Numerical Solution to Non-Uniformity Correction in Wide Aperture Target Magnet

A thesis submitted in partial fulfillment of the requirement
for the degree of Bachelor of Science in
Physics from the College of William and Mary in Virginia,

by

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May 2017

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April 21, 2017

Abstract

In this report, the concept and details of the calculation of a 5 Tesla field with uniformity sufficient for NMR is discussed. The key feature of the magnet that will produce this field is its large opening angle which allows for a significant improvement to particle physics experiments in which particles are scattered from a target. With this wider observation angle, particles scattered at angles where they were previously blocked by the magnet assembly can now be detected. The magnet design is based on two short solenoid coils that produce the field and specially designed correction coils made to achieve the uniformity needed for experimentation.

1 Introduction

In the field of nuclear physics, research regarding the strong nuclear force requires experimentation on polarized nuclear targets. To use these targets, a strong and highly uniform magnetic field is needed to maintain their polarization in the presence of the electron beam whose high luminosity works to depolarize the target material. Uniformity or homogeneity, which refers to the variance of a magnetic field in a particular region, is integral to the process of polarizing the target. Through the process of electron pumping, in which a resonance condition between the spin-orbit frequency and the frequency of the polarizing electromagnetic waves are equal, polarization is transfered from the electrons to the protons in the target. If the uniformity is not high enough, the polarization transfer will not occur. A particular configuration, known as the Helmholtz configuration, is essential for achieving this uniformity requirement. Current interests in nuclear physics lie in the processes in the forward beam direction where the target is transversely polarized. However, due to the requirements of the Helmholtz configuration and the essential components of the magnet, the transverse aperture of the coils is greatly reduced, limiting the detection of escaping particles.

In order to do an effective experiment where wide angle processes in the direction transverse to the target polarization are measured, the opening angle of the magnet must be increased. Figure 1 shows a simplified depiction of how these experiments look. By increasing the distance between the magnets coils, a larger opening can be achieved, however doing this results in an unacceptable loss of field uniformity. Correcting this nonuniformity is possible. This paper proposes the addition of correcting solenoids to the existing magnet design and explores their effectiveness in resolving both the loss in uniformity and the need for a wider opening angle.

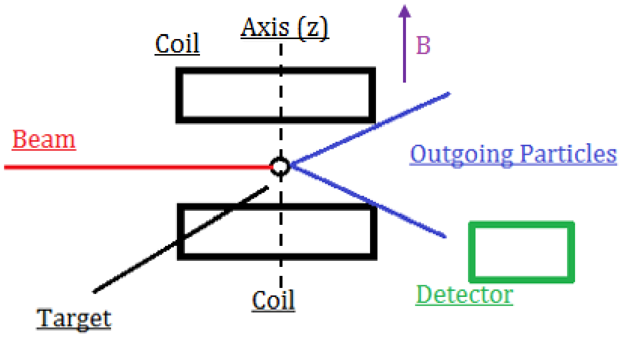


Figure 1: This figure shows the configuration for a typical particle physics experiment

2 Formulations and Relevant Problems

To form a basis of comparison and create a better understanding for the interpretation of results several basic, albeit useful problems from electromagnetism are discussed, as well as a brief description of the useful forms of the Maxwell equations employed. This section relies heavily on the material gathered from item [1] in the reference section.

The Maxwell equations for magnetostatic fields are

$$\boldsymbol{\nabla} \cdot \mathbf{B} = 0 \tag{1}$$

$$\boldsymbol{\nabla} \times \mathbf{B} = \mu_0 \mathbf{J} \tag{2}$$

where $\mathbf{B}$ is the magnetic field, μ_0 is the permeability of free space, and $\mathbf{J}$ represents the volume current density. The magnetic field, $\mathbf{B}$, is a vector field that is a function of (x, y, z), and has components B_x, B_y, and B_z. In magnetostatics, the theory of steady currents, the magnetic field is constant in time. Looking at Equations 1 and 2, the symbol "$\boldsymbol{\nabla}$" is known as the del operator and is defined as $\boldsymbol{\nabla} = \frac{\partial}{\partial x}\,\hat{\mathbf{x}} + \frac{\partial}{\partial y}\,\hat{\mathbf{y}} + \frac{\partial}{\partial z}\,\hat{\mathbf{z}}$ where $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$, $\hat{\mathbf{z}}$ represent unit vectors in their respective coordinate direction. When the del operator acts upon a vector via the dot (scalar) product, such as in (1), the result is a measure of how much the vector diverges from a given point. This quantity is known as the divergence. When del operates on a vector via the cross (vector) product, represented in Equation 2, it yields a measure of how much the vector "swirls" around a given point. This result is know as the curl of the vector. In the particular case when the current, $\mathbf{J}$, equals 0, Equation 2 becomes

$$\boldsymbol{\nabla} \times \mathbf{B} = 0. \tag{3}$$

Equations 1 and 3 can be expressed in terms of their components. Re-writing the Maxwell equations as a sum of their components, Equations 1 and 2 become

$$\boldsymbol{\nabla} \times \mathbf{B} = \sum_{j,k} \delta_{i,j,k} \frac{\partial}{\partial x_j} B_k = \mu_0 \mathbf{J}_i \tag{4}$$

and

$$\boldsymbol{\nabla} \cdot \mathbf{B} = \sum_{i=1}^{3} \frac{\partial}{\partial x_i} B_i = 0. \tag{5}$$

