Energy Loss of an Electric Dipole in a Medium

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Abstract

This thesis presents a reworking of the fundamental underpinnings that describe the loss of energy for a heavy neutrino with an electric dipole moment (EDM) interacting with a target electron, whereas the classic equations - namely the Bethe formula - primarily assume monopolarity. Such altered derivations entail replacing the electric field from a charge with that of a dipole and treating the following change in ionizing energy loss. From there, it can be shown that the usual corrections associated with the formula drop out, and the stopping power itself becomes extremely small. These calculations are fully detailed throughout the thesis. We aim to incorporate this newly-interpreted equation into a coinciding experiment in the works at the LHC under the milliQan Collaboration. Their team hopes to find millicharged particles via weakly ionizing tracks, but our proposition is that the detectors may also be sensitive enough to observe this EDM for the heavy neutrino.

1 Introduction

The Bethe formula describes in essence the stopping power of matter acting on an incident particle traversing through it, interacting to varying extent with its inner electrons. First derived in 1930, several corrections to the formula have since been proposed, which will be discussed in detail later. Our project investigates the possibility that a heavy neutral Dirac fermion could have an EDM such that the neutral particle has ionizing effects on the matter with which it interacts. Taking this into account, it is clear that the concept of stopping power - that is, energy loss - applies to our situation; thus, we posit that a novel construction of the Bethe formula and its corrections as they relate to a charge-dipole interaction will be incredibly insightful.

Derivations for the Bethe corrections have already been done thoroughly, although some of the additional terms (especially those that are logarithmic) lack consistency across non-relativistic and relativistic applications. We will primarily use the relativistic form, but will exclusively stay in the classical and semi-classical realm - although we borrow some terminology from quantum mechanics, overall quantum interactions between the heavy neutrino and electron are not relevant.

1.1 The Goal of the Research

The main purpose of this research is to first demonstrate which fundamental parameters remain fixed in the Bethe formula and its corrections when rederived for a dipole, examining closely how the relationship of energy loss changes as it ceases to describe a purely classical Coulomb collision. Subsequently, with our resulting equation, we should hope to contribute scholarship to the LHC's milliQan Collaboration experiment, as previously mentioned. Of course, our work will not begin to approach the experimental aspects of this field, but it is certainly worth keeping the inspiration in mind for the bigger picture.

2 Theory

2.1 Stopping Power

The fundamental concept surrounding our research is stopping power. This can be best understood by the formula below

$$S = -dE/dx$$

wherein stopping power is represented by S, E is energy (thus $-dE$ representing a loss), and x is a unit distance taken by a traveling particle. More broadly, this relationship describes the extent to which said particle is slowed down as it passes through matter. The subsequent definitions of this energy loss are what constitute the Bethe formula and its respective corrections, so we will delve more deeply into the theory of those later; first, however, it is imperative to gain a visual understanding of the relationship that stopping power has with increasing distance of travel. This is demonstrated in Figure 1 on the following page, using the classic Bragg curve to

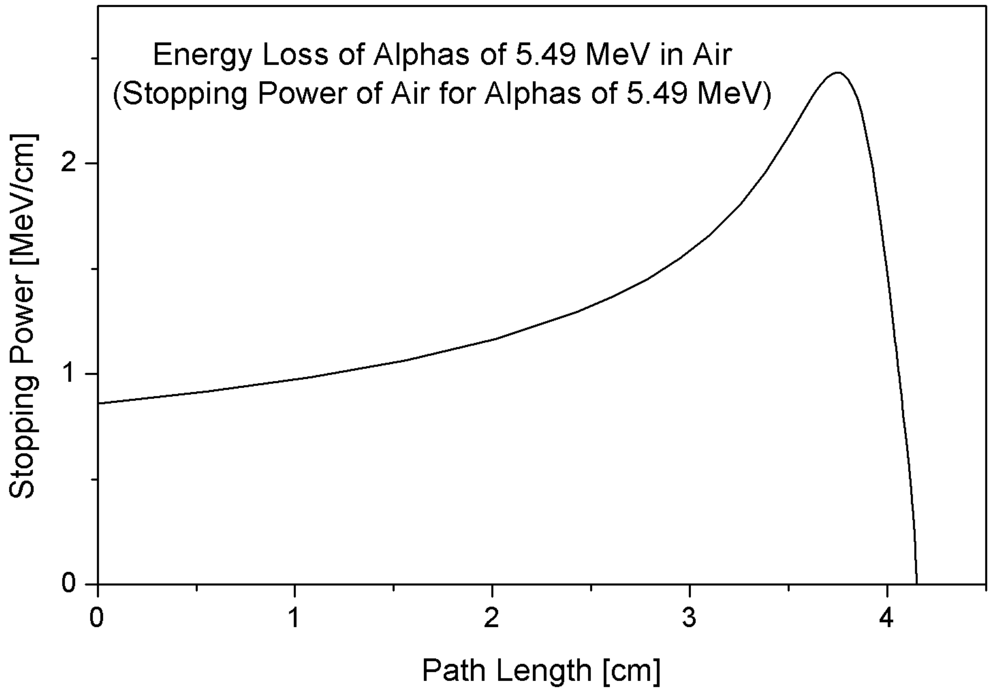

Figure 1: Stopping power vs. penetration depth. This figure clearly displays how distance (expressed in cm) affects the stopping power (expressed in MeV per unit length in cm). One should keep in mind that the highest magnitude of stopping power equates to the greatest rate of energy loss. We see that this rate peaks at around 3.5 cm into the path of the alpha particle, before dipping down to a constant value of energy [1].

represent alpha particles traversing air as a trivial example, in which the stopping power experiences apparently exponential growth before reaching a maximum with respect to path length and then decaying rapidly.

