Using COMSOL to Model a 1-D Lithium-Ion Battery

A thesis submitted in partial fulfillment of the requirement
for the degree of Bachelor of Science in
Physics from the College of William and Mary in Virginia,

by

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Abstract

The William and Mary Department of Physics conducted research to model lithium-ion batteries. We built a one-dimensional model of a lithium-ion battery using a multiphysics program called COMSOL [1]. This model produces graphs of electrolyte concentration, voltage, and discharge rates of the battery over time. To further understand the model, we varied volumetric fraction (analogous to porosity) of the positive and negative electrodes, electric conductivity of the positive and negative electrodes, diffusion coefficient of the battery separator, and discharge current to observe their effects on concentration, voltage, and discharge rates. We found that volumetric fraction of both electrodes, electric conductivity of the positive electrode, and discharge current have significant effects on concentration and voltage, while diffusion has significant effects on discharge rates. The electric conductivity of the negative electrode did not have significant effects on any of the dependent variables. With these results, further understanding of battery models can be gained in hope of eventual construction of efficient lithium-ion batteries for commercial use.

Chapter 1

Introduction

1.1 Goals

In today's energy-dependent world, the ability to generate power is vital to maintain our society's need for electricity. To promote further research in the field of battery development, the ability to model a battery proves useful. To successfully model a lithium-ion battery, this project uses a program called COMSOL, which is a GUI-based modeling platform that can analyze complex physical properties and collect a wide range of data on complicated systems. It was developed in 1986 to assist engineers with product design [1]. With COMSOL, a one-dimensional model of a lithium ion battery can be built and then analyzed by experimenting with the effects of different variables on the battery output.

The goal of this research is to understand the modeling process, successfully build a model of a 1D lithium-ion battery, and test the output of the model by playing with different variables. The model-building process requires an understanding of the differential equations used to describe electrochemical flows and the physics of a battery. Chengjun Xu has made considerable strides in this subject through his research at Tsinghua University in Guangdong Province, China. His research was thoroughly studied in preparation for this project [2] [3] [4].

The ultimate goal in this project is to lay the foundation for future work on creating

alternate and more efficient batteries for commercial use. With a reliable model, further modelling in multiple dimensions can be researched with eventual battery design, testing, and construction.

1.2 Background

Batteries convert chemical energy into electrical energy using electrochemical oxidation-reduction reactions [5]. Our one-dimensional battery is made up of three sections: the negative electrode (made out of a graphite material), the separator (made out of a polymer electrolyte), and the positive electrode (made out of lithium magnesium oxide). The negative electrode gives up electrons and Lithium ions. The electrons traverse across the circuit while the lithium ions travel across the separator to meet at the positive electrode. This process can be seen in Figure 1.1, which comes from the COMSOL 1D battery model information manual [6].
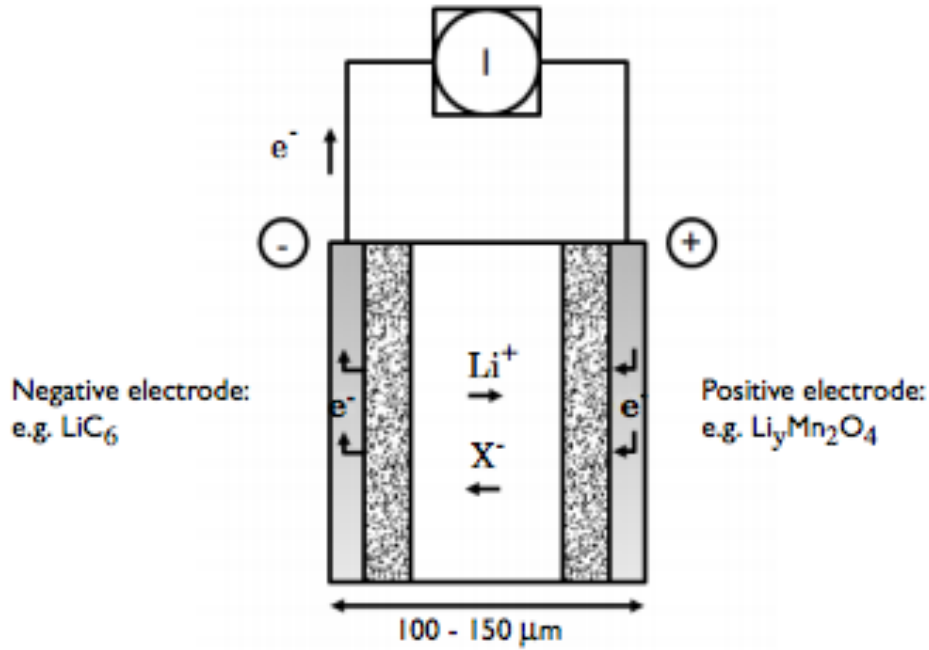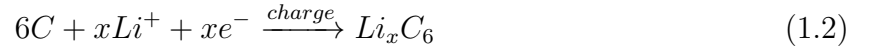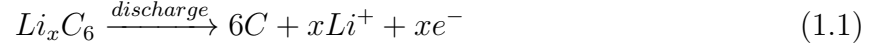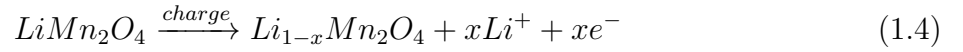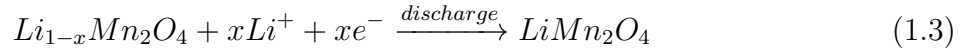

