of the fundamental constituents of matter and their interactions. Yet, what is considered fundamental has changed as our knowledge has expanded. The modern theory of particles physics is referred to as the Standard Model. Developed in the 1970s, the Standard Model is an elegantly simple, but well-tested, theory that has successfully accounted for most experimental results and has predicted a variety of phenomena. The Standard model explains the properties and interactions via a small number of particles of three distinct types: two spin-$\frac{1}{2}$ families of fermions called quarks and leptons, and one family of spin-1 bosons called gauge bosons. Additionally, there is a spin-0 particle, the Higgs boson, which is the origin of mass within the theory. Within this model there are four forces mediated by their corresponding gauge bosons. The strong force is carried by the gluon, the weak force by the $W^\pm$ and $Z^0$, the electromagnetic by the photon ($\gamma$) and gravity. Moreover, every particle has a corresponding antiparticle with all internal quantum numbers inverted.

Relevant to this research are the leptons. Leptons, of which six are known, interact via the weak force and occur in three generations:

\[
\begin{pmatrix}
e^- \\
\nu_e \\
\mu^- \\
\nu_\mu \\
\tau^- \\
\nu_\tau
\end{pmatrix}.
\]

(1)

Each of the charged leptons (electron, muon and tauon respectively) have a corresponding neutrino without any charge and each have corresponding antiparticles with opposite charge. Depending on the mediating boson, there are two possible interactions neutrinos can have with matter. If the boson exchanged is the $W^\pm$, the interaction is called charged current and the neutrino is converted to the charged lepton of its same generation and matter. If the boson exchanged is the $Z^0$, the interaction is called neutral current and the neutrino is not converted at all \[1\].

The Standard Model originally considered neutrinos massless. However, in the 1990s experiments began to convincingly show that neutrinos change flavor (oscillate) as they travel. The phenomena of neutrino oscillations require that neutrinos have mass. This requires physics outside of the Standard Model.

II. NEUTRINO FLUX CALCULATIONS

The idea of neutrino oscillations was first proposed by Bruno Pontecorvo in 1957. One of the phenomena that gives rise to neutrino oscillations is called neutrino mixing and occurs when neutrinos have nonzero masses. Neutrino mixing relies on the assumption that neutrinos do not have definite masses, instead they are a linear combination of three other, definite states $\nu_1$, $\nu_2$ and $\nu_3$. Instead of considering all three flavor states, it is simpler and a nice approximation to consider the mixing between just two flavors (denoted $\nu_\alpha$ and $\nu_\beta$). In order to conserve the orthonormality of states, we write

\[
\nu_\alpha = \nu_i \cos \theta_{ij} + \nu_j \sin \theta_{ij}
\]

(2)

\[
\nu_\beta = -\nu_i \sin \theta_{ij} + \nu_j \cos \theta_{ij}
\]

(3)

where $\nu_i$ and $\nu_j$ are two mass eigenstates. Also $\theta_{ij}$ is the mixing angle that is of great interest to neutrino oscillation experiments to determine. If we consider when there is mixing, such that $\theta_{ij}$ is nonzero, interesting physics follows. Take a $\nu_\alpha$ neutrino produced perhaps by an accelerator with momentum $\vec{p}$ at time $t=0$. In this case, the $\nu_i$ and $\nu_j$ components will have slightly different energies
E_i and E_j due to their different masses. Quantum mechanics dictates that their associated wave functions will have different wave functions and therefore slightly different frequencies giving rise to a phenomena similar to the beats heard when sound waves of different frequencies are superimposed. As a result, the original \( \nu_\alpha \) develops a \( \nu_\beta \) component whose intensity oscillates as it travels through space.

Considering the oscillation between two neutrinos is a good one because the nature of conventional neutrino beams forming \( \nu_\mu \) for long baseline experiments. Omitting the math, the two-neutrino oscillation probability is found to be

\[
P_{\nu_\alpha \to \nu_\beta} = \sin^2(2\theta) \sin^2\left(\frac{\Delta m^2_{ij} L}{4E}\right).
\]

The distance traveled by the neutrino is \( L \), the energy is \( E \) and the squared mass difference \( (m^2_i - m^2_j) \) is \( \Delta m^2_{ij} \). Naturally, the formalism becomes more complicated when a third neutrino is added into the mix. As can be inferred from equation 2 and 3, the parameters for three-neutrino oscillations are three masses \( (m_1, m_2, m_3) \), three mixing angles \( (\theta_{12}, \theta_{23}, \theta_{13}) \) and an odd seventh parameter, a charge-parity violating phase \( (\delta) \).