Here, the symbol $\delta_{i,j,k}$ represents the three dimensional Levi-Civita symbol. The definition of the Levi-Civita symbol is as:

$$\delta_{i,j,k} = \begin{cases} +1 & \text{if}(i,j,k)\text{is}(1,2,3),(2,3,1),(3,1,2) \\ -1 & \text{if}(i,j,k)\text{is}(3,2,1),(1,3,2),(2,1,3) \\ 0 & \text{if} i = j, j = k, k = i \end{cases}$$

This definition shows that, for a cyclic or even permutation of the indices, the value of $\delta_{i,j,k}$ is $+1$, for odd or anti-cyclic permutations, $\delta_{i,j,k}$ = -1, and whenever two or more of the indices are equal $\delta_{i,j,k} = 0$. The indices 1,2, and 3 represent respectively the coordinates x,y,and z. When these definitions of the Maxwell equations are used, say, for the x direction, the result would be μ_0J$_1$ $= \delta_{1,2,3}$ $\frac{\partial}{\partial x_2}$ B$_3$ $+$ $\delta_{1,3,2}$ $\frac{\partial}{\partial x_3}$ B$_2$ $=$ $\frac{\partial}{\partial y}$ B$_z$ - $\frac{\partial}{\partial z}$ B$_y$ $=$ μ_0J$_x$. Thus, the Maxwell equations in three dimensions can be written in general as

$$\mu_0 J_x = \frac{\partial}{\partial y} B_z - \frac{\partial}{\partial z} B_y \tag{6}$$

$$\mu_0 J_x = \frac{\partial}{\partial z} B_x - \frac{\partial}{\partial x} B_z \tag{7}$$

$$\mu_0 J_x = \frac{\partial}{\partial x} B_y - \frac{\partial}{\partial y} B_x \tag{8}$$

$$\frac{\partial}{\partial x} B_x + \frac{\partial}{\partial y} B_y + \frac{\partial}{\partial z} B_z = 0. \tag{9}$$

2.1 Maxwell's Equations in 2 Dimensions

For the two dimensional case, where it is assumed that there is no dependence on z and Equation 3 is used in place of Equation 2, the solutions to the Maxwell equations must satisfy

$$\frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} = 0 \tag{10}$$

$$\frac{\partial B_z}{\partial x} = 0 \frac{\partial B_z}{\partial y} = 0 \tag{11}$$

$$\frac{\partial B_y}{\partial x} = \frac{\partial B_x}{\partial y}. \tag{12}$$

The Solutions in Equation 12 imply that B_z is constant. When obvious symmetry is recognized, such as spherical or cylindrical, it is useful to know the Maxwell equations in each of their respective coordinate systems. In cylindrical coordinates, Equations 1 and 3 become

$$\boldsymbol{\nabla} \times \mathbf{B}(\mathbf{r}, \boldsymbol{\phi}, \mathbf{z}) = \left[\frac{1}{r}\frac{\partial B_z}{\partial \phi} - \frac{\partial B_\phi}{\partial z}\right]\hat{\mathbf{r}} + \left[\frac{\partial B_r}{\partial z} - \frac{\partial B_z}{\partial r}\right]\hat{\boldsymbol{\phi}} + \frac{1}{r}\left[\frac{\partial (r B_\phi)}{\partial r} - \frac{\partial B_r}{\partial \phi}\right]\hat{\mathbf{z}} \quad (13)$$

$$\boldsymbol{\nabla} \cdot \mathbf{B}(\mathbf{r}, \boldsymbol{\phi}, \mathbf{z}) = \frac{1}{r}\frac{\partial}{\partial r}(r B_r) + \frac{1}{r}\frac{\partial B_\phi}{\partial \phi} + \frac{\partial B_z}{\partial z}. \quad (14)$$

In a cylindrical coordinate system, the coordinate z represents height, r represents the radial distance from the z-axis to a point P, and ϕ is the azimuthal coordinate representing the angle between the reference direction on a chosen plane and a line from the origin to the projection of P on that plane.

and in spherical coordinates,

$$\boldsymbol{\nabla} \times \mathbf{B}(\mathbf{r}, \boldsymbol{\theta}, \boldsymbol{\phi}) = \frac{1}{r\sin(\theta)}\left[\frac{\partial}{\partial \theta}(r B_\phi) - \frac{\partial B_\theta}{\partial \phi}\right]\hat{\mathbf{r}} + \frac{1}{r}\left[\frac{1}{\sin(\theta)}\frac{\partial B_r}{\partial \phi} - \frac{\partial}{\partial r}(r B_\phi)\right]\hat{\boldsymbol{\theta}} + \frac{1}{r}\left[\frac{\partial}{\partial r}(r B_\theta) - \frac{\partial B_r}{\partial \theta}\right]\hat{\boldsymbol{\phi}} \quad (15)$$

$$\boldsymbol{\nabla} \cdot \mathbf{B} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 B_r) + \frac{1}{r\sin(\theta)}\frac{\partial}{\partial \theta}(\sin(\theta) B_\theta) + \frac{1}{r\sin(\theta)}\frac{\partial B_\phi}{\partial \phi} \quad (16)$$

