A Toy Model for Dark Photon Compton Scattering

A thesis submitted in partial fulfillment of the requirement
for the degree of Bachelor of Science in Physics from
The College of William and Mary

by

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May 5 2020

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Contents

1 Public Abstract

mmmIn this project we propose a toy model to understand the fundamental physics suggested by scattering employing the dark photon ansatz. Specifically, we investigate the scattering of a dark sector particle off an electron, which is analogous to standard model Compton Scattering. This model encapsulates some of the basic kinematic and dynamic interactions between the dark sector particle and the electron. We use both analytical and computational tools to try and understand the direction and magnitude of relevant parameters and observables in the scattering process, such as the value of the dark photon mass and the differential cross section $\frac{\partial \sigma}{\partial \Omega}$. Finally, we conduct a literature review of several recently proposed investigations into dark sector scattering processes. Several of the major unanswered questions in astrophysics and particle physics are at their core focused on understanding what composes dark matter and how it interacts with the particles that compose the standard model. Understanding at a basic level how dark sector particles interact with the standard model via the electromagnetic force guides both theoretical and experimental calculations in various sectors of physics. A better understanding of dark matter's interaction with the standard model is necessary in solving problems relating to the spiraling rate of galaxies, or why dark matter seems to interact weakly through the four fundamental forces. The broader impact of this project include guidance for experimentalists searching for dark matter in particle collisions, as well as better accuracy in identifying dark sector Compton scattering events.

2 Introduction

The theory of dark matter was first proposed in 1933 by Swiss astronomer Fritz Zwicky, as a way to explain the unexpectedly high rotation rate of galaxies[1]. In order for large scale galaxies to rotate at the rate they do, instead of either rotating more slowly or flying apart altogether, they must contain a large component of unobservable matter. This hypothesized unobservable matter is referred to as dark matter, as it cannot be observed with any of the present array of telescopes. Additional evidence pointing to the existence of dark matter includes increased rates of gravitational lensing and the location of mass during galactic collisions, among other factors. In the standard Lambda-CDM model of cosmology, the energy of the universe is only approximately only five percent observable matter and energy, whereas the universe is hypothesized to contain 20-27 percent dark matter[2]; with the remainder consisting of dark energy. An alternative approach to dark matter includes particles that exhibit a form of dark-electromagnetism. One of the proposed 'dark' particles is the dark photon- a hidden sector particle that is similar to the photon but is connected with dark matter. Like its regular photon counterpart, the dark photon is hypothesized to interact with the electromagnetic force, and is a spin-1 boson. These particles are posited to weakly interact with other charged particles via the electromagnetic and gravitational forces[3,4]. However, unlike the regular photon, the dark photon is posited to have mass in addition to momentum. [5] As dark matter has not been directly observed, it must interact weakly with ordinary matter and radiation, except through the gravitational force. Thus, dark matter is hypothesized to exist in the form of Weakly Interacting Massive Particles, or WIMPS, which are classified as cold, warm or hot based on the ratio of their momentum energy to their mass energy. However, as these particles do not strongly interact with ordinary baryonic matter, calculating any observable for these WIMPS has proven difficult[6].

3 Motivation

Understanding the theoretical kinematic and dynamic interactions between dark particles and baryonic matter is necessary to guide experimentation and place bounds on the mass of individual WIMPs. Some examples of experiments searching for dark matter include visible decay searches, E774 beam dump experiments at fermilab, and pion decay searches at CERN[7,8,9]. The aims of these experiments are to use different analytical techniques across a variety of fields to prove the existence of dark matter, with specific focus on the dark photon. Modeling dark photon interactions employing Compton Scattering, or the elastic scattering of a dark photon off of an electron, we can attempt to place bounds on both the mass, momentum, and allowed wavelengths of the dark photon.

4 Background

To begin we will look at the case of ordinary Compton Scattering using the momentum four-vectors of our constituent particles. In this configuration, we have an incoming photon that strikes a stationary electron, and scatters off at an angle θ. This is shown below in figure one. For purely the kinematics of this interaction, we do not need the Lagrangian interaction between these two particles. However, as it will be useful later, we will note the Lagrangian interaction between a fermion and a vector boson. The electron is a fermion with spin 1/2. A vector boson is a particle that has a associated transformation in its field when acted upon by a transformation matrix. This relation can be expressed as: $A(x) \to \Lambda A(\Lambda x)$. These particles have a spin value of one.
The Lagrangian interaction of a photon is given by the massless Proca Equation, which is $\partial_\mu(\partial^\mu B^\nu - \partial^\nu B^\mu)$. The Lagrangian interaction of an electron is given by . Thus the Lagrangian of the interaction of these two particles can be expressed as . Using the kinematics of the situation, shown below in figure one, we can solve for the differential cross section of the scattered photon[fig1].

In classical scattering theory this is governed by Thompson scattering. The differential cross section of Thompson scattering is given by: $\frac{\partial \sigma_T}{\partial \Omega} = (1/2 * r_0^2)/(1 + cos^2(\theta))$, where $\frac{\partial \sigma_T}{}$ is the differential Thompson cross section and r_0 is the classical radius of the electron, defined as $e^2/m_e c^2$. The cross section is

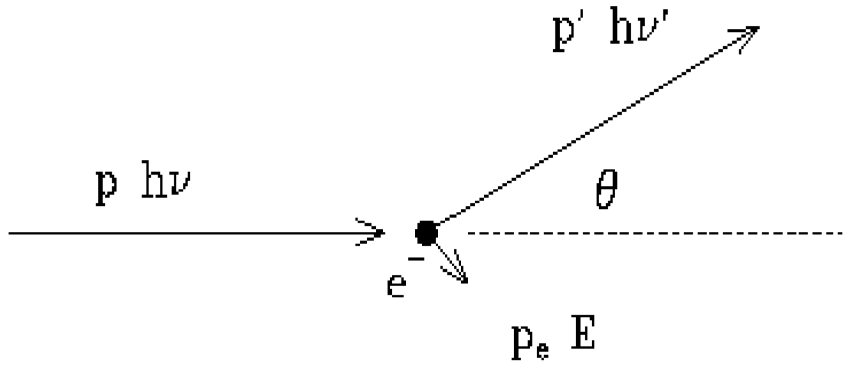


Figure 1: Compton Scattering Kinematics

plotted below in figure two from $0 < \theta < \pi/2$.