Long-baseline oscillation experiments, such as MINOS and NO\(\nu\)A, use a conventional neutrino beam (which is described in the next section) with a near and a far detector. The location of the far detector and energy spectrum of the beam are chosen to match the oscillation maxima of equation 3. Having the two detectors share the same beam systematically reduces the uncertainty due to neutrino flux, cross section model and improves detector performance. Figure 1 shows the the basic concept between a two detector design.

If the cross-section is well understood the oscillation probability may be calculated as

\[
P(E_i) = \left(\frac{N_2}{N_1}\right) \left(\frac{A_1}{A_2}\right) \left(\frac{1}{\phi_2}\right).
\]

Here \( A_1 \) and \( A_2 \) represent the efficiency of each detector, \( N_1 \) and \( N_2 \) are the number of events at a particular energy and \( \phi_1 \) and \( \phi_2 \) are the neutrino fluxes at each detector. This probability is greatly simplified. In reality, the flux uncertainty only partially cancels because the near detector observes a distributed neutrino source and the far detector observes a point source. The cross sections also do not cancel if the detectors are made of different materials.[2]

Additionally, neutrino-nucleus cross sections are not well known and complicate experiments running in the 0.1-20 GeV range. A neutrino cross section is calculated using the following

\[
\sigma(E) = \frac{N(E)}{\phi(E) \cdot T}
\]

where the number of interactions is divided by the flux multiplied the target nuclei. Thus any uncertainty in flux enters directly into the cross section measurement. Because MINER\(\nu\)A is dedicated to study how muon type neutrinos interact in the 1-20 GeV range with various nuclei (carbon, lead, iron and others), the determination of flux is especially relevant. These measurements and others planned for MINER\(\nu\)A are also important for future neutrino oscillation experiments planned with beams of energies 1-20 GeV. In this region at the transition between elastic and inelastic scattering, neutrino cross-sections are difficult to predict theoretically and are poorly measured. Results from the MINER\(\nu\)A experiment will significantly reduce errors from unknown neutrino cross-sections in the MINOS and NO\(\nu\)A experiments.[3] In the future, DUNE will involve a two-detector system and a liquid argon detector. Therefore, it will be vitally important to understand neutrino-nucleus interactions. Large errors in cross section measurements lead to systematic uncertainties in oscillation measurements.[4]
FIG. 1: These plots, provided by the MINERνA Collaboration, show data from low energy runs at Fermilab of the reconstructed data vs the simulation. The left is a histogram plot and the right is a ratio plot showing the data dived by the simulation.

Figure 1, from the MINERνA Collaboration and provided by Dr. Kordosky, shows the relationship between neutrino flux from data and the corresponding simulation. It is clear that there is a stark disagreement in certain energy ranges. The amount and variation to which the data and simulation disagree is a good indicator that that this is indeed a flux issue. An error in cross section measurements would be a constant shift and would not be so varied as we see here. Moreover, similar data from the NOνA Collaboration show similar trends which leads us to believe that this disagreement is in fact an issue related to flux measurements. And, as stated before, it is vitally important that we be able to accurately and precisely predict neutrino flux. And to the best of our ability, we want neutrino flux from the simulation to agree with data. These plots graphically show the energy ranges where we will seek to improve the simulation.

III. THE NUMI BEAMLNE

This section is not intended to be an all inclusive detailed explanation of the NuMI beamline. Instead, I intend to give a short, but detailed, explanation on how the NuMI beamline takes protons from the main injector and creates a focused neutrino beam.