Figure 1.1: Mechanics of a one-dimensional battery

The chemical reactions at the negative electrodes during charging and discharging phases is given by [7]:

$$Li_xC_6 \xrightarrow{discharge} 6C + xLi^+ + xe^- \tag{1.1}$$

$$6C + xLi^+ + xe^- \xrightarrow{charge} Li_xC_6 \tag{1.2}$$

The chemical reactions at the positive electrode during charging and discharging phases is given by [7]:

$$Li_{1-x}Mn_2O_4 + xLi^+ + xe^- \xrightarrow{discharge} LiMn_2O_4 \tag{1.3}$$

$$LiMn_2O_4 \xrightarrow{charge} Li_{1-x}Mn_2O_4 + xLi^+ + xe^- \tag{1.4}$$

Several governing equations are used by the COMSOL model for prediction. In the electrolyte region, the governing equations are:

$$\frac{\partial c_l}{\partial t} + \frac{\partial N_l}{\partial x} = R_l \tag{1.5}$$

$$i_l = -\sigma_l \frac{\partial \phi_l}{\partial x} + \frac{2\sigma_l RT}{F}(1 + \frac{\partial \ln f}{\partial \ln c_l})(1 - t_+)\frac{\partial \ln c_l}{\partial x} \tag{1.6}$$

$$N_l = -D_l \frac{\partial c_l}{\partial x} + \frac{i_l t_+}{F} \tag{1.7}$$

The variables in these governing equations are defined as follows:

- c_l is the electrolyte salt concentration

- N_l is the number of ions

- R_l is the contact resistance between the electrode and current collectors [7]

- i_l is the ion transfer current across the electrode

- σ_l is the electric conductivity of the electrolyte

- ϕ_l is the electrolyte potential

- R is the universal gas constant

- T is the temperature of the isothermal battery

- F is the Faradic constant

- $\frac{\partial \ln f}{\partial \ln c_l}$ is the activity dependence. "f", in this case, is the mean molar activity coefficient, which is assumed to be constant [7]

- t_+ is the transport number

- D_l is the diffusion coefficient

Equation 1.5 tells us that the change in electrolyte salt concentration with respect to time and the change of number of ions with respect to battery length equal a constant, the contact resistance between the electrode and current collectors. Equation 1.6 describes electrical current flow across the battery. It shows that current comes from the potential gradient and diffusion due to the concentration gradient. According to David Linden, the transport number here "determines the fraction of the electric energy resulting from the displacement of the potential from the equilibrium value" [5]. This number essentially regulates the rate of electrochemical transformation. In other words, it tells us what fraction of Lithium ions

travel across the separator. Equation 1.7 defines the lithium ion current flow as equal to the concentration current density added to a fraction of the net current flow adjusted for units.

In the porous electrode regions, the governing equations are:

$$\frac{\partial \epsilon_l c_l}{\partial t} + \frac{\partial N_l}{\partial x} = R_l \tag{1.8}$$

$$i_l = -\sigma_{l,eff}\frac{\partial \phi_l}{\partial x} + \frac{2\sigma_{l,eff}RT}{F}(1 + \frac{\partial lnf}{\partial lnc_l})(1 - t_+)\frac{\partial lnc_l}{\partial x} \tag{1.9}$$

$$N_l = -D_{l,eff}\frac{\partial c_l}{\partial x} + \frac{i_l t_+}{F} \tag{1.10}$$

Here, ϵ_l is the volumetric fraction (analogous to porosity), $\sigma_{l,eff} = \epsilon_l^{1.5} \sigma_l$ is the effective electrical conductivity in the porous electrode, and $D_{l,eff} = \epsilon_l^{1.5}D_l$ is the effective diffusivity coefficient in the porous electrode. These effective values are used to account for both porosity and tortuosity. The factor of 1.5 is the Bruggeman coefficient, which is set to 1.5 to model a packed bed of spherical particles [6]. Bruggeman's coefficient essentially models the change in electric conductivity and diffusivity caused by tightly packed spheres touching each other.

The model used for this experiment comes from the COMSOL website. It is a one-dimensional, isothermal system that models electronic current conduction in the electrodes, ion transport across the battery, material transport in the electrolyte, and Butler-Volmer electrode kinetics using experimentally measured discharge curves for the equilibrium potential [6]. A picture of the model as it appears in the COMSOL program can be seen in Figure 1.2. The model is split into three sections in the following order: the negative electrode of length 10^{-4} meters, the separator of length 0.52×10^{-4} meters, and the positive electrode of length 1.83×10^{-4} meters.
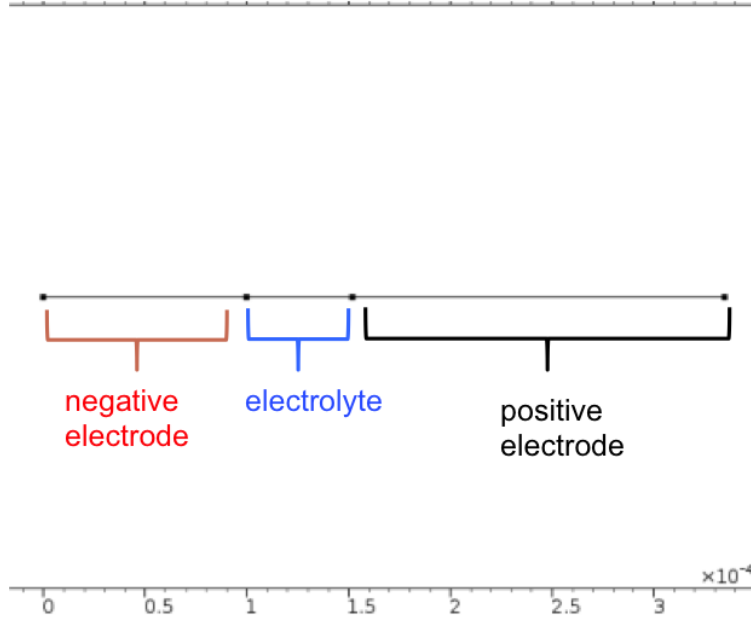
Figure 1.2: The one-dimensional battery as it appears on COMSOL, with labels of the parts of the battery