However, this scattering cross section is only valid for low incoming photon

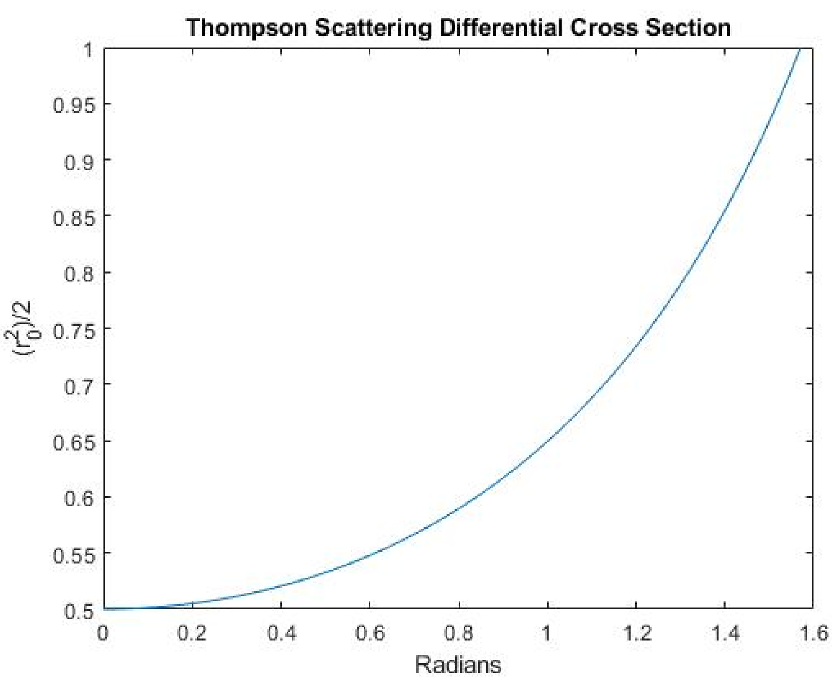


Figure 2: Thompson Cross Section from 0 to $\Pi/2$

energies, or when $h\nu << m_e c^2$. At higher photon energies, the kinematics and cross section change due to higher energy quantum effects. The emergence of these quantum effects give rise to Compton scattering. The energy shift in the outgoing photon is easily calculated using the kinematics of the configuration. This energy shift is given by the formula:
$E_{outgoing} = \frac{E_{incoming}}{1+\frac{E_{incoming}}{m_e c^2}(1-cos(\theta))}$
In our treatment throughout this thesis we will use the standard four-vector notation, and we shall write $x^\mu = (E, \overrightarrow{p})$, where $\mu = (0,1,2,3)$. This indicates a four-vector with an energy component $x^0 = E$ and momentum components $p^j = (1,2,3)$. Additionally, we will follow convention and label the tensor that describes Minkowski time-energy as $g_{\mu,\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$

The covariant vector is defined as: $p_\mu = g_{\mu,\nu} p^\nu$. Thus, multiplying the two vectors, we find the covariant vector is equal to: $x_\mu = (ct, \overrightarrow{-p})$. While this is not the main thrust of the paper, we can express QED Compton scattering using Feymann diagrams. The two tree level Feymann diagrams representing Compton scattering are displayed below in figure 3

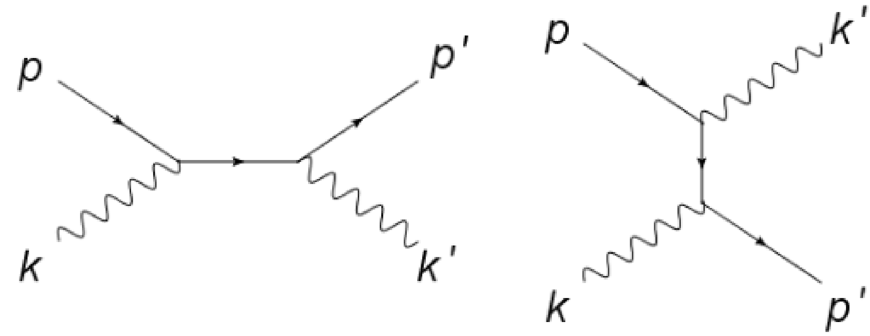


Figure 3: Tree Level QED Compton Scattering Feymann Diagrams

5 Dark Photon Compton Scattering

Now let us look at the case of a dark photon scattering off of an electron. After striking the electron, the dark photon is converted to a regular photon, and scatters at an angle θ. This interaction is displayed below in figure three.

We can express the Lagrangian of this particle interaction as the Lagrangian between a 'dark'vector boson and a real fermion. Thus, we can express the Lagrangian as . Let us look at the kinematics of the situation. We can express the energy momentum four vectors of each of our constituent particles as:

$\overrightarrow{x_{dark}} = [\frac{E_{dark}}{c}, \overrightarrow{P_{dark}}]$

$\overrightarrow{x_{ei}} = [m_e c, \vec{0}]$

$\overrightarrow{x_{photon}} = [\frac{E_{photon}}{c}, \overrightarrow{\frac{E_{photon}}{c}}]$

$\overrightarrow{x_{ef}} = [\frac{E_{ef}}{c}, \overrightarrow{P_{ef}}]$ (1)

As momentum is conserved and $\Delta P = 0$, the sum of the initial momenta must be equal to the sum of the final momenta. Thus, we can write the sum of the four vectors as:

$$\overrightarrow{x_{dark}} + \overrightarrow{x_{ei}} = \overrightarrow{x_{photon}} + \overrightarrow{x_{ef}} \tag{2}$$

Subtracting the four momentum of the outgoing photon from both sides, we are left with the equation:

$$\overrightarrow{x_{dark}} + \overrightarrow{x_{ei}} - \overrightarrow{x_{photon}} = \overrightarrow{x_{ef}} \tag{3}$$

Now that we have isolated the four momentum of the outgoing electron, we can square both sides of our equation. Doing so gives us:

$$(\overrightarrow{x_{dark}} + \overrightarrow{x_{ei}} - \overrightarrow{x_{photon}})^2 = \overrightarrow{x_{ef}}^2 \tag{4}$$

Multiplying the left hand side out, we arrive at the equation:

$$\overrightarrow{x_{dark}}^2 + \overrightarrow{x_{ei}}^2 + \overrightarrow{x_{photon}}^2 + 2\overrightarrow{x_{dark}} \cdot \overrightarrow{x_{ei}} - 2\overrightarrow{x_{dark}} \cdot \overrightarrow{x_{photon}} - 2\overrightarrow{x_{ei}} \cdot \overrightarrow{x_{photon}} = \overrightarrow{x_{ef}}^2 \tag{5}$$