There are four types of experiments studying neutrino oscillations: solar, atmospheric, reactor and accelerator. Among the four neutrino sources, accelerator neutrino beams provide the greatest control over the source. The concept behind accelerator neutrino beams are actually quite simple. A proton beam strikes a thick nuclear target, producing secondaries, such as pions and kaons. Those secondaries leave the target, boosted in the forward direction but with some divergence (transverse momentum). The mesons (a quark and an antiquark) are permitted to drift in free space and decay into neutrino tertiaries.[5] This basic type of beam most commonly sees pions as secondaries, and the neutrino tertiaries are predominately muon type neutrinos. Pions most often decay into a muon and a muon type neutrino. Although some pions will decay into an electron, an electron type neutrino and a muon type neutrino.

The NuMI beam was built for the MINOS experiment but has since been used for MINERνA, NOνA, LArIAT and other neutrino experiments at Fermilab. A schematic view of the NuMI beam can be seen in the appendix under figure 8. NuMI is an acronym for neutrinos at the Main Injector and uses a 120 GeV proton beam from the Fermilab Main Injector (an accelerator worthy of its
The proton beam is incident on a narrow graphite target that must be capable of withstanding the 400kW output from the proton beam while maximizing the amount of secondary hadrons produced. The target consists of 47 fins, each of which is 20 mm long (along the beam direction), 15 mm tall, and 6.4 mm wide and is just under 1 meter in length. The proton beam strikes the target within a baffle carrier system which can move together with the target while minimally affecting the incoming protons.

The secondary mesons produced from interactions within the target are then focused by two magnetic horns in series. The horns increase secondary particle flux in the desired energy range and provide flexibility in choosing that energy. Moreover, the target to horn difference and horn to horn distance is able to changed with minimal effort. The NuMI horn’s inner conductors have a parabolic radial profile and can be thought of as linear lenses. The horns are pulsed with a half-sine wave having a duration of 2.3 ms to produce toroidal magnetic focusing fields of up to 3 T. Moreover, the angle that the mesons are produced is inversely proportional to their longitudinal momentum, but directly proportional to their transverse momentum. The different trajectories of the mesons and the varying degrees to which they are focused cause distinct portions of the neutrino spectrum. The trajectories can be divided into five categories. The first is unfocused mesons which travel down the neck of both horns and have a high momentum (and correspondingly low transverse momentum). The next two categories are horn 1 only and horn 2 only. Horn 2 only mesons have a larger momentum than their counterparts; whereas, horn 1 only particles have a larger transverse momentum and tend to decay into neutrinos only. The next possibility is underfocused mesons these have an intermediate energy slightly less than that of unfocused mesons. The final trajectory possible is overfocused mesons that are over-corrected by the Horn 1 and that are also corrected by the Horn 2.[6] Figure 9, in the appendix, diagrams this effect nicely and shows how each trajectory contributes to the energy spectrum of the neutrinos that is observed.

After the focusing process, the mesons travel down a 675 meter long decay pipe where, as the name suggests, the mesons are permitted to decay into neutrinos as described by the processes mentioned earlier. The beam is now focused but still a combination of hadrons, some leptons and neutrinos. The beam continues through 240 meters of rock which stops all the particles but the neutrinos. On the other side of the layers of rock there is a cavern which houses the MINOS near and MINERνA detectors which observe this flux of neutrinos.

IV. G4NUMI SIMULATION

I will now follow with, again, a short description of the g4numi simulation: the program that simulates the NuMI beam and creates outputs of the neutrino flux. In the next section, I will go into more detail on what parts of the NuMI beamline were of interest to my research, what I changed in g4numi and, finally, how the flux was affected.

The physics and geometry of the NuMI beamline are modeled in an entirely GEANT4 based Monte Carlo simulation called g4numi. GEANT4 is a C++ based toolkit for the simulation of particles traveling through matter and is especially flexible allowing many user customizations. A Monte Carlo simulation is a computer algorithm that utilizes repeated random sampling to obtain numerical results. The primary output of the g4numi code are files representing the decays of secondaries that give rise to neutrinos. The data within these files were analyzed using the data analysis framework ROOT which could create understandable outputs.

Needless to say, the implementation of the NuMI beamline, with its various components, necessarily implies the need for modeling. This means the translation of a set of engineering blue prints to a given set of volumes within GEANT4. This requires some level of simplification and approximation.
Moreover, there is an evident need for very high levels of accuracy for the simulation of neutrino flux as mentioned in the previous section. As the figure 1 shows there is disagreement between the data and simulations. And yet, despite the general success of g4numi simulation, as it greatly improved from its predecessors, there is a need for further refinement.