The model comes with default properties associated with the materials used to construct it. In our model, the negative electrode has a volumetric fraction of $\epsilon_l^{neg} = 0.357$ and an electrical conductivity of $\sigma_l^{neg} = 100 S/m$. The positive electrode has a volumetric fraction of $\epsilon_l^{pos} = 0.444$ and an electrical conductivity of $\sigma_l^{pos} = 3.8 S/m$. The electrolyte has a diffusion constant of 7.5e-11 m^2/s.

The electrical conductivity values are not as high as they are for metals, which have values on the order of magnitude of 10^7 S/m [8]. At the same time, the electrical conductivities are not as low as they are for insulators, as materials such as wood have electrical conductivities on the order of magnitude of $10^{-16} S/m$ [8]. The electrical conductivity of the positive electrode is closer to the electrical conductivity of sea water, which is 4.8 S/m [8]. The diffusion coefficient is also on the lower side. The diffusion of gas through water is on the order of magnitude of $10^{-9} m^2/s$ [9].

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Chapter 2

Methods

Much of the fall semester was spent researching the theory behind the mechanics of electrochemical flows in batteries. Three research papers written by Chengjun Xu, *Dynamics of Electrochemical Flows I: Microscopic Governing Equations of Flows* [2], *Dynamics of Electrochemical Flows II: Electrochemical Flows-Through Porous Electrode* [3], and *Dynamics of Electrochemical Flows III: Closure Models* [4] were carefully studied. Several potential partial differential equations were identified for possible use in the COMSOL model. However, the efforts from the fall semester proved inconclusive as instructions for building models can be easily found on the COMSOL and the equations found in Xu's papers were largely unused in the COMSOL models.

The second semester was, at first, spent building a three-dimensional model of a lithium ion battery. This was done using instructions and on-line literature from the COMSOL website [10]. Unfortunately, the three-dimensional battery took roughly 30 minutes to perform any calculations. It was also an immensely complex model with so many moving parts that it was difficult to pin down what was going on and what the important variables were. Therefore, the decision was made to reduce the scope of the model from a three-dimensional lithium-ion battery to a one-dimensional lithium-ion battery.

The one-dimensional model was built with the assistance of online literature and forums

on the COMSOL website [6]. It has a calculation run-time of thirty seconds, vastly speed-
ing up the time needed to perform calculations compared to the three-dimensional model.
The variables required and the equations needed to successfully model the battery are also
vastly reduced in numbers and complexity. With this new, simplified model operating more
efficiently than previous models, we were able to run tests to determine the effect on battery
output.

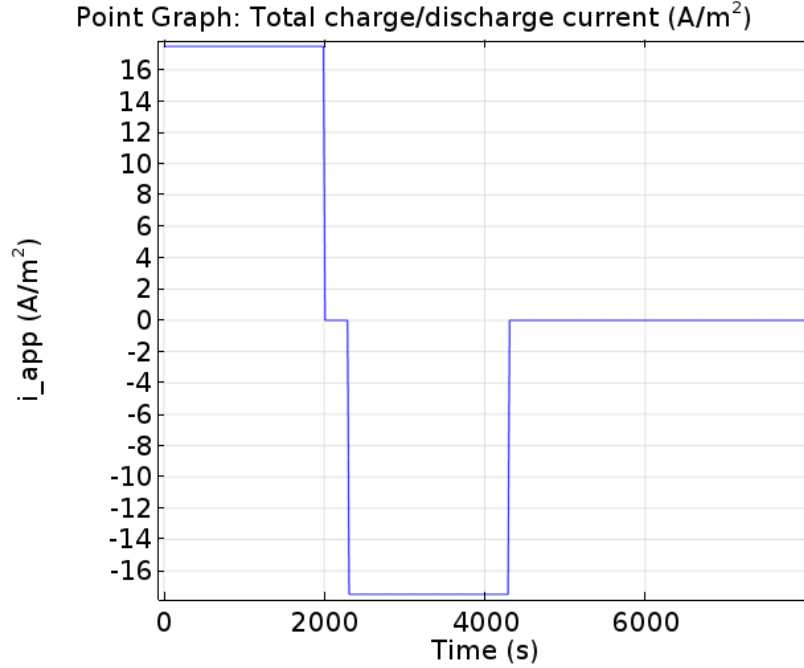To test the model, we ran the battery through a discharge/charge cycle seen in Figure
2.1.