The magnitude of any momentum four vector is defined as $x^\mu x_{mu}$. **Check this notation)** Thus, the magnitude of any massive momentum four vector is equal to $-\frac{(((mc^2)^2+p^2c^2)^{1/2})^2}{c^2} - p^2c^2$. Simplifying, this leaves us with $|x^2| = -m^2c^2$. However, the magnitude of any massless momentum four vector is equal to zero, as $x^\mu x_{mu}$ gives $p^2c^2 - p^2c^2 = 0$. Thus the magnitude of the four momentum of a photon is equal to zero. Substituting this relation into equation 5 gives us:

$$m_{dark}^2 c^2 2 - m_e^2 c^2 + 0 + 2\overrightarrow{x_{dark}} \cdot \overrightarrow{x_{ei}} - 2\overrightarrow{x_{dark}} \cdot \overrightarrow{x_{photon}} - 2\overrightarrow{x_{ei}} \cdot \overrightarrow{x_{photon}} = -m_e^2 c^2 \tag{6}$$

Now we will calculate the cross terms in our equation. Multiplying the energy-momentum four vectors together gives us:

$$2\overrightarrow{x_{dark}} \cdot \overrightarrow{x_{ei}} = 2 * [\frac{E_{dark}}{c}, \overrightarrow{P_{dark}}] \cdot [m_e c, \overrightarrow{0}] = 2E_{dark}m_e \tag{7}$$

Similarly, multiplying the terms for the real and dark photon together gives us:

$$-2\overrightarrow{x_{dark}} \cdot \overrightarrow{x_{photon}} = -2[\frac{E_{dark}}{c}, \overrightarrow{P_{dark}}] \cdot [\frac{E_{photon}}{c}, \overrightarrow{\frac{E_{photon}}{c}}] = -2(\frac{E_{dark}E_{photon}}{c^2} - \frac{P_{dark}E_{photon}cos(\theta)}{c}) \tag{8}$$

Finally, multiplying together the four momentum of the initially stationary electron with the outgoing photon four momentum gives the following:

$$-2\overrightarrow{x_{ei}} \cdot \overrightarrow{x_{photon}} = -2[m_e c, \overrightarrow{0}] \cdot [\frac{E_{photon}}{c}, \overrightarrow{\frac{E_{photon}}{c}}] = -2m_e E_{photon} \tag{9}$$

Now, substituting the results of equations 7,8, and 9 back into our result for equation 6 gives us:

$$-m_{dark}^2c^2 - m_e^2c^2 + -2m_eE_{photon} - 2(\frac{E_{dark}E_{photon}}{c^2} - \frac{P_{dark}E_{photon}cos(\theta)}{c}) + 2E_{dark}m_e = -m_e^2c^2 \tag{10}$$

Now, we can simplify this equation by adding $m_e^2c^2$ to both sides, and grouping

like terms. Doing so gives us:

$$-m_{dark}^2c^2 - 2(m_eE_{photon} + (\frac{E_{dark}E_{photon}}{c^2} - \frac{P_{dark}E_{photon}}{c}cos(\theta)) - E_{dark}m_e) = 0 \tag{11}$$

We can solve equation 11 above for the energy of the outgoing photon, E_{photon}. First, we add $m_{dark}^2c^2 - 2E_{dark}m_e$ to each side. This gives us:

$$-2(m_eE_{photon} + (\frac{E_{dark}E_{photon}}{c^2} - \frac{P_{dark}E_{photon}}{c}cos(\theta))) = m_{dark}^2c^2 - 2E_{dark}m_e \tag{12}$$

Now, we can factor out E_{photon}. This gives us:

$$-2(E_{photon}(m_e + (\frac{E_{dark}}{c^2} + \frac{P_{dark}}{c}cos(\theta)))) = m_{dark}^2c^2 - 2E_{dark}m_e \tag{13}$$

Dividing by $-2(m_e + (\frac{E_{dark}}{c^2} - \frac{P_{dark}}{c}cos(\theta))$ gives:

$$(E_{photon}) = \frac{-(m_{dark}^2 c^2 - 2E_{dark}m_e)}{2(m_e + (\frac{E_{dark}}{c^2} - \frac{P_{dark}}{c}cos(\theta)))} \tag{14}$$

Now we can divide both the numerator and denominator by a factor of m_e.

$$(E_{photon}) = \frac{-(\frac{m_{dark}^2 c^2}{2m_e} - E_{dark})}{(1 + (\frac{E_{dark}}{m_e c^2} - \frac{P_{dark}}{m_e c}cos(\theta)))} \tag{15}$$

Now that we have our expression for the energy of the outgoing photon, we can check to make sure than in the limit that $m_{dark} \to 0$ that we recover the case of Compton scattering. As $m_{dark} \to 0$, the $P_{dark} \to \frac{E}{c}$. This leaves us with the familiar Compton scattering case of $E_{outgoing} = \frac{E_{incoming}}{1+\frac{E_{incoming}}{m_e c^2}(1-cos(\theta))}$

Additionally, now that we have the energy of the outgoing photon in terms of our other parameters, we can find the differential scattering cross section of this configuration. As this interaction occurs at relativistic speeds, we will use the Klein-Nishina differential cross section **(note: in QED, Thompson scattering is obtained at higher scattered photon frequencies. at lower frequencies one obtains Compton scattering- think about how that translates for the dark photon domain)**. The Klein-Nishina differential cross section is defined as: $\frac{\partial\sigma_{kn}}{\partial\Omega} = \frac{r_0^2}{2}\frac{E_{dark}^2}{E_{photon}^2}(\frac{E_{photon}}{E_{dark}} + \frac{E_{dark}}{E_{photon}} - sin(\theta)^2)$

Substituting the relation between the energy of the regular photon and the dark photon we found in equation 15, we can solve for the Klein-Nishina differential cross section in this instance. Noting that substituting in the relation we found in equation 15, we find the differential cross section is:

$$\frac{\partial\sigma_{kn}}{\partial\Omega} = \frac{r_0^2}{2}\frac{(p_{dark}^2 c^2 + m_{dark}^2 c^4)}{(\frac{-(\frac{m_{dark}^2 c^2}{2m_e} - E_{dark})}{(1+(\frac{E_{dark}}{m_e c^2} - \frac{P_{dark}}{m_e c}cos(\theta)))})^2}[(\frac{(p_{dark}^2 c^2 + m_{dark}^2 c^4)^{1/2}}{\frac{-(\frac{m_{dark}^2 c^2}{2m_e} - E_{dark})}{(1+(\frac{E_{dark}}{m_e c^2} - \frac{P_{dark}}{m_e c}cos(\theta)))}} + \frac{\frac{-(\frac{m_{dark}^2 c^2}{2m_e} - E_{dark})}{(1+(\frac{E_{dark}}{m_e c^2} - \frac{P_{dark}}{m_e c}cos(\theta)))}}{(p^2 c^2 + m_{dark}^2 c^4)^{1/2}} - sin(\theta)^2] \tag{16}$$