Because of the nature of my research, the next section can get rather computer technical due to the nature of the g4numi simulation. So the most efficient plan of action is to describe what geometric aspects of the NuMI beamline were of particular interest and then discuss what changes were made to the g4numi simulation in order to resemble these geometries. From there I will discuss (and show) how this change in the simulation affected the neutrino flux. My intent will be to not get into great depth on the configuration of the g4numi simulation, but rather to focus on the impact certain geometrical changes have on flux and their broader implications. The main area of interest for this research was investigating geometric changes to the magnetic horns and the proton target. The magnetic horns and target can be considered the most crucial components to the g4numi simulation. Interestingly enough, much of this type of research and analysis was done for the low energy runs of the NuMI beam. However, this analysis has not yet been done for the medium energy target and thus there even more of a need for this research.

V. RESULTS

![Graph showing neutrino flux with different settings for maxDev.](image)

FIG. 2: This shows the neutrino flux when the parameter maxDev is changed from 500 µm to 5µm.

The first geometric aspect of the simulation that was investigated was the modeling of the magnetic horns. From previous investigations into the magnetic horns it was found that a significant discrepancy at approximately 4 GeV had been observed. At a transverse momentum of 0.25 GeV, approximately 10 GeV neutrino’s tracks are grazing the neck of the horn, resulting in some focusing, or none at all, depending critically on the position of the track with respect to the inner conductor. And recall from figure 1, there is a significant discrepancy in the 10 GeV range between the data and the simulation. In the previous, default version of the g4numi simulation the inner conductor was modeled by a G4Polycone whose segmentation is computed by requiring a fixed precision on the radius and thickness of 500 µm. This is represented by the parameter, maxDev, within the
FIG. 3: This shows the neutrino flux when the momentum of the proton beam is offset in the x-axis by 60 micro-radians.

FIG. 4: This shows the neutrino flux when the spot size of the proton beam is shifted from $1.1 \times 1.1 \text{ mm}^2$ to $1.4 \times 1.4 \text{ mm}^2$.

g4numi code. The newer, alternate version of the simulation, uses a similar algorithm to determine the segmentation and, thereby, the radii for the set of G4Cone instances. Essentially, the alternate version of the g4numi simulation improves upon various aspect of the magnetic horn geometry in various ways, one of which is allowing more segmentations when defining the inner conductor of the magnetic horns.[7]

When increasing the number of G4Cone instances by decreasing maxDev within the alternate version of the g4numi code, at a certain level of segmentation when modeling the inner conductor the neutrino flux should converge. So I proceeded by decreasing this parameter from its default value of 500 $\mu$m to a value of 5 $\mu$m. What we found was that maxDev does not control the segmentation of the inner conductor within the alternate version. In fact, maxDev only controls the segmentation
modeling the layer of water within the inner conductor. This is the water that cools the magnetic horns as they are powered and consequently heated. The layer of water lays along the inner radial profile of the inner conductor. Disappointingly, both Dr. Kordosky and myself misinterpreted previous research. In the nominal version of g4numi maxDev controlled the segmentation of the inner conductor. And in the alternate (newer version) maxDev is still a parameter, but it no longer controls the segmentation of the inner conductor. However, this change to maxDev in the alternate version did have some impact on the neutrino flux, which can be seen in figure 2. In this figure the baseline simulation is shown in black and the modified simulation is shown in red. The plot on the left is a histogram of flux at various energies; the plot on the right is ratio showing the modified version divided by the baseline version. The main profile of flux is in the 1-12 GeV range, and higher energies see significantly less neutrinos and lead to low statistics (the rapid up and down motion of the plot). This description of the figures is consistent for all plots in this results section. Error bars have not yet been calculated for these graphs. The improved segmentation of the water layer did lead to a constant 1-2% increase in neutrino flux in the 1-11 GeV range. The conclusion we can draw from this is that further study is needed on the modeling of the magnetic horns within the alternate version of the g4numi simulation. And perhaps it is not a single parameter that controls the modeling of the inner conductor of the magnetic horns.