Figure 2.1: Current charging cycle applied to battery

The cycle begins with the battery discharging for 2000 seconds, then functioning as an
open circuit for 300 seconds, then charging for 2000 seconds before returning as an open
circuit. In Figure 2.1, i_app represents the total charge/discharge current function, which is
a step function of magnitude of the default discharge current. The default discharge current

equals 17.5 A/m^2, implying the charging current is -17.5 A/m^2, which is roughly equivalent to the current output of a watch battery.

With the battery operating using this discharge cycle, the next step was to alter different variables to observe the effect on the concentration gradient, cycle voltage, and discharge-curves of the battery. The concentration, voltage, and discharge curves were of interest because of their potential usefulness in constructing lithium-ion batteries in future research. The variables altered were the volumetric fraction, ϵ_l, of the positive and negative electrodes, the electric conductivity, σ_l, of the positive and negative electrodes, the diffusion coefficient, D_l, of the electrolyte, and the discharge current, i_{1C}. These variables were chosen based on their relationship to concentration, voltage, and discharge seen in the governing equations. They were also the variables that COMSOL allowed for manipulation without the system crashing. A helpful table on which variables were tested on which output can be seen in Table 2.1 below.

Table 2.1: List of Results from Tests on 1D Battery

	Volumetric Fraction	(Neg) Electrical Conductivity	(Pos) Electrical Conductivity	Discharge Current	Diffusion Coefficient
Concentration Gradient	See Figure 3.2	No Impact	See Figure 3.4	See Figure 3.6	No Impact
Cycle Voltage	See Figure 3.3	No Impact	See Figure 3.5	See Figure 3.7	No Impact
Discharge Curve	No Impact	No Impact	No Impact	No Impact	See Figure 3.8

The purpose of this analysis was to discover which terms are important and which can be disregarded. In other words, we want to know how different properties of the battery affect the battery performance. This analysis also gives key insights into the internal physics of the battery model.

Chapter 3

Results

3.1 Volumetric Fraction (ϵ_l)

First, we examine the effects that meddling with volumetric fraction values of the positive and negative electrodes (ϵ_l^{pos} and ϵ_l^{neg}) has on electrolytic salt concentration gradient, voltage output, and discharge curve graphs. We start by looking at the effects on the concentration gradient graphs. The default, base values of volumetric fraction are $\epsilon_l^{pos} = 0.444$ and $\epsilon_l^{neg} = 0.357$, which correspond to the physical properties of our electrodes. The salt concentration graphs display the concentration of electrolytic salt along the length of the battery at various times. The times of 1200s, 2200s, 2400s, 4300s, 4400s, and 6000s were chosen because they are key times that correspond with transitions in the discharge cycle seen in Figure 2.1. The cross section of the electrolytic salt concentration with the default volumetric fraction values can be seen in Figure 3.1.

When examining a variable along the x-axis in this battery, it is important to remember from Figure 1.2 that the negative electrode is 10^{-4} m long, the separator is 0.5×10^{-4} m long, and the positive electrode is 1.83×10^{-4} m long in that order. So the transitions from the negative electrode to separator to positive electrode are easily observed in Figure 3.1 at x=1 and x=1.5. Examining the figure, we see the concentration of the electrolytic salt is higher
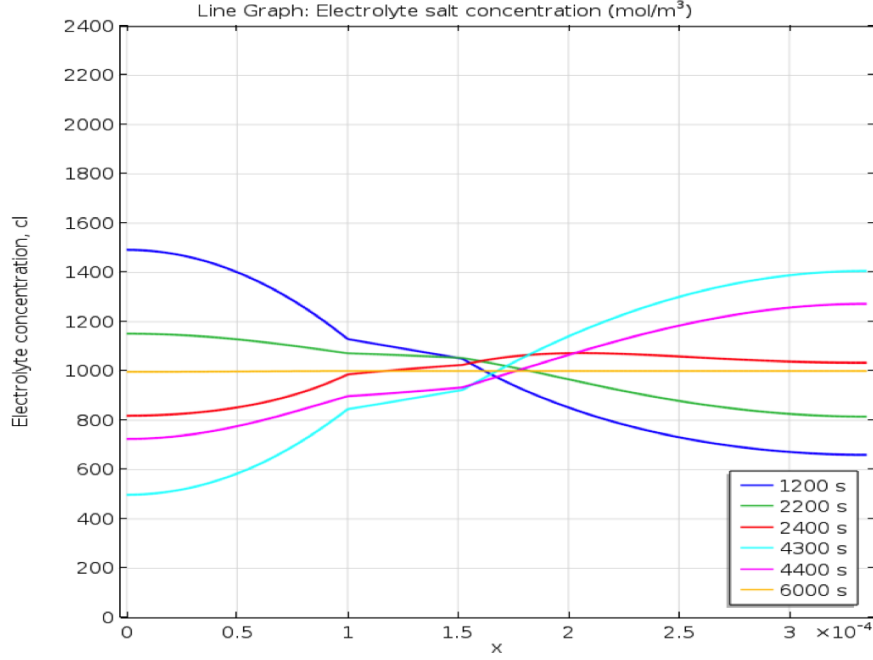
Figure 3.1: Electrolytic salt concentration at different times. Default values of volumetric fraction were used, i.e $\epsilon_l^{pos} = 0.444$ and $\epsilon_l^{neg} = 0.357$.

in the negative electrode during the discharge phase at t=1200 and t=2200 seconds. When the circuit is open, the concentration gradient moves towards a constant concentration along the whole profile. This constant concentration value can be seen at t=6000 seconds when the circuit has been open for 1700 seconds. During the charging phase, the concentration profile skews towards the positive electrode, as seen in the figure at 2400, 4300, and 4400 seconds.