2.2 Bethe Formula and Corrections

To apply the concept of the now better-understood stopping power to a charged incident particle interacting with electrons in the medium, we return to the Bethe formula. This equation has endured several revisions that we will soon treat - namely the shell, the density effect, Barkas, and Bloch corrections - but the basic formula derived from J. D. Jackson's *Classical Electrodynamics* is given below [2]:

$$\frac{dE}{dx} = 4\pi NZ\frac{z^2e^4}{mv^2}lnB$$

where

$$B = \frac{b_{max}}{b_{min}} = \frac{\gamma^2mv^3}{ze^2\omega}$$

In these equations, N is the number of atoms per unit volume, Z is the number of electrons per atom, b is the impact parameter of the incident particle with upper and lower limits, ze is the charge of the incident particle with e being the fundamental charge of an electron and z being the orientation of the charge transverse to the plane of motion, m is the mass of the electron, ω is the characteristic atomic frequency of motion, and v is the incident particle's velocity. In the definition of B we see the relativistic aspect of the formula with γ being the Lorentz factor, equal to $\frac{1}{\sqrt{1-\beta^2}}$ with $\beta = v/c$ i.e. the velocity of the particle divided by the speed of light. The logarithmic term is in fact what first gives way to one of the major corrections, and for this reason the Bethe formula is frequently displayed as a deconstructed expression of two parts: one being the consistent terms - in this fundamental case, $4\pi NZ\frac{z^2e^2}{mv^2}$ - and the other being a series of all applicable corrective terms, condensed into what is called the stopping number, $L(\beta)$, which will be expanded upon later.

2.2.1 Shell Correction

The shell correction to the Bethe formula accounts for a bound electron velocity that is not significantly less than that of the particle; in other words, the electron is not stationary as is assumed in the default Bethe formula. The shell correction is represented in the first term of the stopping number, usually referred to as the primary stopping number L_0 in the series $L(\beta) = L_0(\beta) + Z_1L_1(\beta) + {Z_1}^2L_2(\beta) + ...$

The shell correction itself is expressed as C/Z, best defined as an average over the contributions of several shells for the target atom. According to the work of James F. Ziegler, this term contributes up to a 6% correction to the stopping power [3]. There are several versions of the shell correction itself, but we will mainly concern ourselves with the model for a hydrogenic wave function, since it best approximates the incident particle interacting with an individual target electron as opposed to a gas, etc.

A recurrent theme that will arise with many of our corrections is the extremely high energy of our system. The shell correction is mostly suited for stopping powers in the energy range of 1-100 MeV, and our theorized heavy neutrino has an expected mass of 5-1000 GeV. While the shell correction is ordinarily the most significant of the corrections among classical phenomena, the weak interaction of the semi-classical state we describe, coupled with decreasing contributions from the other corrections to be described below, lead this term to play a far lesser role in energy loss of the EDM.

2.2.2 Density Effect

The density effect, expressed as $\delta/2$, primarily acts as a high-energy corrector for polarization in the target atom. In one of the seminal works of Ugo Fano, it is noted that the density effect, in describing the interactions of atoms with an electromagnetic field, stops the relativistic rise of stopping power [4]. Thus it is useful to include such a correction in our Bethe formula reworkings for the ionization plane present in the LHC experiment.

This term is particularly crucial when the kinetic energy of a particle exceeds its rest mass. The issue for our case, however, is that the density effect describes the interactions of many atoms in a polarized field of a dielectric material - something unconsidered by Bethe or Bloch - with an incident particle as well as with each other. Additionally, Ziegler et al. seem to imply that the correction primarily holds for transverse excitations, and for longitudinal excitations the effect is "zero-energy" [3]. Hence we believe that this term, too, will become negligible.

2.2.3 Barkas Correction

Unlike the previous two corrective terms, which were the most significant and centered around L_0 which treats Z to one order of magnitude, the Barkas correction employs the secondary term of the expanded stopping number, $Z_1 L_1(\beta)$. Here Z_1 is used to distinguish the atomic number of the incident particle from that of the target, Z_2. From the L_1 term of this expansion, Barkas calculates stopping power using the incident particle's charge $(ze)^3$, that is raising the Born approximation to be proportional to the third-order exponential [5]. This correction aims to define the target electrons' response to the incoming particles in which their orbits are affected; at very high energies, however, we know this term drops out due to the large velocity of the ion such that initial motion of the electrons cannot be catalyzed [3]. Better yet, in our specific case, the overall charge number of our incident particle (a dipole) is zero, and as such the term becomes immediately irrelevant. However, it is still crucial to gain a complete understanding of the motivations underlying each major correction so as to have a more robust defense against them.