The other prominent aspect of the NuMI beamline that is of interest is the proton beam and the target that it strikes. The proton beam is 120 GeV. And there were two central design principles for the NuMI proton beam line: safe and low-loss transmission of a very high-power proton beam and accuracy and stability of targeting. Fractional losses over the 350 m beam line were required to be kept below $10^{-5}$. The physics of the MINOS experiment required the beam to have an angular stability of $\pm 60$ micro-radians, and a positional stability of $\pm 250 \mu m$ at the target.[6] Within the g4numi simulation, these were all values that were kept as constant. So in order to measure the scale to which these values can impact neutrino flux measurements, I altered the simulation the with these values set to their maximum known uncertainties.

The proton beam which strikes the target is simulated as traveling entirely in the z-direction (and in fact defines it) with a momentum of 120 GeV and strikes the target. In reality, the beam is not entirely in the z-direction, and the angular stability of $\pm 60 \mu rad$ represents how the beam jitters, so to speak, and has a momentum not entirely in the z-direction. To test the maximum effect this may have, I offset the proton beam beam momentum by $60 \mu rad$ in the x-direction. The results can been seen in figure 3. This has less than a 1% effect on the neutrino flux, and can thus be ruled out as a viable source of error.

Furthermore, the proton beam extracted from the Main Injector has a Gaussian profile symmetric in the x and y directions. The g4numi code maintain the proton beam profile (I will call this spot size for the remainder of the paper) of 1.1 by 1.1 mm$^2$ which is comparable with target size, causing some of the beam to miss the target altogether. The spot size at the NuMI target is not exactly measured, but needs to be extrapolated from the profile monitor which is 6 m upstream of the target.[8] However, we know from various medium energy runs, that the spot size was not constant and could often reach values of 1.4 mm in the x and/or y directions. Again to simulate the maximum effect this may I have I changed the beam incident on the target to have a spot size of 1.4 by 1.4 mm$^2$. The result can be seen in figure 4. Although this is an overestimate, it is clear that there is a significant difference in the 8-12 GeV range where there is an increase in flux as compared to the baseline. This represents an increase in neutrinos that are unfocused and focused very little. These higher energy neutrinos travel down then neck of the horn and are evidently detected in larger quantities when the spot size is increased. What can we conclude from this? Well this is not an effect that can be ignored like the proton momentum above. The spot size is something we know can and has changed from run to run at Fermilab. Moreover, its affect, as shown, must be considered
when doing detailed flux calculations. There are two such approaches that may be considered when dealing with the affect. One: the simulations can be altered to have the spot size set at its average value throughout all the medium energy runs and account for the uncertainty that the spot size causes separately after the fact. Two: The g4numi code could be altered to the point that when running a series of simulations the spot size is allowed to vary at the same rate we know it does within the experiment. Although this is a perhaps a more involved approach, all the uncertainties that the spot size causes on neutrino flux would be accounted for within the simulation.

FIG. 5: This shows the neutrino flux when the position of the proton beam is offset by 0.25mm in the x-direction.

FIG. 6: This shows the neutrino flux when the position of the proton beam is offset by +0.25mm in the y-direction.

The last set of conditions I considered was the position where the proton beam strikes the target. As the figure 10 shows in the appendix, the proton beam does not strike the medium energy target
FIG. 7: This shows the neutrino flux when the position of the proton beam is offset by -0.25mm in the y-direction.

in its center. G4numi, by default, simulates the proton beam as being symmetric about the x-axis and hitting the upper portion of the target. However, the known uncertainty in this beam position is 250 mm. How the proton beam strikes the target can have very drastic affects on the amount of neutrinos produced and their energies. To account for this effect, in continuing with the same method as the last two cases, I altered the beam position separately in the x and y directions by 0.25 mm. Because the beam is symmetric about the x-axis I need only simulate it either as either offset positive or negative x-direction. And because the beam is not symmetric in the y-direction, I simulated the beam as offset by 0.25 mm in the positive and negative y directions. The results can be seen in figures 5, 6 and 7.