Now that the default setup for concentration gradient has been established, it is time to alter volumetric fraction values. The results of increasing and decreasing the volumetric fraction of the positive and negative electrodes can be seen in Figure 3.2 below.
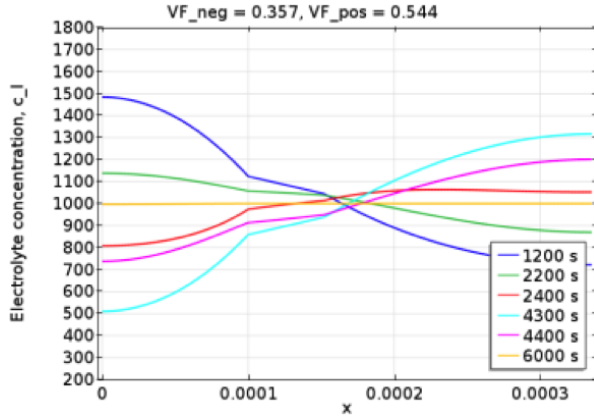
Comparing Figure 3.2 to the default volumetric constant values found in Figure 3.1, we can see that the volumetric constant has an inverse relationship with concentration. In other words, as ϵ_l^{pos} and ϵ_l^{neg} increase, the concentration in the regions of the porous elec-
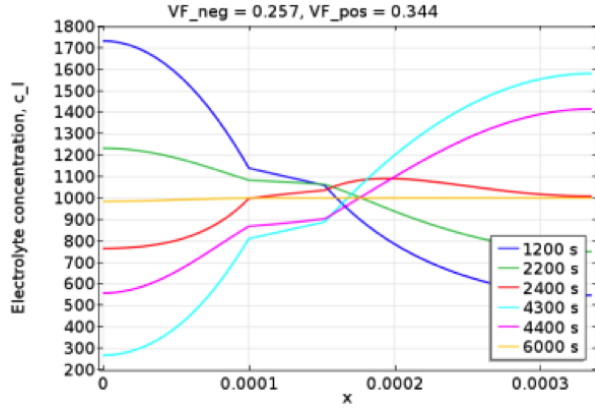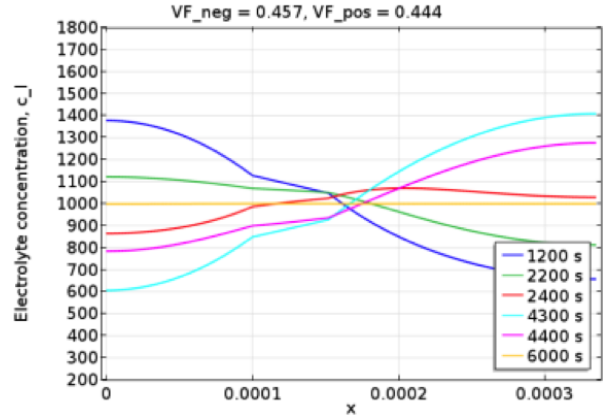
(a) ϵ_l^{pos} and ϵ_l^{neg} reduced by 0.2
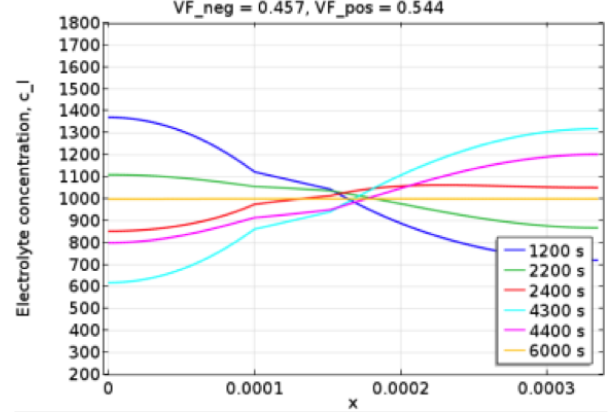
(b) ϵ_l^{pos} and ϵ_l^{neg} increased by 0.2

(c) ϵ_l^{pos} increased by 0.2 only

(d) ϵ_l^{neg} increased by 0.2 only

Figure 3.2: Effects of varying the volumetric fraction, ϵ_l, of the positive and negative electrodes on the concentration gradient profile

trodes decreases. Moreover, the effect of changing the porosity of an electrode only affects the concentration within that respective electrode. This can be seen in Figure 3.2c and d. When only one electrode increases in volumetric fraction, the concentration values in that electrode are reduced, while the concentration in the unchanged electrode are unaffected. It is important to note that the concentration along the battery separator is not affected and so the boundaries between the electrodes and the separator remain at a constant concentration despite changes in volumetric constant. An explanation for the behavior in Figure 3.2

could be that as volumetric fraction increases, there is more space between particles in the electrode. This could make it harder for charge to carry through the electrodes, resulting in a decreased potential difference across the battery. As a result, less salt particles are pulled into the electrodes by the potential difference across the battery at higher volumetric fractions.