Barkas' reasoning for proposing such a corrective term was founded on the intermediate-range behavior of the electron's kinetic energy (as will be discussed in greater detail later). He posited that raising the Born approximation to the next-higher-order would correct the stopping power for an incident particle velocity roughly equal to that of the target, even if the incident particle is much heavier. While we are indeed treating rather heavy particles, the difference in velocities - whether the electron is free and stationary, or moving non-relativistically - would yet again render the term insignificant, even if the charge number were still in play.

2.2.4 Bloch Correction

Lastly, where the Barkas correction utilizes the L_1 term of the stopping number expansion to treat z^3, Bloch continued to the L_2 term to increment the incident particle's charge one order higher. This $(ze)^4$ charge is a part of what is overall a very small correction, especially for low energies, but was found to be necessary only for large impact parameters; otherwise, a model closer to Bohr's harmonic oscillator is more appropriate. This correction is posited to be on the order of

$$L_2 \propto (\frac{Z_1\alpha}{\beta})^2(\frac{r_0^2}{2b^2})/ln(\frac{r_0 v_1}{bv_0})$$

as discussed again by Zeigler [3]. Once again, we see that the term is dependent on Z_1 which for our intents and purposes is zero, and so we have another correction which not only becomes very small but in fact directly drops out.

All of these corrections can be collectively compared with the classic stopping power equation, showing how well they approximate each other beyond the exceptional cases, in Figure 2. With extremely small energies, the pure Bethe formula approximates a much lower range of stopping power than what is observed, whereas the corrections - when universally applied - demonstrate a closer alignment with the experimental values.

3 Research Methods

3.1 Previous Work

Since our thesis is predominantly derivation-based, it would be helpful to walk through at least the fundamental reworkings of the Bethe formula for an EDM, which has

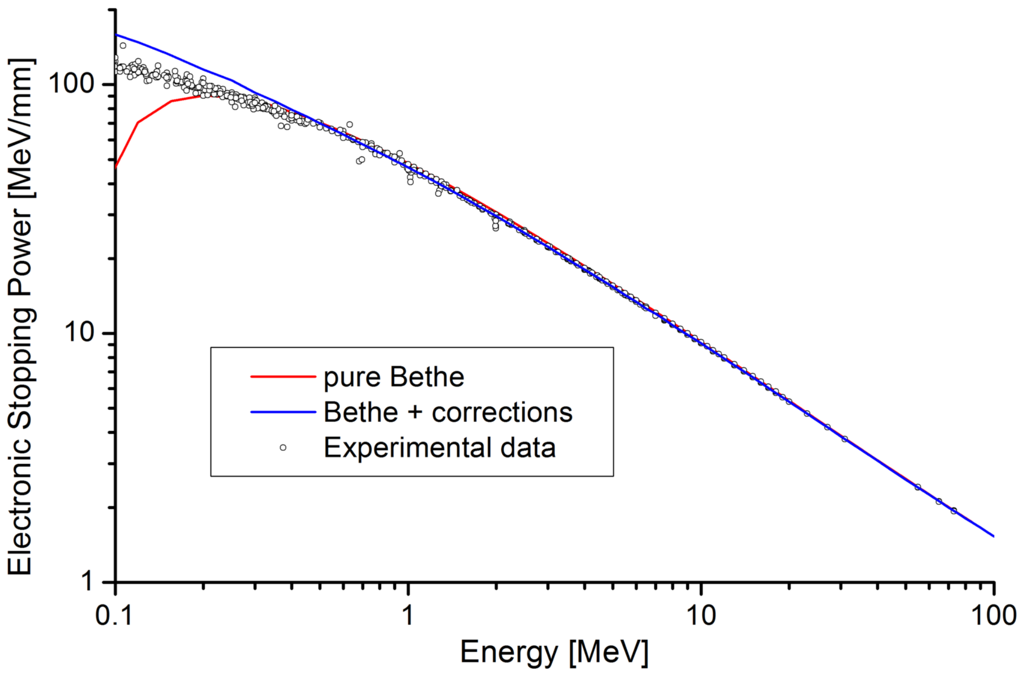

Figure 2: Here we see the relationship between electronic stopping power in MeV/mm and energy in MeV. As energy increases, stopping power falls (mostly) linearly on a log-log scale; however, looking more closely at high stopping numbers for low energies, we see where the corrections differ from the pure Bethe formula. The most significant contributions, as we know, come from the primary terms i.e. the shell and density corrections [8].

mostly been demonstrated in the work of Sher and Stevens [6]. We begin by considering a heavy neutrino moving in the x-direction and an electron at the impact parameter $y = b$. An impulse is given to the electron described by the equation

$$\Delta\vec{p} = \int e\vec{E}dt$$

We then suppose there exists a dipole in the z-direction, that is, transverse to the plane of the particle's and electron's motion, such that in this direction is the only non-zero electric field component. Thus, with the time of the closest approach $t = 0$,

$$E_z = \frac{eD}{4\pi\epsilon_0}(b^2 + v^2t^2)^{-3/2}$$

where eD is the size of the EDM (e = charge, D = distance between particles) and ϵ_0 is the permittivity of free space. From here, we can return to the impulse equation as follows:

$$\Delta\vec{p} = \int eE_z dt = \frac{e^2D}{4\pi\epsilon_0}\int(b^2 + v^2t)^{-3/2}dt = \frac{e^2D}{4\pi\epsilon_0}\sqrt{\frac{b^2}{v^2}\frac{2}{b^3}} = \frac{e^2D}{4\pi\epsilon_0}\frac{2}{vb^2}$$

If we next assume there is a dipole in the y-direction, we see by trigonometry that $E_x = \frac{eD}{4\pi\epsilon_0 r^3}(3\sin\theta\cos\theta)$ and $E_y = \frac{eD}{4\pi\epsilon_0 r^3}(3\cos^2\theta - 2)$ with $r^2 = b^2 + v^2t$ and $\tan\theta = \frac{b}{vt}$. Accounting for symmetry, our $\Delta\vec{p_x}$ vanishes and $\Delta\vec{p_y} = \frac{e^2D}{4\pi\epsilon_0}\frac{2}{vb^2}$. Lastly, for an x-orientation, both electric fields integrate to zero and there is no net momentum transfer.