The results for moving the position of the beam are interesting but not necessarily very revealing. Certainly when we offset the position of the beam by the maximum possible amount, 250 microns, we see significant effects. When the beam is moved in the x-direction, the modified simulation produces a histogram that seemingly improves the neutrino flux in a way that matches the trends of figure 1. The affects of the histograms for offsets in the y-direction do not point to any specific trends. Clearly higher statistics are needed in this area of research. But it is noteworthy that the simulation does not account for any instability for the proton beam even though there is some known instability within the description of the proton beam. Further research into this area should focus on gaining higher statistics and finding to what extant the beam actually shifts in either the x or y directions over the course of runs.

VI. THE FUTURE

Certainly the results (or lack thereof) from modeling the inner conductor of the magnetic horn are leaving much to be desired. Due to a misinterpretation of previous research, the maxDev parameter does not make the changes to the geometry of the magnetic horns that I had sought to make. So certainly further investigation is needed into remodeling the inner conductor of the magnetic horns within the alternate version of the g4numi simulation. So there is still much uncertainty as to the effect altering the geometry of the magnetic horns may have.
The research I have done with respect to the proton beam and the target are my most significant research. From the alterations to the simulation I made, I was able to rule out the angular instability of the proton beam as a viable source of error. It is always a positive when you can rule out an aspect of the simulation as a significant source of error. Perhaps the most significant effect I found from altering the simulation was the effect of changing the spot size. The change from $1.1 \times 1.1 \text{ mm}^2$ to $1.4 \times 1.4 \text{ mm}^2$ seems to have a significant effect in the 10 GeV range. Although this discovery does not help to reckon the flux disagreement between simulation and data, this very real effect is something that must be accounted for when considering flux measurements. The effects beam position has can also be significant when at the maximum quantity of 0.25 mm. However, in order to better understand this change we would want to know to what extent the beam position changes over the course of a series of runs. Certainly the beam is not offset by 0.25 mm in any one direction. Further investigations into studying beam position should want, first of all, higher statistics within the simulation and, secondly, to knowledge as to what degree the beam position actually changes.

When Dr. Kordosky first proposed this research topic (albeit after a change in project four weeks into the REU), he first wished to investigate the maxDev parameter, but had other inclinations as to what may be causing the all too evident flux discrepancy. Perhaps optimistically, he, as did I, hoped that there may be a single parameter which causes the vast amount of the neutrino flux error. That sort of outcome would have been much to the benefit of all those associated with neutrino experiments MINERνA, MINOS, NOνA and future experiments such as DUNE. Yet, that was not the case. There was no silver bullet; there was no simple solution. And I think we both realistically expected as much. The reality is, that although the g4numi simulation is effective, it is not perfect. There are a variety of reasons for why the flux from the simulation and data do not agree. In the future we hope to both recognize and account for these uncertainties and accordingly alter g4numi. The research I have conducted this summer builds off of previous research, and is a good start into investigating various effects of changing the g4numi simulation. But plenty of work is still required and yet to come in the future.

VII. APPENDIX

This appendix contains various useful images and diagrams that I could not fit into my paper above. The same titles and numbering scheme used above is referenced in this section below.

![FIG. 8: This a general schematic of the NuMI beamline.](image)

The next set of figures are produced within the simulation and show where all protons are incident within the area where the target is. These plots are not referenced within my paper, but are helpful illustrations. The plots are a tomography of the target, so to speak, and help to illustrate how
FIG. 9: This shows the different trajectories and energies of mesons after they are produced from protons interacting with the target.

the spot size changes and how the beam position changes within the context of the target. I will include four figures (the spot size change and the three beam position changes), and in each figure the baseline will be shown on the left and the altered version on the right to show the changes made.
FIG. 10: This shows a cross section of the low and medium energy proton targets. The blue dot represents where the proton beam strikes the target.

FIG. 11: This shows where the protons are incident for the baseline simulation (left) and when the spot size is broadened (right).

FIG. 12: This shows where the protons are incident for the baseline simulation (left) and when the proton beam is shifted 0.25 mm in the x-direction (right).
FIG. 13: This shows where the protons are incident for the baseline simulation (left) and when the proton beam is shifted +0.25 mm in the y-direction (right).

FIG. 14: This shows where the protons are incident for the baseline simulation (left) and when the proton beam is shifted -0.25 mm in the y-direction (right).
ACKNOWLEDGMENTS

I wish to thank my advisor Dr. Kordosky. This work was supported by NSF REU grant PHY-1359364.

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