Next, we tested the effects of volumetric fraction on output voltage. The output voltage with respect to time at the five volumetric fraction value combinations in Figures 3.1 and 3.2 can be seen in Figure 3.3.
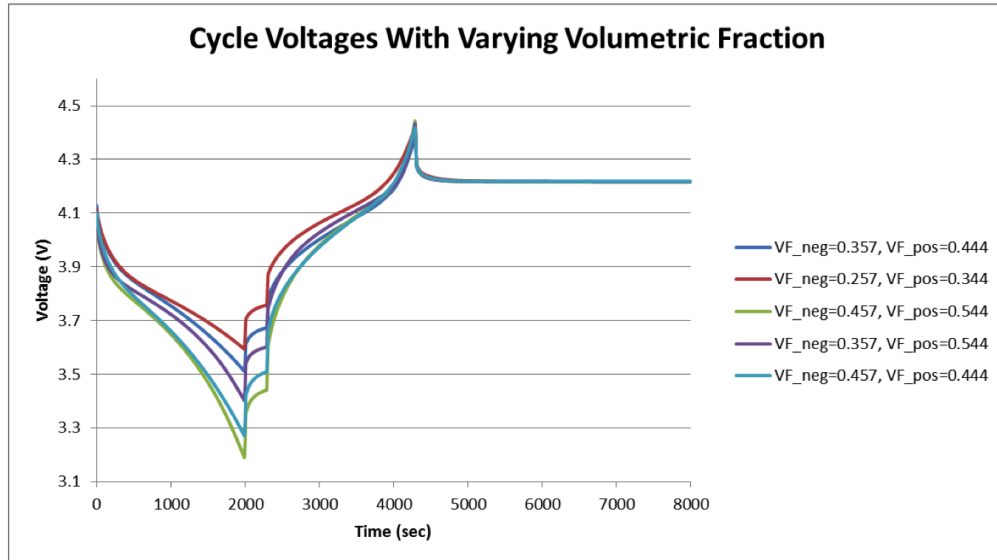

Figure 3.3: Effects of volumetric fraction of positive and negative electrode on output voltage

It is important to understand what this graph displays. Looking at a single curve, clear discharge, open circuit, and charging sections can be seen, and the times of transition correspond to the transitions seen in Figure 2.1. As the battery discharges, voltage steadily decreases until t=2000 when the battery switches to an open circuit. At that moment, there is an instantaneous jump of voltage, followed by a slower increase of voltage during the open circuit period caused by a decrease in concentration [6]. The charging process causes the

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voltage to spike, then gradually increase until the circuit is opened again at t=4300.

Once the cause of the shape of each curve is understood, we can move onto analyzing the difference in curves with changing volumetric fraction. Each shift to the volumetric fraction causes changes to voltage values and the charging and discharging rates. When both positive and negative electrode volumetric fraction values are higher, the voltage dips lower during the discharge phase. Then, there is a greater jump in voltage at t=2000 seconds when the volumetric fraction is large for both electrodes than when it is small. Interestingly, however, the output voltage reaches the same maximum value at t=4300 for all volumetric fraction values. This can be explained in a similar way as the explanation for the concentration gradient graphs. A higher volumetric fraction could increase the open space in an electrode and therefore make it harder for charge to travel through the electrodes. As a result, the battery cannot output as much voltage with a high volumetric fraction in the positive and negative electrodes.

Lastly, when volumetric fraction was tested on discharge curves, no impact was seen. As a result, it can be concluded that volumetric fraction did not have a significant effect on the discharge curve graphs.
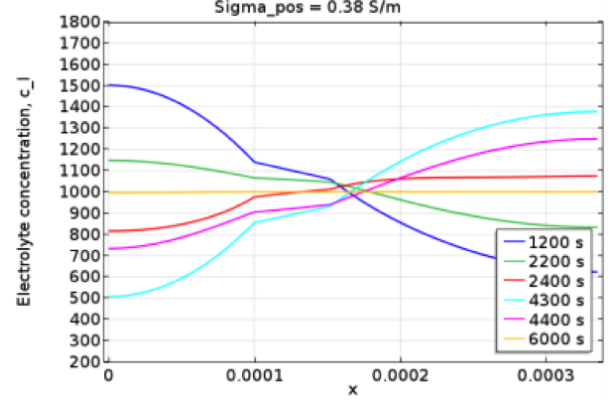
3.2   Electric Conductivity (σ_l)