We can thus generalize our impulse to contain all results if it exists in the plane perpendicular to the neutral particle's motion such that

$$|\Delta\vec{p}| = \frac{e^2D}{4\pi\epsilon_0}\frac{2}{vb^2}$$

being aware of the fact that this impulse simply vanishes if the dipole is oriented parallel to the plane of the particle's motion. Since we have 2 directions to which this definition can apply, the net average impulse is half of this result. Plugging this equation into the change in energy, we see that

$$\Delta E = \frac{|\Delta\vec{p}|^2}{2m} = \frac{e^4D^2}{2m(4\pi\epsilon_0)^2(vb^2)^2}$$

since the electron is moving non-relativistically. However, recall that $\Delta E_{max} = 2m\gamma^2v^2$ such that $b^2_{min} = \frac{e^2D}{4\pi\epsilon_0(2m\gamma v^2}$. This is necessary to set upper and lower limits on the impact parameter in particular, which is something discussed more extensively by Jackson [2]. That said, we can now yield a comprehensive formula for stopping power that accounts for an EDM, relativistic interactions between incident and target particles, and a minimum impact parameter through cylindrical integration:

$$\frac{dE}{dx} = 2\pi NZ \int_{b_{min}}^{\infty} \Delta E(b) b db = \pi NZ (\frac{e^2}{4\pi\epsilon_0}) D\gamma$$

with N = neutron number and Z = nuclear charge. Our goal from here will be to follow this example for each Bethe correction and see to what extent it augments or diminishes the resulting loss of energy.

3.2 Fano Derivation

Now that we have a fundamental understanding of how a dipole interacts with a target electron, we can aim for a more comprehensive and robust formulation of this relationship in terms of energy loss. In order to achieve such a feat, we must consult Ugo Fano's works once again. Fano describes the differential cross section for the inelastic collision with respect to the momentum transfer $\vec{q}$ of the incident particle - in our case, the heavy neutrino - to the electron which is unbound and initially at rest using the formula

$$d\sigma_n = \frac{m}{2\pi\hbar^4 v^2} |(\vec{p'}, n|V|\vec{p}, 0)|^2 (1 + \frac{Q}{mc^2}) dQ$$

where m is the electron mass, v is the velocity of the incident particle, V is the electromagnetic interaction (potential) between particles, $\vec{p'}$ is the final momentum of the incident particle and $\vec{p}$ is the initial momentum [5]. The variable Q here represents the kinetic energy of the unbound electron with momentum $\vec{q} = \vec{p} - \vec{p'}$ which, for relativistic calculations, is given by $Q(1 + Q/2mc^2) = q^2/2m$. It is important to note that this framing of stopping power is semi-classical to even quantum mechanical, in its classification of the particle interactions in terms of impulse or momentum transfer rather than the non-observable impact parameter. Since we are mainly concerned with the high-Q range of energy transfer, our value of Q in fact approximates the total