The Maxwell equations in spherical coordinates are particularly useful in finding the magnetic field of a dipole. In this case, the magnetic field has no dependence in the ϕ direction. Under these conditions, where the derivative with respect to ϕ ($\frac{\partial}{\partial \phi}$) is zero, the Maxwell equations simplify to:

$$\boldsymbol{\nabla} \times \mathbf{B}(\mathbf{r}, \boldsymbol{\theta}) = \frac{1}{r\sin(\theta)}\left[\frac{\partial}{\partial \theta}(r B_\phi)\right]\hat{\mathbf{r}} - \frac{1}{r}\left[\frac{\partial}{\partial r}(r B_\phi)\right]\hat{\boldsymbol{\theta}} + \frac{1}{r}\left[\frac{\partial}{\partial r}(r B_\theta) - \frac{\partial B_r}{\partial \theta}\right]\hat{\boldsymbol{\phi}} \quad (17)$$

$$\boldsymbol{\nabla} \cdot \mathbf{B} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 B_r) + \frac{1}{r\sin(\theta)}\frac{\partial}{\partial \theta}(\sin(\theta) B_\theta). \quad (18)$$

As a result of Equation 1, a vector potential can be introduced. The magnetic field written in terms of the vector potential $\mathbf{A}$ is shown as

$$\mathbf{B} = \boldsymbol{\nabla} \times \mathbf{A}. \quad (19)$$

The vector potential can be specifically formulated for a dipole. For a dipole with magnetic dipole moment $\mathbf{m}$, the vector potential can be expressed as

$$A_{dip} = \frac{\mu_0}{4\pi}\frac{\mathbf{m} \times \hat{r}}{r^2} = \frac{\mu_0}{4\pi}\frac{m\sin(\theta)}{r^2}\hat{\phi}. \quad (20)$$

By using the spherical form of Maxwell's equations (Equation 10), and substituting in for $\mathbf{A}$ in Equation 13, the magnetic field of a dipole with magnetic dipole moment $\mathbf{m}$ is:

$$\mathbf{B}_{dip} = \nabla \times \mathbf{A} = \frac{\mu_0 m}{4\pi r^3}\left(2\cos(\theta)\hat{r} + \sin(\theta)\hat{\theta}\right). \quad (21)$$

2.2 Magnetic Field of Solenoid

A solenoid is a tightly wound coil of wire carrying a current that produces a magnetic field. This kind of magnetic source, seen in Figure 2, is often used because of the high uniformity and homogeneity of the field inside of the coils. In addition, the magnetic field inside of a solenoid has no dependence on it's distance from the axis, or on the solenoid's cross sectional area. To calculate the magnetic field for such a source, Ampere's Law (Equation 2) can be used. The integral form of Ampere's law can be written as:

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{enc} \tag{22}$$

where I_{enc} is the current enclosed by an arbitrary loop of current known as an Amperian loop. For the case of an infinitely long solenoid, for which fringe effects can be ignored, a square Amperian loop of length L can be placed in such a way that half of it is inside while the other is left outside. Note that the two sides of the loop perpendicular to the field and the side outside of the solenoid have negligible contributions to the field. By observing that the loop intersects with nL turns of wire (where n is the number of turns per unit length $\frac{N}{L}$), the enclosed current can be determined. If the wires carry a current I, and knowing the line integral is the algebraic sum of currents flowing through the loop, the enclosed current becomes $I_{enc} = nLI$. From this, the magnetic field for the interior of an infinitely long solenoid is determined to be

$$B = \mu_0 I n. \tag{23}$$

To show how this might look when using practical units, take the case in which there is an infinitely long solenoid with current I=1 A, and number of turns per unit length n=100 m^{-1}. If μ_0=4$\pi\times$10^{-7} $\frac{kg}{A^2 s^2}$, plugging these values in to Equation 23 yields B=4$\pi\times$10^{-5}T where T represents units of Tesla. Alternatively, in units of Gauss the value of the magnetic field becomes B=4$\pi\times$10^{-1} G.

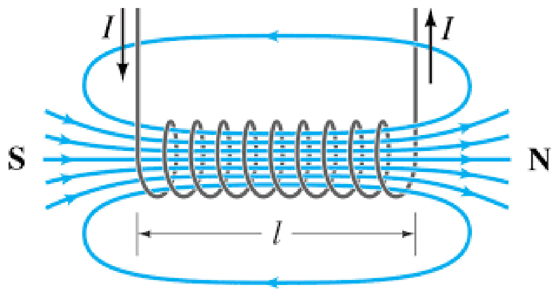


Figure 2: This figure shows the magnetic field of a typical solenoid. See reference [3].