Next, we look at the effect of changing the electric conductivity of the negative and positive electrodes. After running tests, the effects of varying the electric conductivity of the negative electrode was found to be negligible for all graphs. Therefore, it can be said that the electrical conductivity of the negative electrode does not have a significant effect on concentration gradient, output voltage, or discharge curves. However, altering values of electric conductivity in the positive electrode did have effects on concentration gradient and output voltage. The results of varying the positive electrode's electric conductivity on
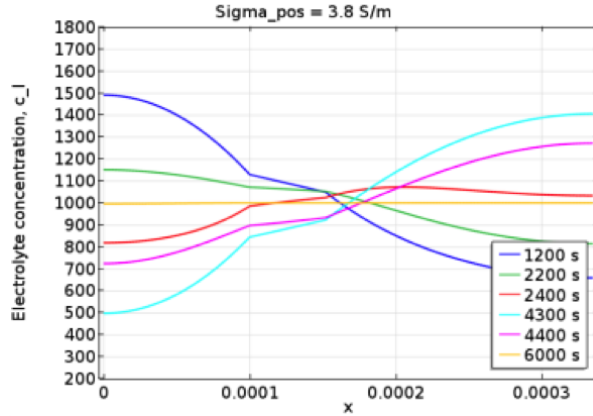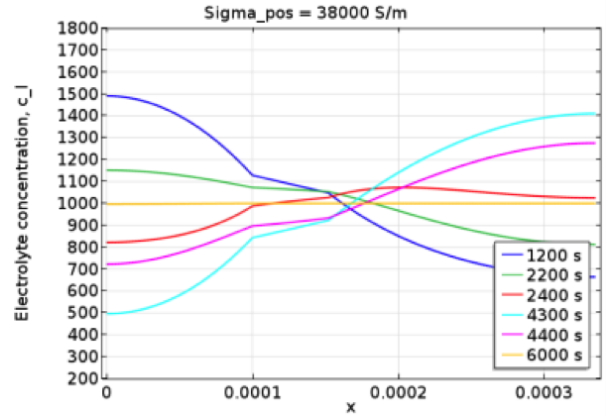
concentration gradient can be found in Figure 3.4.
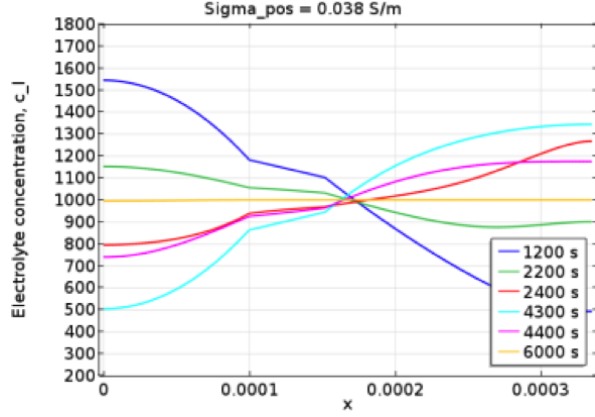
(a) Concentration Gradient at σ_l^{pos}=0.038 S/m

(b) Concentration Gradient at σ_l^{pos}=0.38 S/m

(c) Concentration Gradient at σ_l^{pos}=3.8 S/m

(d) Concentration Gradient at σ_l^{pos}=38000 S/m

Figure 3.4: Effects of varying the Electrical Conductivity, σ_l, of the positive electrode on the concentration gradient

Four values of σ_l^{pos} are graphed at different orders of magnitude. Observing these graphs, it can be seen that a low electric conductivity causes irregularities in the concentration gradient in the positive electrode. As the conductivity increases, these irregularities become less pronounced until changes in large electric conductivities have little effect on the concentration profile. The difference between $\sigma_l^{pos} = 3.8$ S/m and $\sigma_l^{pos} = 38000$ S/m is very small, so we can assume that when electrical conductivity is large, changes in electrical

conductivity do not change the concentration gradient. This makes sense because at small electrical conductivities, the electrode cannot conduct electricity as efficiently, which cause irregularities in the concentration gradient. However, at large conductivities, charge can flow freely through the electrodes so changes do not have much of an effect since there is little resistance to movement.

Next, the electric conductivity of the positive electrode was tested on the cycle voltage, and the effect can be seen in Figure 3.8. As with concentration, the greatest effects occurred at low electric conductivities, while changes in high electric conductivities do not change the output. The lower conductivity also increases the range of voltages in the cycle and changes the shape of the discharging and charging phases. This makes sense because at lower conductivities, it is harder for ions to flow through the electrode, so the battery must expend more voltage to keep the discharge and charging current at a constant value. As a result, the battery must discharge more voltage and charge more voltage, which can be seen in Figure 3.5
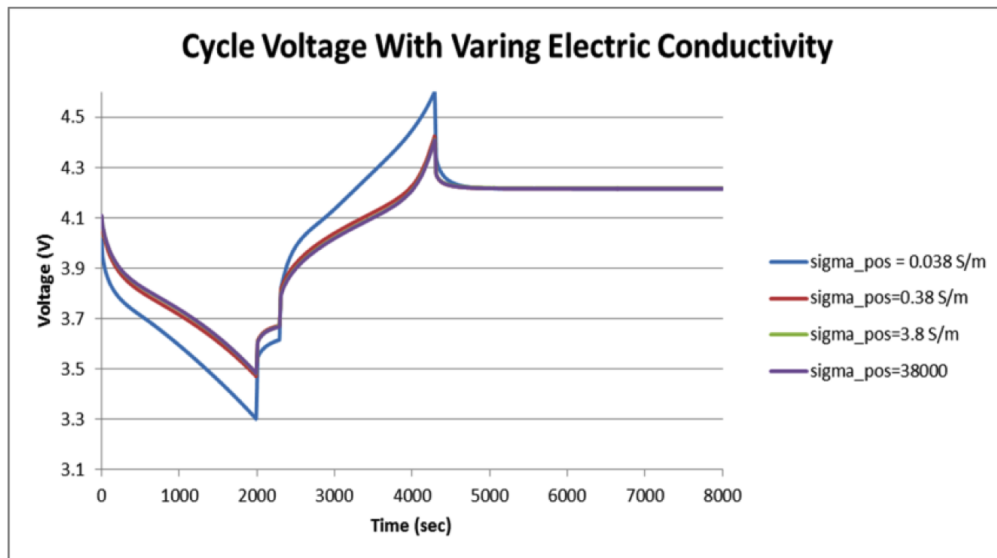

Figure 3.5: Effects of electric conductivity, σ_l, of the positive electrode on output voltage