To solve this equation, we can multiply either side by $\partial\Omega$ and integrate along the solid angle. Doing so gives the solution for σ_{kn}. As there is no ϕ dependence in any of our terms, integrating over ϕ will simply give us a factor of 2π. This reduces our integrals to:

$$\int \partial\sigma_{kn} = 2\pi \int [\frac{r_0^2}{2} \frac{(p_{dark}^2 c^2 + m_{dark}^2 c^4)}{(\frac{-(\frac{m_{dark}^2 c^2}{2m_e} - E_{dark})}{(1+(\frac{E_{dark}}{m_e c^2} - \frac{P_{dark}}{m_e c} cos(\theta)))})^2} [(\frac{(p_{dark}^2 c^2 + m_{dark}^2 c^4)^{1/2}}{\frac{-(\frac{m_{dark}^2 c^2}{2m_e} - E_{dark})}{(1+(\frac{E_{dark}}{m_e c^2} - \frac{P_{dark}}{m_e c} cos(\theta)))}} + \frac{\frac{-(\frac{m_{dark}^2 c^2}{2m_e} - E_{dark})}{(1+(\frac{E_{dark}}{m_e c^2} - \frac{P_{dark}}{m_e c} cos(\theta)))}}{(p^2 c^2 + m_{dark}^2 c^4)^{1/2}} - sin(\theta)^2]] \tag{17}$$

Integrating over θ proves more challenging to accomplish analytically. However, if we assume the cold scattering case, or that the rest mass of the dark photon is much greater than its momentum ($m_{gamma} >> p_{gamma}$), we can greatly simplify this expression.

6 Computational Techniques

To evaluate the results, we employed computational techniques to evaluate and plot the outcomes of our calculations. In order to evaluate and plot the parameters we had previous obtained expressions for, such as the differential cross section and total cross section, we used the technical computing program Wolfram Mathematica (henceforth abbreviated to Mathematica).
To plot the differential cross section displayed above in equation 16, we inputted the expression above into Mathematica. However, to graphically display the output, we needed to ascribe numerical values to some of the parameters in equation 16. To easily adjust these parameters, we used the 'Manipulate' function, which allows for the user to plot the output of a range of different values for a parameter. For this problem, we used the manipulateplot function to vary the values of **(how? what are the values?)** the mass of the dark photon, its momentum, and the momentum of the outgoing electron. We varied the values of the mass between zero and 100GeV, the momentum of the dark photon between 1eV and 10MeV **change these values**, and the momentum of the outgoing electron between 1eV and 10MeV. We plotted the differential cross section from $\theta = 0$ to $\theta = \pi$. The program we wrote to accomplish this is shown below in figure 4:
To decrease the length of expression above, we have let b be the incoming momentum of the dark photon, c be the mass of the dark photon, and f be the momentum of the outgoing electron. We defined a function 'g' that is equal to this expression, with four variables: b,c,f and x. We then used the manipulate function to plot our expression 'g' between the values of $x = 0$ and $x = \pi$ for different values of b,c, and f. The units for b,c, and f are eV, as we are using natural units. Implied on the units of the differential cross section are a factor of $\frac{r_0^2}{2}$ multiplied by a coupling constant. The differential cross section using our manipulateplot function is shown below in figure 5.

$$\frac{\left(b^2+c^2\right)\left(-\frac{\frac{c^2}{1\,022\,000}-\sqrt{b^2+c^2}}{\left(c^2+f^2\right)^{0.5`}\left(\frac{\sqrt{b^2+c^2}}{511\,000}-\frac{b\cos(x)}{511\,000}+1\right)}+\frac{\left(b^2+c^2\right)^{0.5`}\left(\frac{\sqrt{b^2+c^2}}{511\,000}-\frac{b\cos(x)}{511\,000}+1\right)}{\frac{c^2}{1\,022\,000}-\sqrt{b^2+c^2}}-\sin^2(x)\right)}{\left(\frac{\sqrt{b^2+c^2}}{\frac{\sqrt{b^2+c^2}}{511\,000}-\frac{b\cos(x)}{511\,000}+1}-\frac{c^2}{1\,022\,000}\right)^2}$$

Figure 4: Differential Cross Section Expression in Mathematica's Traditional-form

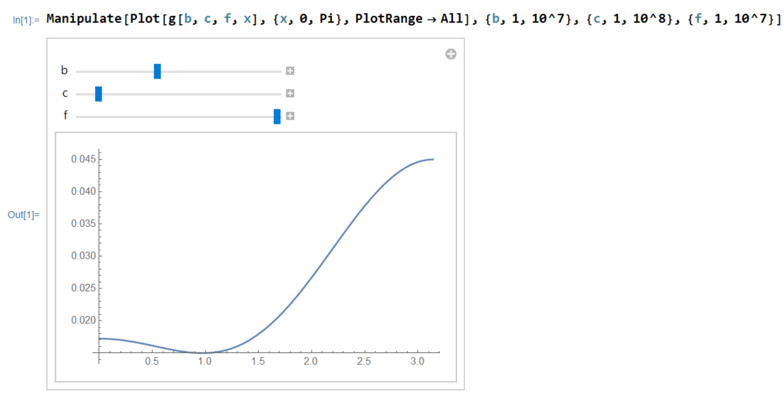


Figure 5: Differential Cross Section Expression in Mathematica's ManipulatePlot

While the nuanced features change depending on the inputted values for the mass of the dark photon, its momentum, and the outgoing momentum of the electron, there are some general trends in the differential cross section that we can draw from these plots. Even as we adjust the values of our input parameters, the greatest value of the differential cross section is either near $\theta = 0$ or $\theta = \pi$. These values represent either forwards or backwards scattering. In between these two maxima, there is a region where $\sigma \approx 0$. This region is 'lateral scattering', or scattering that is not predominantly in forwards or backwards. Scattering is generally much less likely to occur in this region. A good check of our code is to see when we reduce the mass of the dark photon to zero (our C parameter), and scatter at relatively low energies, the differential cross section reduces to a form that resembles Thompson scattering. This low energy approximation is shown below in figure 6. In this approximation the dark photon momentum is approximately equal to the mass of the electron(511000 ev), and the recoil energy of the electron is much less (10000ev) than the momentum of the photon. Now we can reintroduce the mass term back into our scattering cross section and view how it alters the scattering profile for different mass values. The scattering cross sections for both low (much less than m_e) and high values (much greater than m_e) of the dark photon mass are shown below in figures 7 and 8. In the differential cross section, the role of the dark photon mass is to decrease the scattering amplitude (total cross section) and to increase the probability of scattering at a low angle.