In the case of a short solenoid, the magnetic field along the axis can be determined using the equation:

$$\mathbf{B} = B_\infty \frac{\Delta\Omega}{4\pi} \tag{24}$$

where B_∞ is the magnetic field of an infinite solenoid and $\Delta\Omega$ represents the solid angle of the solenoid. A solid angel is the 3D analog of the 2D plane angle. It is defined by the surface area of a unit sphere contained by a closed curve (or object) that is subtended by the vertex of the angle [2]. The solid angle can be seen as a measure of how large the object looks as observed from that vertex.

2.3 Magnetic Field of a Current Carrying Ring

In order to find the magnetic field a distance x above a ring of current with radius R, the Biot-Savart law can be used. The Biot-Savart law is stated as

$$\mathbf{B(s)} = \frac{\mu_0}{4\pi}\int \frac{\mathbf{I}\times\hat{\mathbf{r}}}{r^2}d\mathbf{l} = \frac{\mu_0}{4\pi}I\int \frac{d\mathbf{l}\times\hat{\mathbf{r}}}{r^2} \tag{25}$$

where the integral is taken along the path of current, in the direction of flow. The quantity d**l** represents an element of length along the wire that contains some current, and **r** is a vector pointing from the source of current (in this case d**l**) to a point **s** at which we would like to find the field. Assume that the ring is aligned along the x-axis. The y and z components cancel out and leave only a x component of the magnetic field. Thus,

$$dB_x = \frac{\mu_0 I dl \sin(\theta_1)}{4\pi r^2}\cos(\theta_2) \tag{26}$$

where θ_1 is the angle between **r** and d**l** (90°), and θ_2 is the angle between d**B** and dB_x. The quantity d**l** and be written in cylindrical coordinates as d**l** = R dϕ, thus changing the bounds of the integral to 0 to 2π. This results in a magnetic field of

$$\mathbf{B} = \int dB_z = \frac{\mu_0 I\cos(\theta_2)}{4\pi r^2}\int_0^{2\pi} Rd\phi = \frac{\mu_0 I\cos(\theta_2)2\pi R}{4\pi r^2} \tag{27}$$

By recognizing certain geometries, it can be determined that $\cos(\theta_2)=\frac{R}{r}$, and from the Pythagorean theorem r is found to be $r=\sqrt{x^2+R^2}$. After making some substitutions, the magnetic field a distance x above a current carrying ring is

$$\mathbf{B} = \frac{\mu_0 I}{2}\frac{R^2}{(x^2+R^2)^{3/2}}. \tag{28}$$

2.4 Magnetic Field Between Two Current Loops

In the case of a single current loop, it was found that the magnetic field can be described by Equation 28. To find the field between two such loops of current the principle of superposition can be applied. If the loops are of radius R, have current I moving in the same direction, and are separated by a distance d where the z-direction is along their common axis and z=0 is directly between them, the expression for the net magnetic field along the axis becomes

$$B = \frac{\mu_0 I R^2}{2}\left[\frac{1}{\left(R^2+(z-\frac{d}{2})^2\right)^{3/2}} + \frac{1}{\left(R^2+(z+\frac{d}{2})^2\right)^{3/2}}\right]. \tag{29}$$

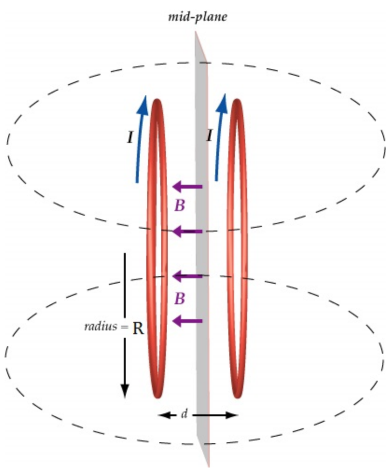
The first term in the expression represents the field contribution from the top loop, while the second term represents the field contribution from the bottom. When these loops are in a particular configuration, known as a Helmholtz pair, non-uniformity is minimized. This configuration requires that the derivative as well as the second derivative of the magnetic field along the axis is 0. In the first case, the derivative becomes:

$$\frac{\partial B}{\partial z} = \frac{\mu_0 I R^2}{2}\left[-\frac{3}{2}\frac{2z-d}{\left(R^2+(z-\frac{d}{2})^2\right)^{5/2}} - \frac{3}{2}\frac{2z+d}{\left(R^2+(z+\frac{d}{2})^2\right)^{5/2}}\right] \tag{30}$$

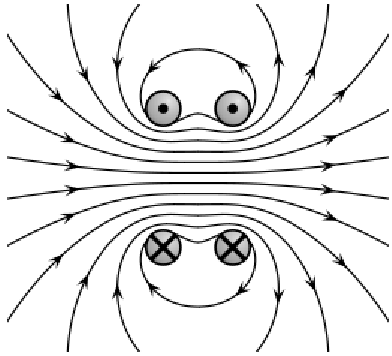
which at z=0 becomes $\frac{\partial B}{\partial z}$=0. The second derivative, at z=0, is then

$$\frac{\partial^2 B}{\partial z^2} = \frac{\mu_0 I R^2}{2}\left[\frac{15}{2}\frac{d^2}{\left(R^2+\frac{d^2}{4}\right)^{7/2}} - \frac{6}{\left(R^2+\frac{d^2}{4}\right)^{7/2}}\right]. \tag{31}$$

Setting this expression equal to zero and then solving for d gives the condition for a Helmholtz pair, which is that at $d = R$, the uniformity of the field is at a maximum.