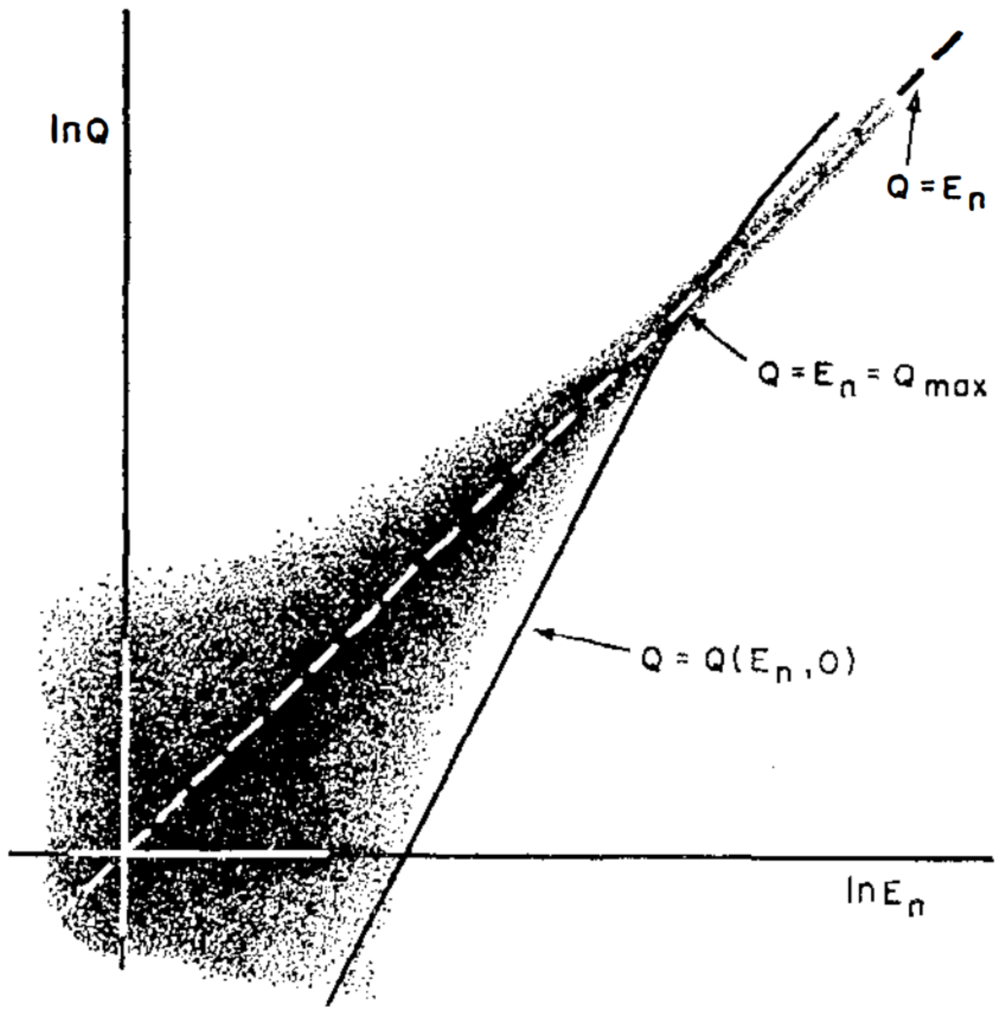

Figure 3: This plot displays the relationship between energy transfer of the collision and the kinetic energy of a free electron with a given momentum. In the high-Q range, both values approximate one another, beginning at the intersection of the functions at $Q = E_n = Q_{max}$ and upwards in the y-direction, where the energy loss ceases having a significant effect on the collision. Likewise, the intermediate range constitutes the Q function lying entirely below the linear strip, and the low values of Q are out of range [5].

energy transfer E_n. This claim may be observed in Figure 3, which plots the energy transfer versus the kinetic energy logarithmically.

Ordinarily, for a simple charge-charge interaction, calculating the matrix elements of the entire interaction - that is, accounting for potential and momentum transfer - is trivial. One need only do a Fourier transform of the electric potential and integrate the longitudinal excitation Fourier component. Once this is complete, plug said matrix element back into the differential cross-section and solve for the stopping power. For a charge-dipole interaction, however, that first step - the Fourier transform - is not so straightforward. Therefore we must first reach an awareness of the potential experienced between these two agents.

3.2.1 Charge-Dipole Interaction

The Coulomb potential between a charge and dipole has been derived by Masahiro Yamamoto at Kyoto University in the following steps [9]:

$$V_{cd} = \frac{z_i(-z_j)e^2}{4\pi\epsilon_0}\frac{1}{|\vec{r_j} - \vec{r_i}|} + \frac{z_i z_j e^2}{4\pi\epsilon_0}\frac{1}{|\vec{r_j} + \vec{p_j} - \vec{r_i}|}$$

where z_i and z_j are the charge numbers for the electron and dipole fields, respectively, $|\vec{r_j} - \vec{r_i}| = \vec{r_{ji}}$ and $|\vec{r_{ji}}| = r_{ji}$ then we have

$$V_{cd} = -\frac{z_i z_j e^2}{4\pi\epsilon_0}(\frac{1}{r_{ji}} - \frac{1}{\sqrt{r_{ji}^2 + 2\vec{r_{ji}}\cdot\vec{p_j} + p_j^2}})$$

$$= -\frac{z_i z_j e^2}{4\pi\epsilon_0 r_{ji}}[1 - \frac{1}{(1 + 2\vec{r_{ji}}\cdot\vec{p_j}/r_{ji}^2 + \vec{p_j^2}/r_{ji}^2)^{1/2}}]$$

Using the Taylor series expansion of $(1+x)^{-1/2}$ we then see that

$$V_{cd} \approx -\frac{z_i z_j e^2}{4\pi\epsilon_0 r_{ji}}[1 - 1 + \frac{1}{2}(\frac{2\vec{r_{ji}}\cdot\vec{p_j}}{r_{ji}^2} + \frac{\vec{p_j^2}}{r_{ji}^2}) - \frac{3}{8}(\frac{2\vec{r_{ji}}\cdot\vec{p_j}}{r_{ji}^2} + \frac{\vec{p_j^2}}{r_{ji}^2})^2]$$

We will primarily be working with situations where the radius of separation between the charge and dipole is much greater than the dipole distance itself, i.e. where $r_{ji} >> p_j$, then we conclude with the following interaction:

$$V_{cd} = -\frac{z_i z_j e^2}{4\pi\epsilon_0}\frac{\vec{r_{ji}}\cdot\vec{p_j}}{r_{ji}^2} = \frac{z_i z_j e^2}{4\pi\epsilon_0}\frac{\vec{r_{ij}}\cdot\vec{p_j}}{r_{ij}^3} = \frac{z_i e}{4\pi\epsilon_0}\frac{\vec{r_{ij}}\cdot\vec{\mu_j}}{r_{ij}^3}$$

with dipole moment $\vec{\mu_j} = e z_j \vec{p_j}$.