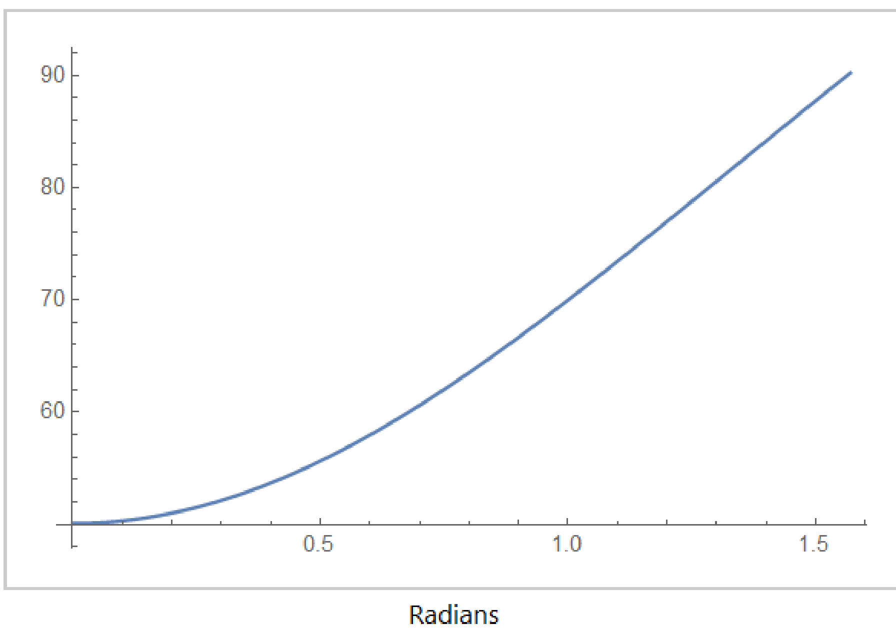


Figure 6: Differential Cross Section with $m_{dark} = 0$ in Mathematica's ManipulatePlot

As is shown in the figure above, there is agreement (up to a scale factor) between the theoretical differential cross section of Thompson scattering and our code.

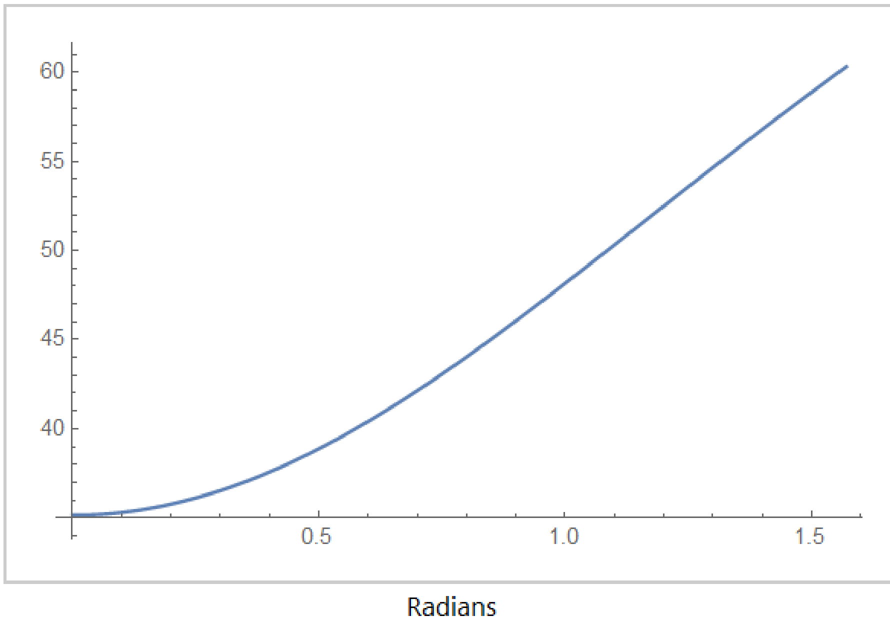


Figure 7: Differential Cross Section Expression with low m_{dark} in Mathematica's ManipulatePlot

Using a low value of $m_{dark} = .1MeV$, the differential cross section takes the same form as our Thompson scattering calculation, but scaled down by a factor that increases as the value of m_{dark} increases. Thus the total cross section of our scattering decreases as m_{dark} increases.

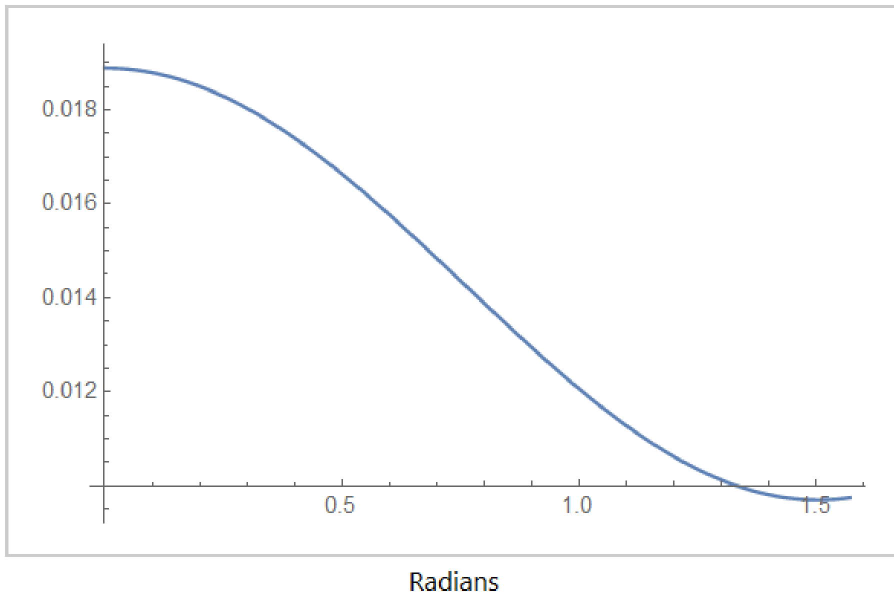


Figure 8: Differential Cross Section Expression in with high m_{dark} Mathematica's ManipulatePlot

When the dark photon mass is increased to much greater than both its momentum and the mass of the electron ($m_{dark} = 10MeV$), the form of the differential cross section changes drastically, as forward scattering becomes far more likely than back-scattering. We can compare both the high and low mass cases to the no mass case to visualize how the differential cross section changes. A comparison of the low and high mass cases to the no mass case is displayed below in figures nine and ten. Note the low mass case is a linear scale and the high mass case is logarithmic.