(a) This figure shows the Helmholtz configuration. See reference [4].



(b) This figure shows a field diagram for a Helmholtz pair. Note that at the center, the field lines are parallel, implying uniformity of the field. See reference [5].

Figure 3

3 Two Coil Magnet For Polarized Target

In particle physics experiments that utilize polarized targets and have high beam intensity, a high magnetic field is required to polarize the target and maintain this polarization under the depolarizing effects due to interaction with the beam . In this case, a field of 5 T is needed [6]. For the particular polarized targets used in this study, a high field uniformity of $\approx 10^{-4}$ in a cylindrical region of length $3cm$ and diameter of $1cm$ is required [6]. The size of this region is determined based on the current designs of the targets.

To perform an efficient physics experiment, there must be not only beam-target interaction, but the product of this interaction (scattered particles) must be able to be observed, in this case, via a large acceptance detector. New interests lie in the physics of processes in the transverse direction. In the current configuration of the magnets, only a small opening angle in this transverse direction is available to allow particles to pass without being blocked by the apparatus itself. In order for more of the particles produced by the beam-target interaction to be observed, a wider opening angle is required. The task of developing a system with a wider angle of observation is particularly difficult if the target is polarized transversely relative to the beam direction and the particle of interest is moving at an angle of 30∘ around the beam, as is the case here (Bogdan Wojtsekhowski, personal communication, Nov. 15, 2016).

There are several magnets used in similar polarized target experiments that are comparable to the one discussed here. Argonne National Laboratory's R & A magnet is designed to produce a field of 2.54 T with relative field homogeneity of ±1.2 G in a 5 cm diameter sphere [7]. The magnet construction allows for a transverse opening of 23°. This magnet can be seen in Figure 4 where the it should be noted that the spacing between the two coils is rather small.

3.1 Explaining Requirements for Uniformity

Uniformity of the magnetic field is a key component in experiments involving polarized targets. The term uniformity refers to the variance of the magnetic field in a specific region. In this case, the specific region is the volume of a polarized target. Uniformity can be quantified by the expression $\frac{\Delta B}{B}$ which yields a value for the uniformity. In other words, the uniformity is the ratio of the change in the field over a specified region and the total field.

For the experiments of interest, the uniformity condition $\frac{\Delta B}{B} \leq 10^{-4}$ must be met [6]. This high degree of uniformity is necessary for achieving target polarization. Polarization is a property of particle spin, and refers to the degree to which the spin of the particles in a system are pointed in a particular direction. Complete polarization would occur when all of the particles have their spins oriented in the same direction. In these experiments, the goal is to create polarized protons. Target materials such as NH_3 have free protons which are easier to polarize than neutrons and bound nucleons. The physical mechanism responsible for creating these polarized protons has to do with the spin-rotation frequency and the frequency of the polarizing electromagnetic waves. The condition for polarization, known as the resonance condition, occurs when the frequency of the proton's spin-rotation is equal to the frequency of the EM waves and can be seen in equation

$$\mu B = \hbar\omega \tag{32}$$

where μ is the effective magnetic moment , B is the local magnetic field, $\hbar$ is Planck's constant, and ω is the frequency of the EM waves. This process of creating polarized protons is known as proton pumping. The magnet discussed in this study is responsible for the pumping of electrons in the target material. Once the electrons have been polarized, the polarization can be transferred to the protons. The high uniformity condition is not important for the polarizing

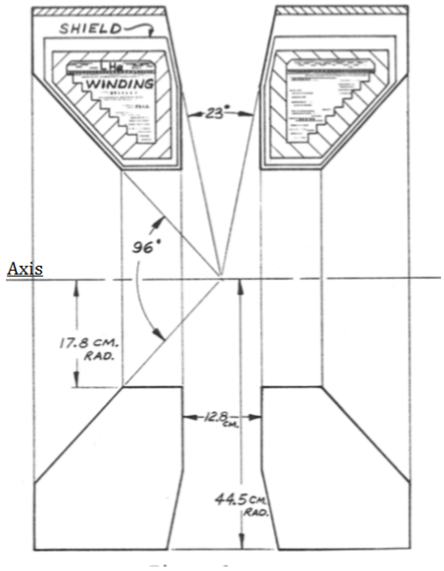


Figure 4: This figure shows a cross-sectional view of a magnet configuration similar to one discussed in study. The particular magnet shown is from a study performed at Argonne National Lab [7].

of the electrons, but if the uniformity is too low, then the transfer of polarization between the electrons and protons will not occur.