3.2.2 Fourier Transform

In order to be able to relate this potential felt between the charge and dipole back to stopping power, we return to the steps first detailed at the beginning of this subsection. We are required to complete a Fourier transform of the electromagnetic interaction (again, only the longitudinal excitation), which is carried out in the universal language below.

$$V(k) = \frac{1}{(2\pi)^3} \int d^3 r V(r) e^{ikr\cos\theta}$$

The reader may recall that our predefined potential $V(r) = \frac{z_i z_j e^2}{4\pi\epsilon_0} \frac{\vec{r_{ij}} \cdot \vec{p_j}}{r_{ij}^3}$ where $\vec{p_j}$ is the separation distance, or position of charge j with respect to i. However, this distance is fixed and infinitesimally small compared to the radius of interaction, so we will treat it simply as a constant D, for the dipole size. If we exploit geometry in setting up our field such that the dipole is moving in the x-direction and our corresponding impulse is oriented in the transverse direction, then we can define the product of $\vec{r_{ij}} \cdot D$ as simply $|\vec{r_{ij}}| \sin\theta |D|$. This angular relationship is illustrated in Figure 4. Thus, returning to our Fourier transform,

$$V(k) = \frac{1}{(2\pi)^3} \frac{z_i z_j e^2}{4\pi\epsilon_0} D \int 2\pi r_j^2 \frac{\sin^2\theta}{|r_i - r_j|^2} e^{ikr_j\cos\theta} dr_j d\theta$$

We are integrating over r_j here because r_i is our target electron coordinate, thus being fixed at the origin such that it drops out. Now that this clarification has been made, we may continue just referring to our integration variable as r, while maintaining an explicit knowledge of what this refers to. Therefore

$$V(k) = \frac{z_i z_j e^2}{32\pi^4\epsilon_0} D \int_0^\infty 2\pi [\frac{\pi J_1(kr)}{kr}] dr$$
$$= \frac{z_i z_j}{16\pi^2\epsilon_0} \frac{1}{k} e^2 D$$

where J_n is a Bessel function of the n^{th} kind. This result is relatively neat, although where typical utilizations of Fourier's trick result in a $1/k^2$ term for the potential between two charges, we see here how that of the charge-dipole interaction begins to depart from its conventional structure.

3.2.3 Matrix Integration

In the case of a charge-charge interaction, integration of the momentum-potential matrix element takes the form below [5].

$$|(\vec{p'}, n|V|\vec{p}, 0)| = \frac{ze^2}{2\pi^2}\int d^3\ \vec{k}\frac{(p'|e^{-i\vec{k}\cdot\vec{r}}|p)(n|\sum_j e^{i\vec{k}\cdot\vec{r_j}}|0)}{k^2}$$

$$+\sum_s \frac{(\vec{p'}|\vec{\alpha}\cdot\hat{\vec{A}}_s e^{-i\vec{k}\cdot\vec{r}}|\vec{p})(n|\sum_j \vec{\alpha_j}\cdot\hat{\vec{A}}_s e^{i\vec{k}\cdot\vec{r_j}|0)}}{k^2-(E_n/\hbar c)^2}$$

In practice, we will only have use for the first half of the integral, as the latter refers to the emission and reabsorption of photons within the Coulomb gauge representation, which is beyond our paygrade. For our case, we must multiply the term in the integrand by our $V(k)$ factor - that is, the $1/k$ Fourier term, which we will be treating here as a vector. Additionally, in the formula above, one might note that the second exponential in the numerator is being summed over some j. This is a count of electrons contributing to energy loss of the incident particle, and because we are only analyzing the effects of a single electron, we will continue with only one exponential.

$$(\vec{p'}, n|V|\vec{p}, 0) = \frac{z_i z_j e^2}{16\pi^2\epsilon_0}\int d^3\vec{k}(\vec{p'}|e^{-i\vec{k}\cdot\vec{r}}|\vec{p})\frac{1}{\vec{k}}e^{i\vec{k}\cdot\vec{r}}$$

Lastly, incorporating conservation of momentum into the relationship such that $(\vec{p'}|e^{-i\vec{k}\cdot\vec{r}}|\vec{p})$ $= (2\pi)^3\delta(\vec{k}+\frac{\vec{p'}}{\hbar}-\frac{\vec{p}}{\hbar})$ where δ is the Dirac delta function, we can take advantage of integration tricks (that is, replacing k with $\frac{\vec{p'}-\vec{p}}{\hbar}=\frac{\vec{q}}{\hbar}$) to demonstrate the complete interaction matrix element:

$$(\vec{p'}, n|V|\vec{p}, 0) = \frac{z_i z_j}{2\epsilon_0}\pi e^2 D e^{\frac{i\vec{q}\vec{r}}{\hbar}}\frac{\hbar}{\vec{q}}$$