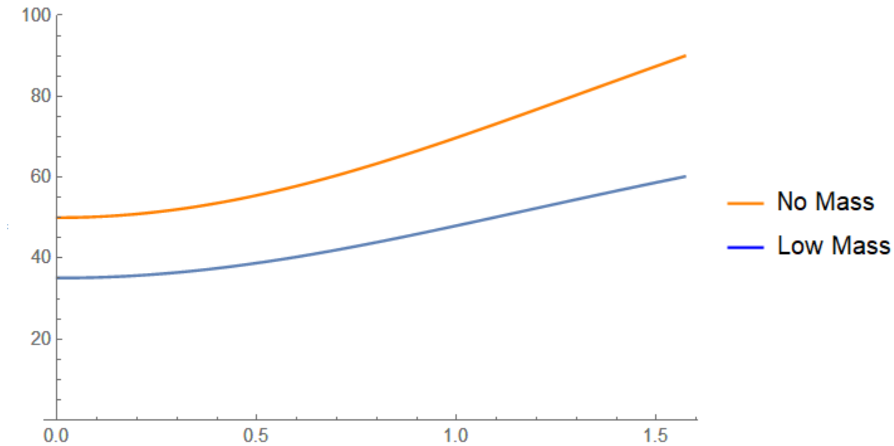


Figure 9: Low Mass (.1MeV) vs No Mass Differential Cross Section

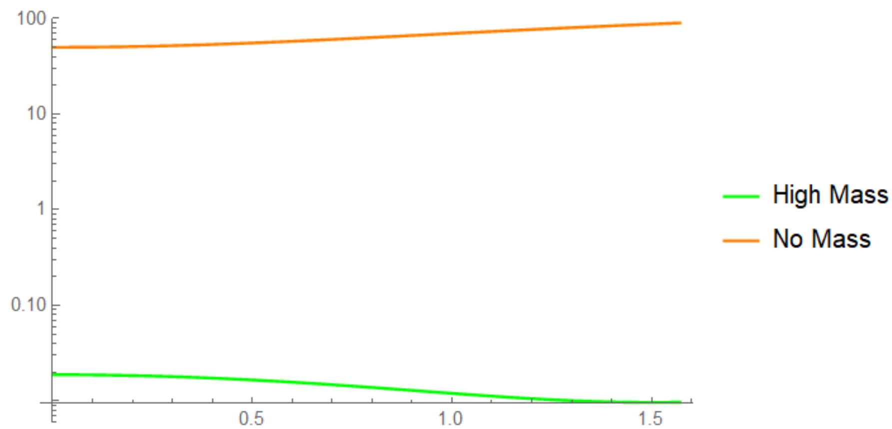


Figure 10: High Mass (10MeV) vs No Mass Differential Cross Section

Furthermore, using the numerical integration techniques on mathematica, we can find the total cross section as a function of the values of the dark photon momentum, mass, and electron recoil momentum. Again using the Manipulate function, we can view the θ component of our $\frac{\partial\sigma}{\partial\omega}$ integral. Thus, we can calculate the total cross section σ of our scattering scenario. The ϕ component gives us an additional factor of 2π as there is no ϕ dependence in our integral. Finally, we need to multiply back in the factor of $\frac{r_0^2}{2}$. The numerical integration code and a sample output are shown below in figures 11 and 12.

```
Manipulate[N[Integrate[g[b, c, f, x], {x, 0, Pi}]], {b, 1, 10^7}, {c, 0, 10^8}, {f, .01, 10^7}]
```

Figure 11: Numerical Integration using Mathematica's Manipulate Function

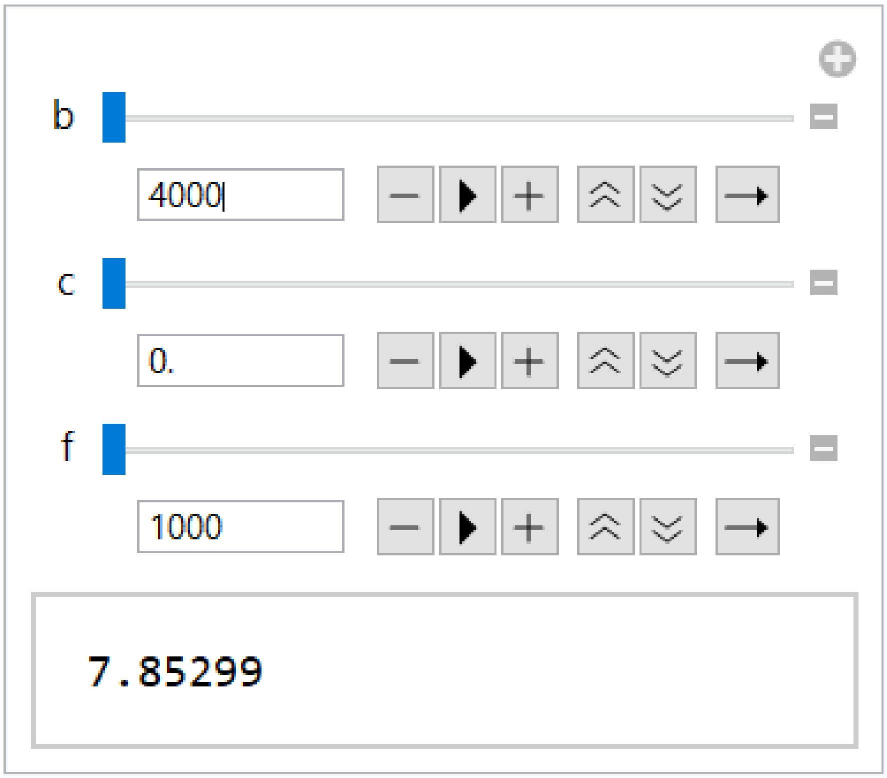


Figure 12: Sample Numerical Integration Output Using Mathematica's Manipulate Function

7 Further Work

Additionally, while this is outside of the scope of our results, we can fit our data to experimental compton scattering results to determine a potential maximum value of the dark photon mass. We can include our dark photon cross section as a perturbation affecting the experimental cross section. For a specific angle θ, we can determine the maximal value of the dark photon mass that is within the error bars. Repeating this process on a continual scale, we can determine the lowest value of the dark photon mass that perturbs the scattering cross section out of its original error bars. Furthermore, to increase the accuracy of the model, we can compute the dynamic terms arising from the Lagrangian governing this interaction. Including dynamic terms from the Lagrangian will alter the cross section, as these calculations include coupling parameters we did not include in our analysis. These coupling parameters, conventionally denoted as ϵ or α define the strength of the interaction between the dark sector and standard model of physics.