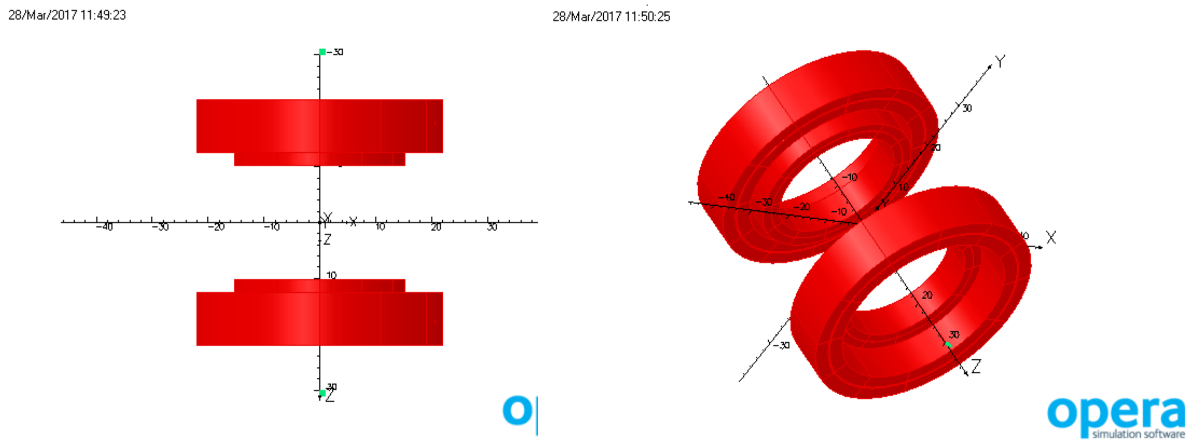
4 Correction to Magnet For Wider Observation Angle

In order to create a wider opening angle for the magnet system, the distance between the primary components of the magnet (the coils) will be increased and corrective coils will be added. As was shown in Section 2.4, the distance separating a Helmholtz pair needed to achieve a high field uniformity is small (equal to the radius of the coils). Thus, when the distance between the coils of the magnet is increased, the non-uniformity of the field increases as well. In order to correct the loss of uniformity, new coils of a specific geometry and current density are added. When the field is small, say on the order of a few Gauss, the calculation of the field can be done analytically, but in this case the field is on the order of 5 T and the geometry is complicated. Due to the high field and complex geometry, a numerical solution for the magnetic field is required. Using a magnetostatics calculation program known as TOSCA, the parameters of the existing coils, along with the specifications of the new correction coils will be used to find a solution.

4.1 Numerical Tools Used in Solution

To find a solution to a field of such a high order, numerical devices are required. For this purpose, the magnetostatics program known as Tosca was used. Tosca uses finite-element analysis to analyze and find solutions to 3 dimensional electrostatic and magnetostatic fields. Finite element analysis is a numerical method used to find solutions to boundary condition problems for partial differential equations. The method breaks a problem up into smaller pieces, called finite elements, and the equations modeling theses elements are then reassembled into a larger system that models the problem as a whole. Using the finite element method allows the user to find solutions to a diverse range of problems, including ones involving complex geometries. Tosca specifically uses a formulation based on reduced and total scalar potentials to solve even non-linear problems [8].

TOSCA's ability to solve systems with complicated geometries makes it a perfect choice to calculate the numerics for the magnet. To test that TOSCA was yielding acceptable results, the relevant physics problems mentioned in Section 2 were used. These problems, whose solutions are well known, were solved analytically and then solved again using Tosca. The results from both calculations were then compared and were found to be in agreement. TOSCA was then used to model the magnet of interest, seen in Figure 5 without the corrective coils, as well as the magnet with the newly designed corrective coils.

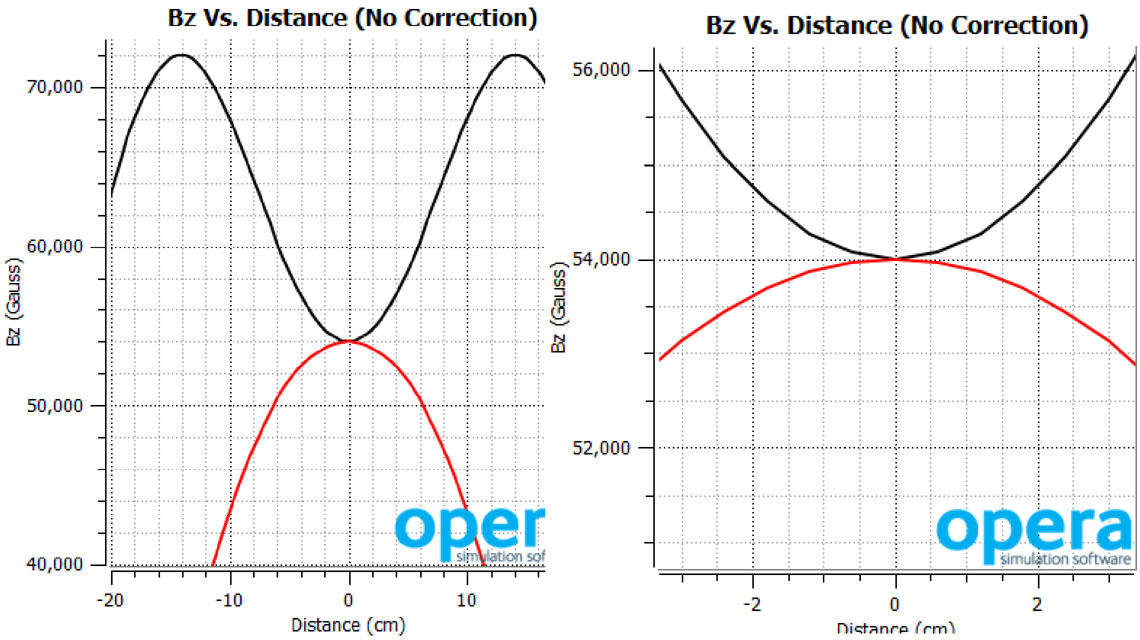


(a) (b)