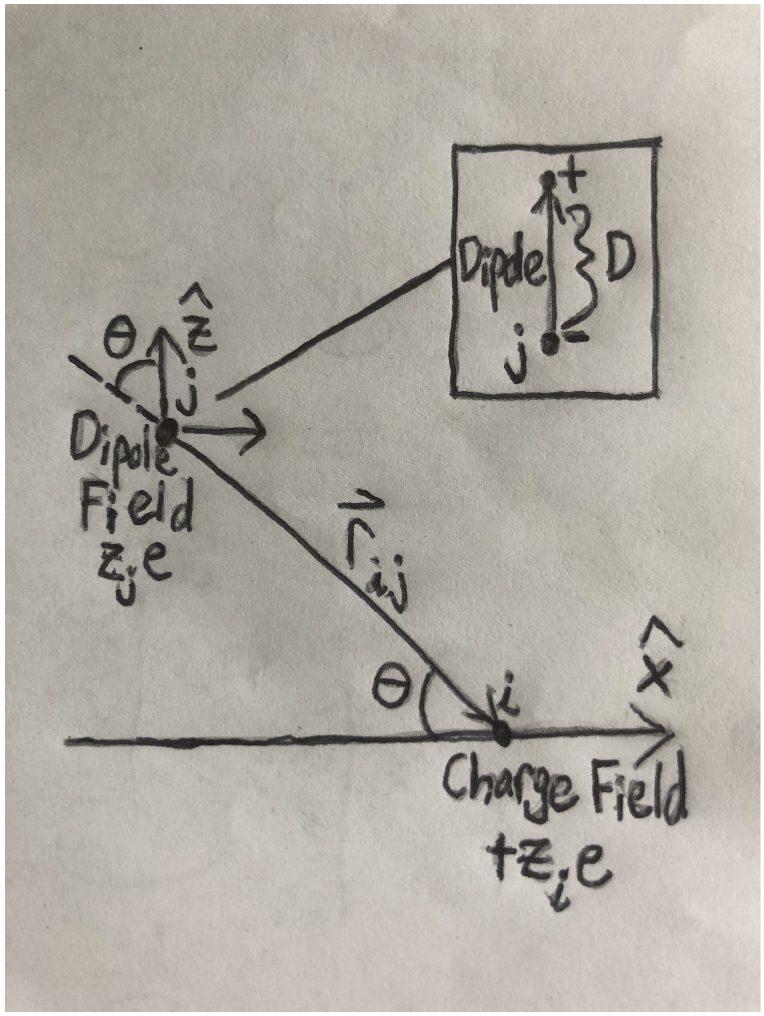

Figure 4: The dipole field is given by positive and negative charges at position j with a separation distance D, and charge $z_j e$. The (electron) charge field is a distance $\vec{r_i j}$ away at position i, with angular separation θ. Here we can see that a dipole oriented in the $\hat{z}$-direction is moving in the $\hat{x}$-direction, simplifying the geometry for our EDM calculations [9].

We now have an equation that is only dependent on the momentum transfer and dipole moment. The distance vector between the charge and dipole has returned, but only briefly - the next section will resolve this conflict.

3.2.4 Cross Section and Stopping Power

The final steps in deriving the basic stopping power formula for an electric dipole entail inserting our previously-calculated matrix element into the differential cross section, and then integrating this quantity with the summed energy transfer within the high-Q approximation. Thankfully, as I teased at earlier, we can now be rid of that pesky exponential and the r vector generally, by way of an absolute square. Remembering Fano's equation for the cross section,

$$d\sigma_n = \frac{m}{2\pi\hbar^4 v^2}|(\vec{p'}, n|V|\vec{p}, 0|^2(1 + \frac{Q}{mc^2})dQ$$

we obtain a more applied version

$$\begin{aligned} d\sigma_n &= \frac{m}{2\pi\hbar^4 v^2}|\frac{z_i z_j}{2\epsilon_0}\pi e^2 D e^{\frac{i\vec{q}\vec{r}}{\hbar}}\frac{\hbar}{\vec{q}}|^2(1 + \frac{Q}{mc^2})dQ \\ &= \frac{m}{v^2}\frac{z_i^2 z_j^2}{8\epsilon_0^2}\frac{\pi e^4 D^2}{\vec{q}^2\hbar^2}(1 + \frac{Q}{mc^2})dQ \end{aligned}$$

where the exponential has vanished due to its complex conjugate and our differential cross section is merely a function of the variable q. In order to simplify here, it is crucial to recall the mathematical connection between kinetic energy and momentum transfer or impulse, modeled by $q^2 = Q(1 + Q/2mc^2)2m$ for relativistic cases, or $q^2 \sim 2Qm$ for non-relativistic. Of course, our concerns are only semi-classical, and so in this range of high-energy (hence also high-velocity) incidence, we must stand with the relativistic interpretation. Our expression then is expressed as

$$d\sigma_n = \frac{\pi z_i^2 z_j^2 e^4 D^2}{16Q\hbar^2 v^2 \epsilon_0^2}dQ$$

To translate this into something more intelligible to stopping power, we'll first sum the cross section multiplied by energy transfer in the high-Q approximation, in which $E_n \sim Q \sim Q_{max}$ and $Q_{max} = \frac{2mv^2}{(1-v^2/c^2)}$.

$$\sum_n E_n d\sigma_n = \frac{\pi z_i^2 z_j^2 e^4 D^2}{16\hbar^2 v^2 \epsilon_0^2}dQ$$