8 Physics Papers

As a component of this research, we have investigated current theoretical predictions in other recently published physics papers. While these papers may not cover the exact same ground that we are treading, we have felt they are similar enough to merit inclusion in this section. Authors such as the late Ann Nelson employ theoretical models to derive bounds on the mass of the dark photon, and potential signatures by which the particle can be measured. Generally speaking, these papers cover QED searches for dark matter, with particular emphasis on QED processes involving the dark photon. However, the calculations are similar (if not much more sophisticated), and shine light on other methodologies of searching for dark matter.
The first paper is entitled "Searches for Dark Photon and Dark Matter Signatures Around Electron-Positron Colliders", by Xin Chen et al.The Feynman diagram displaying the first order process is shown below in figure 13.

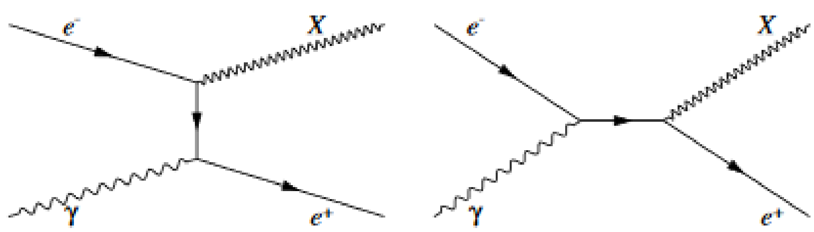


Figure 13: Feymann Diagram for Electron-Positron Annihilation Process. Taken from Chen et al.

In the feymann diagram above, a positron and electron annihilate to create a dark photon and photon. The regular photon is coupled via the electromagnetic force, but the dark photon is coupled to this interaction via a 'dark mediation force' that weakly couples the dark photon to the standard model. Additionally, this paper posits that the dark photon can decay back into particles in the standard model, through some unknown decay process. Furthermore, if both the coupling constant and mass of the dark photon are large, it can decay into a pair of neutral or charge dark sector particles. More specifically, the dark photon can decay into either a pair of standard model fermions or two dark scalar particles. Of the two dark sector particles, one must be in the ground state and one must be in an excited state. The excited dark matter particle eventually decays into a fermion-antifermion pair. The partial decay width of this process is given by the product of the branching ratio β and the total decay with Γ. The partial decay width of an excited dark matter particle decay into a ground state scalar and an fermion-antifermion pair is displayed below in figure 14.

$$\Gamma(X \to f\bar{f}) = \frac{1}{3}\epsilon^2 Q_f^2 \alpha m_X \left(1 + \frac{2m_f^2}{m_X^2}\right)\left(1 - \frac{4m_f^2}{m_X^2}\right)^{\frac{1}{2}}, \quad (4)$$

$$\Gamma(X \to \phi_1\phi_2) = \frac{g_D^2}{48\pi} m_X \left[1 - \frac{2(m_1^2 + m_2^2)}{m_X^2} + \frac{m_1^4 + m_2^4 - 2m_1^2 m_2^2}{m_X^4}\right] \cdot \left[1 - \frac{(m_1 + m_2)^2}{m_X^2}\right]^{\frac{1}{2}} \left[1 - \frac{(m_1 - m_2)^2}{m_X^2}\right]^{\frac{1}{2}}, \quad (5)$$

Figure 14: Partial Decay Rate of Excited Dark Photon to an Exited and Ground State Scalar or an Fermion-AntiFermion Pair. Taken from Chen et al.

In the equation above, m_f represents the charged fermion mass, α is the fine structure constant, Q_f is the charge of the fermion, g_d is the dark coupling parameter

Finally, from this paper we can express the expected number of signal events from electron-positron collisions. The number of expected events is given by the luminosity multiplied by the cross section, the branching ratio, ϵ is a coupling parameter of this interaction, d is the flight length of the dark photon, and L is 1.3m for this experiment. The luminosity of this process is displayed below in figure 15.

$$N_{sig} = \mathcal{L}\sigma(e^+e^- \to \gamma X) BR(X \to \phi_1\phi_2)\epsilon_A \left(e^{-\frac{6}{d}} - e^{-\frac{6+L}{d}}\right)$$

Figure 15: Number of Expected events. Taken from Chen et al.

Additionally, another paper we reviewed discusses the production mechanisms and constraints on the dark photon at electron-positron colliders. This paper, entitled "Production and Constraints for a Massive Dark Photon at Electron-Positron Colliders" by Jiang et al. This paper proposes that dark particles are coupled to the standard model via mediator particles that function similarly to the photon or W and Z boson. This paper discusses two potential mediators: the dark photon and a dark scalar mediator (a particle with spin zero). For the purposes of this analysis we will focus on the paper's treatment of the dark photon. In this process, the electron and positron annihilate into a quark, antiquark, and dark photon. In this two jet system, the first order feymann diagrams for this process are shown below in figure 16.

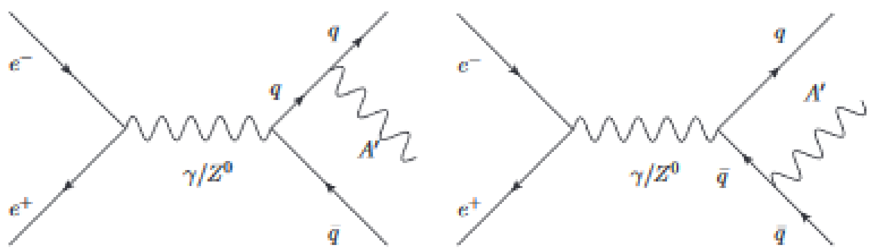
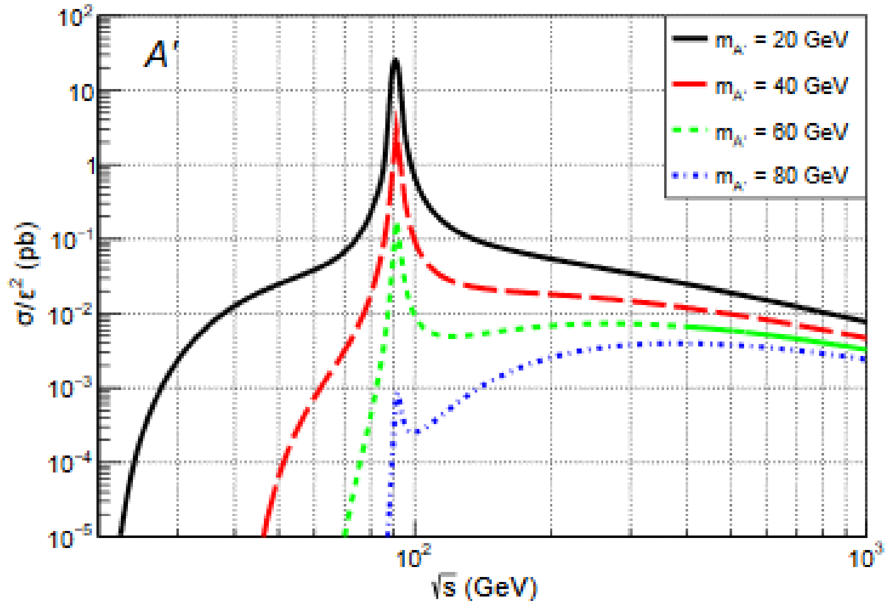