Figure 5: This figure shows a model of the uncorrected coil system generated using TOSCA

5 Non-uniformity of Standard Magnet

The magnet to be used in this study was designed to produce a maximum field of 5 T. The original spacing between the coils was 10 cm, resulting in a transverse aperture of $\pm 17°$ or a total angle of $34°$. In order to achieve a significant increase in this angle, the distance between the coils was doubled to 20 cm and the cross-section along the axis of the coils (z-axis) was doubled as well. This yielded a new open angle of $\approx 50°$. A model of the coils created using TOSCA can be seen in Figure 5. By moving the coils apart, a significant loss in uniformity was experienced. In Fig. 6, a plot of the z component of the magnetic field along the axis of the coils and the beam direction can be seen. This plot shows the loss of uniformity in the region of interest (2 cm-3 cm at the center). Using the expression for uniformity noted in Section 3.1 ($\frac{\Delta B}{B}$), the new uniformity of the system was found. For a target size of $\pm$ 1 cm with a variation on the order of



(a) (b)

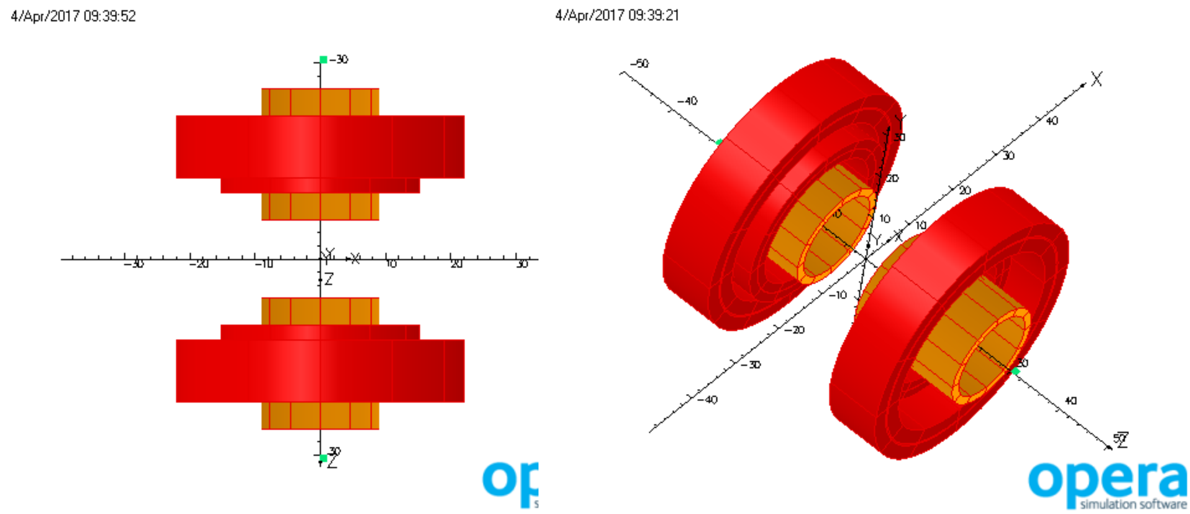
Figure 6: This figure shows plots of the B_z component of the magnetic field plotted against distance along the axis. The black line represents measurements along the axis of the coils (z-axis), and the red represents measurements along the direction of the beam (x-axis).

200 G (gauss), the uniformity is estimated to be $\frac{\Delta B}{B}= 200G/54000G = 3.7\times10^{-3}$, which does not meet the condition of 10^{-4}.

6 Correction to Non-uniformity of Standard Magnet

By increasing the distance between the coils of the magnet without scaling the entire system, a larger opening angle was achieved. However, these changes resulted in a substantial reduction in the field uniformity such that the current uniformity will not support the necessary polarization of the target. Without a properly polarized target, no experiments can be performed. To solve this, TOSCA was used to develop a pair of correction solenoids. In Figure 7, the system with the correction coils can be seen, where the correction coils are the entities highlighted in orange. These solenoids have the following dimensions: an inner radius of R_{inner}=7.5 cm, an outer radius of R_{outer}=9 cm, and length along the axis z=20 cm. While the physical dimensions of the coils are the same, they have equal but opposite current densities of $\mathbf{J}=\pm 29770.3$ A/cm^2.

Using TOSCA, plots of the B_z field component were made. These can be seen in Figure 8 where the black line represents the B_z component along the axis of the coils (z-axis), and the red line represents the same component measured along the direction of the beam (x-axis). Once again utilizing the expression for uniformity mentioned in Section 3.1, the uniformity for a central region of ±1 cm was determined. The result of this calculation was $\frac{\Delta B}{B}$=0.3 G/41000 G$\approx 1\times10^{-5}$,



(a) (b)

Figure 7: This figure shows the corrected coil system. The new correction solenoid is seen highlighted in orange.

which is an order of magnitude smaller than the required uniformity condition of $\frac{\Delta B}{B} \leq 10^{-4}$.