The basic stopping power formula for the high-Q range, then, is simply taking the integral of this result within the appropriate limits and generalizing it with a density of atoms per unit volume, expressed as N:

$$-\frac{dE}{dx} = N\int_{Q_2}^{Q_{max}} \sum_n E_n d\sigma_n = N\frac{\pi z_i^2 z_j^2 e^4 D^2}{16\hbar^2 v^2 \epsilon_0^2}\int_{Q_2}^{Q_{max}} dQ$$

$$= N\frac{\pi z_i^2 z_j^2 e^4 D^2}{16\hbar^2 v^2 \epsilon_0^2}[Q_{max} - Q_2]$$

We now have a description of the energy loss experienced by a heavy neutrino with an EDM as it interacts with a free electron initially at rest via momentum transfer. The range of kinetic energies possible for this theorized interaction is wide, such that we can effectively claim Q_2 to be zero. Additionally, if we wish to express Q_{max} mathematically rather than numerically, one may recall that we were indeed given a general definition as $Q_{max} = \frac{2mv^2}{(1-v^2/c^2)} = 2mv^2\gamma^2$ where γ is the Lorentz factor. Electing to utilize this expression in our stopping power formula, we see that

$$-\frac{dE}{dx} = N\frac{\pi z_i^2 z_j^2 e^4 D^2}{16\hbar^2 v^2 \epsilon_0^2} 2mv^2\gamma^2 = N\frac{m\pi z_i^2 z_j^2}{8\hbar^2 \epsilon_0^2} e^4 D^2 \gamma^2$$

It may be apparent that this result is structurally unorthodox, with the electron mass having returned to the scene, as well as the dipole moment and Lorentz factor being the square of their original representation in our earlier work [6]. That said, these are all constants, and we can set $\hbar$ as being equal to 1, etc., such that even when we do treat the energy loss quantitatively it still exists in a minuscule range. The reasons why this approach differs so drastically from both Fano and Sher will be discussed in the following section.

4 Results

The numerical integration yields a result that precisely expresses the loss of energy in such a high-energy system that - realistically, for this approximation - is almost negligible with regards to the rest of the interaction. This is similar to the conclusion reached by Fano [4]. Furthermore, we have found that, in addition to the ionization and logarithmic terms dropping in the initial derivations, the corrective terms become similarly negligible even when accounting for an electric dipole. This insignificance was almost exclusively hypothesized to arise from the high-Q range of our EDM, as well as from our lack of transverse excitations in the Coulomb gauge representation of interaction (that is, we were not describing a system in which virtual photons were being emitted and reabsorbed.)

The final integral at which we arrived differed from Sher's formulation in some interesting respects. As I've already stated, the Fano derivation utilized the momentum transfer variable as a quantum replacement for the impact parameter, whereas Sher remained purely in the semi-classical realm by keeping the impact parameters in the calculation. One benefit of the former approach is that it includes an observable phenomenon, and so the weak ionization loss affecting the EDM of a heavy neutrino moving much more rapidly than its target electron will make greater use of the dipole derivation I have completed in this paper. That is to say, if the milliQan group were to seek out this additional exotica, the formulation we have here should, in theory, apply to their experiment. The primary concern, then, is simply that at this range of energy transfer, the stopping power might be too weak to rely on.

Additionally, it is necessary to note that, at the time of Fano's writing this, field theory had scarcely been developed, and as such we had little to no knowledge of all the ways in which a non-classical interaction between two subatomic particles could be carried out. Unlike a Coulomb collision, this charge-dipole interaction through weak and electromagnetic forces is far more short-range and without the specified impact parameter b which is so often seen in classical derivations. While an impulse or momentum transfer has the advantage of being an observable parameter, this term drops out for very high energies.

While our final revised Bethe formula leaves something to be desired, a more detailed analysis could be done by taking a less generalized approach directly from the Coulomb collision derived to an EDM, as well as following works with a more contemporary comprehension of field theory interactions supported by experimental evidence. However, this result - along with corresponding dimensional analysis and numerical analysis of constants - was less important in our paper than was making the argument that the original corrections which became crucial to applications of the Bethe formula would inevitably become negligible. We believe this claim to by now be well supported, as the entirety of the $L(\beta$ stopping number has vanished from our equation, leaving only constants with no room for situational approximations.

5 Conclusion/Outlook

Our findings prove not only the possibility but also the convenience of detecting an EDM in the use of weakly ionizing tracts for sensing a heavy neutral Dirac fermion. From Sher's paper, the proposed size of the electric dipole for this heavy neutrino should reach a magnitude of around 10^{-16} e cm, coming from a calculated $D\gamma$ value of $8e{-}17$ where γ is between 10-100 [6]. Given that the LHC's detectors are sensitive to magnitudes as small as 10^{-17}, this provides a great motivation for the milliQan group to seek other exotica while their experiment remains ongoing such that it is even more fruitful than they might have hoped [10].

Should the LHC choose to examine the possibility of utilizing our theoretical formulations to detect a heavy neutrino with an electric dipole oriented transverse to its plane of motion, they would have to ensure that their system reflects that described in our derivations. In other words, if their system contains multiple atomic electrons, moving at roughly the same velocity as the incident particle, then they would have to reinsert the corrections that we were able to disregard. Lastly, they must be aware that the neutral particle will lose very little energy in this process except over longer observation periods, and thus will remain fairly unchanged and mostly non-ionizing - but hopefully not unseen.

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