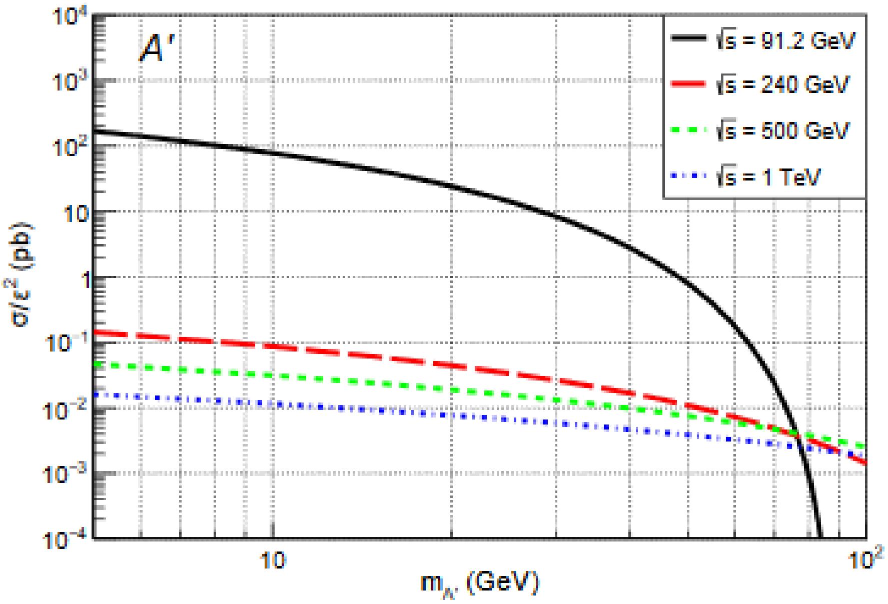
Figure 16: Feymann Diagram of Dark Photon (A') Creation. Taken from Jiang et al.

As is shown in the diagram above, the creation of the dark photon A' can occur via either the electric or weak interaction, as the mediator is either a photon or Z^0 boson. The cross sections of the scattering amplitude are suppressed by a factor of the ϵ^2 where epsilon is the coupling**(you can use this coupling from the Lagrangian to determine how much your Compton scattering cross section is suppressed)**. It is important to note that this coupling is mass dependent, and the scattering amplitude changes with the energy of the system, (as the mass of the dark photon is equivalent to energy given by E=mc^2. We can identify the presence of the dark photon by reconstructing the missing momentum of the system (according to the conservation of four momentum).In their paper, Chen et al. discuss the how the differential cross section changes for different masses and center of mass energies (which are invariant quantities?). Figures 15 and 16 below display the differential cross sections as a function of dark photon mass and center of mass energy $\sqrt{s}$. As is shown in figures 17 and 18, the reduced cross section decreases as a function of dark photon mass, as the coupling parameter ϵ changes with mass. The scattering cross sections all have a peak at 91.2 GEV, as this is the input energy of the system (is this true, maybe rephrase it).



(a)

Figure 17: Reduced Cross Section of A' scattering as a function of $\sqrt{s}$. Taken from Jiang et al.



(c)

Figure 18: Reduced Cross Section of A' scattering as a function of m_A. Taken from Jiang et al.

Additionally, this model presents an alternatative, non-leptonic, decay mode for the dark photon. Chen et al. propose that the dark photon can decay into either a standard model pair of quarks, or a dark matter pair. The decay rate, Γ for each of these processes is shown below in figure 19. The branching ratio is easily calculated using these decay rates, and is shown below in figure 20.

$$\Gamma(A' \to \chi\bar{\chi}) = \frac{g_\chi^2(m_{A'}^2 + 2m_\chi^2)\sqrt{m_{A'}^2 - 4m_\chi^2}}{12\pi m_{A'}^2},$$

$$\sum_q \Gamma(A' \to q\bar{q}) = \sum_q \frac{\varepsilon^2 e^2 c_q^2(m_{A'}^2 + 2m_q^2)\sqrt{m_{A'}^2 - 4m_q^2}}{4\pi m_{A'}^2},$$

Figure 19: Decay rate γ of the dark photon for different decay modes. Taken from Jiang et al.

$$\mathrm{Br}(A' \to \chi\bar{\chi}) = \frac{\Gamma(A' \to \chi\bar{\chi})}{\Gamma(A' \to \chi\bar{\chi}) + \sum_q (A' \to q\bar{q})},$$

Figure 20: Branching Ratio of dark photon decay modes. Taken from Jiang et al.

$m_{A'}$	20 GeV	30 GeV	40 GeV	50 GeV	60 GeV
ε	0.0030	0.0067	0.012	0.019	0.027
$\mathrm{Br}(A' \to \chi\bar{\chi})$	0.996	0.985	0.955	0.898	0.809

Figure 21: Branching Ratio of dark photon decay modes.

However, as we stated before, the coupling factor ϵ depends on the mass of the dark photon, so the branching ratios will differ depending on the mass of the dark photon **(you can use this coupling in your Compton cross section results)**. As the mass of the dark photon increases, the coupling constant increases as well, and the branching ratio to a dark matter pair decreases (as the factor of ϵ^2 in the standard model branching ratio increases). For a specific value of g_x and m_χ, the respective dark sector branching ratios are displayed below in figure 21 as a function of dark photon mass.

9 Conclusion

In this paper, we find an approximate expression for the differential cross section of the dark photon in the hot scattering case, where the momentum of the particle is much greater than the mass. We found the likeliest modes of scattering are either extreme forward or back scattering, with $\theta \approx 0, 180$. However, the scattering amplitude changes with the mass and momentum of the incoming photon. Tying in our findings with recent physics literature, we can search for the existence of the dark photon at electron-positron collisions. Using our findings, we propose the detectors in these experiments are shifted to extreme scattering angles, as these have the greatest likelihood of recording a dark photon event.

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11 List of Figures

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