7 Conclusions

Particle physics experiments that utilize polarized targets necessitate a highly uniform magnetic field in order to achieve polarization, and then maintain this polarization in the presence of the depolarizing effects of radiation produced by high beam intensities. Currently, the Nuclear Physics community has interest in the processes in the forward beam direction where the target is transversely polarized with respect to the direction of the beam. Due to the requirements of the Helmholtz configuration, which provides the conditions for high field uniformity, and the necessary construction elements of the polarizing magnets, the transverse aperture of the coils is small and limits the detection of escaping particles.

An obvious solution to this problem would be to simply move the coils further apart. By doubling the distance between the coils of the magnet from 10 cm to 20 cm and increasing the length if the coils along their axis, the opening angle in the transverse direction saw significant improvement, increasing from 34° to ≈50°. However, by changing these parameters without scaling the entire system, the necessarily high uniformity of the field was sacrificed, dropping to 3.7×10^{-3} which is a factor of 10 from the uniformity requirement of $\frac{\Delta B}{B} \leq 10^{-4}$. To solve this problem, the magnetostatics calculation program TOSCA was used to model the system and determine a numerical solution. With TOSCA, a solution was found using two correction solenoids with equal but opposite current densities. With the addition of these new coils, the uniformity saw a dramatic improvement to $\approx 1 \times 10^{-5}$.

In the past, many experiments have been performed where the target polarization (polarization of the proton) was in the direction of the beam. Using the results of these experiments, nuclear physicist are able to better understand the

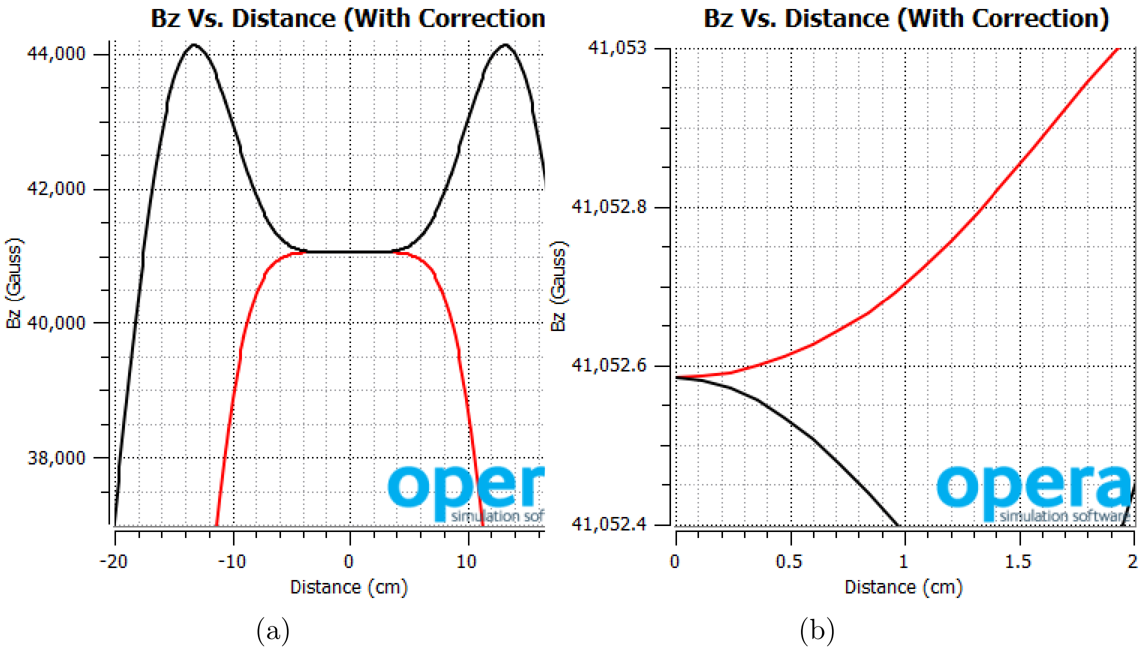


Figure 8: This figure shows plots of B_z versus distance after the addition of the correction coil. Black line is along axis of coils, red is along beam direction. The uniformity was calculated using data from 8(b)

forces holding quarks together, such as in the study of quantum chromodynamics or QCD. To have a complete understanding of these forces, it is necessary to perform experiments in which the target polarization is transverse to the beam direction. With the addition of these correcting coils, the physics of processes in the forward beam direction for a transversely polarized target can now be explored. By adding these solenoids, it prevents the complete redesign of the magnets and allows for data collection in regions that were previously unavailable.

8 Acknowledgements

I would like to acknowledge Bogdan Wojtsekhowski for his tremendous help, guidance, and contribution to this project, as well as for the many enlightening conversations that resulted in it's completion. I would also like to acknowledge Todd Averett for a number of helpful discussions, and for keeping the project on track so that deadlines were met and the work was finished on time.

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