Lastly, the electric conductivity of the positive electrode was tested on the discharge curves, however, no significant change could be detected. Therefore, we can say that electric conductivity in the positive electrode does not have a major effect on discharge curves.
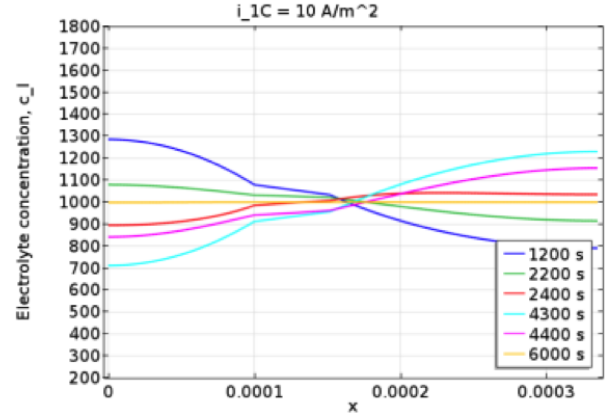
3.3 Discharge Current (i_{1c})

Next, we examine the effect of changing the discharge current on the concentration gradient, output voltage, and discharge curves. Remember, the discharge current is the constant current flowing through the circuit during discharge, seen in Figure 2.1. The default value for discharge current is 17.5 A/m^2.

Four graphs of concentration gradient are provided in Figure 3.6 corresponding to values of 1, 10, 17.5, and 27 A/m^2. While it may be difficult to see because the vertical axes are equal for all four graphs, as discharge current increases, the shape of the discharge curves stay the same but the concentration values magnify like an expanding accordion. Thus, i_{1C} more or less preserves the shape of the concentration gradient at various times, but the range of concentration values is drastically reduced with a decrease in i_{1C}. This makes sense because if little current is need from the battery, a large change in the concentration gradient along the length of the battery is not needed to output the necessary current. But if the circuit requires a lot of current, a much larger gradient from the positive to negative electrode will result.
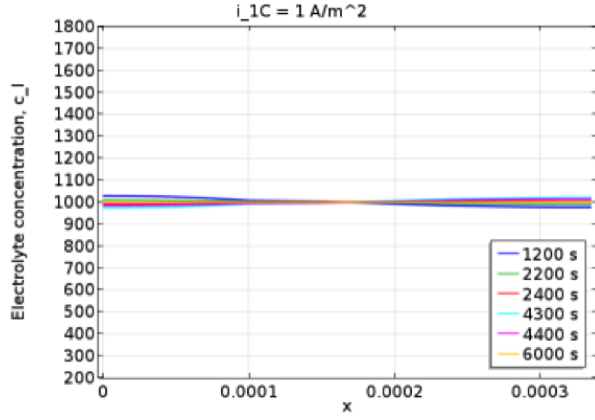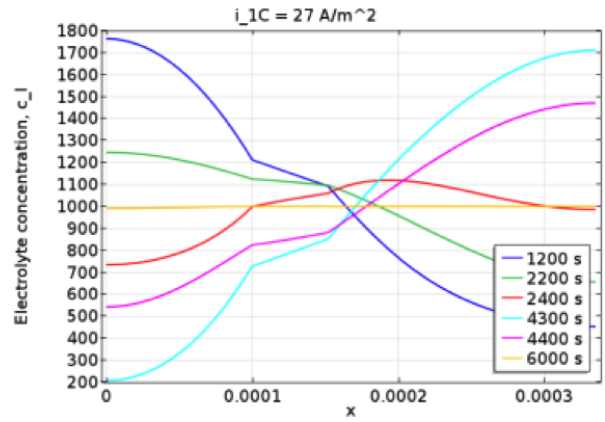
(a) Concentration Gradient at i_{1C}=1 A/m^2

(b) Concentration Gradient at i_{1C}=10 A/m^2
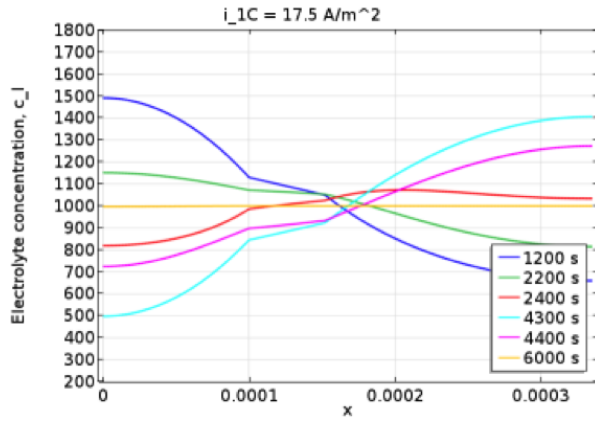
(c) Concentration Gradient at i_{1C}=17.5 A/m^2

(d) Concentration Gradient at i_{1C}=29 A/m^2

Figure 3.6: Effects of varying the discharge current, i_{1C}, on the concentration gradient

Next, the discharge current was tested on cycle voltage, as seen in Figure 3.7. With increasing i_{1C}, the discharge and charging voltage curves become less linear and span a wider voltage range. The range of voltages corresponding to $i_{1C} = 1$ A/m^2, is only 0.12V, while the range is 1.9V for $i_{1C} = 29$ A/m^2. The voltage spike from switching from discharging to open circuit also increases as i_{1C} increases. This makes sense because from ohm's law, we know that voltage and current are directly proportional. Therefore, when the circuit requires more current, the battery will discharge more voltage as seen by the curve at $i_{1C} = 27A/m^2$.
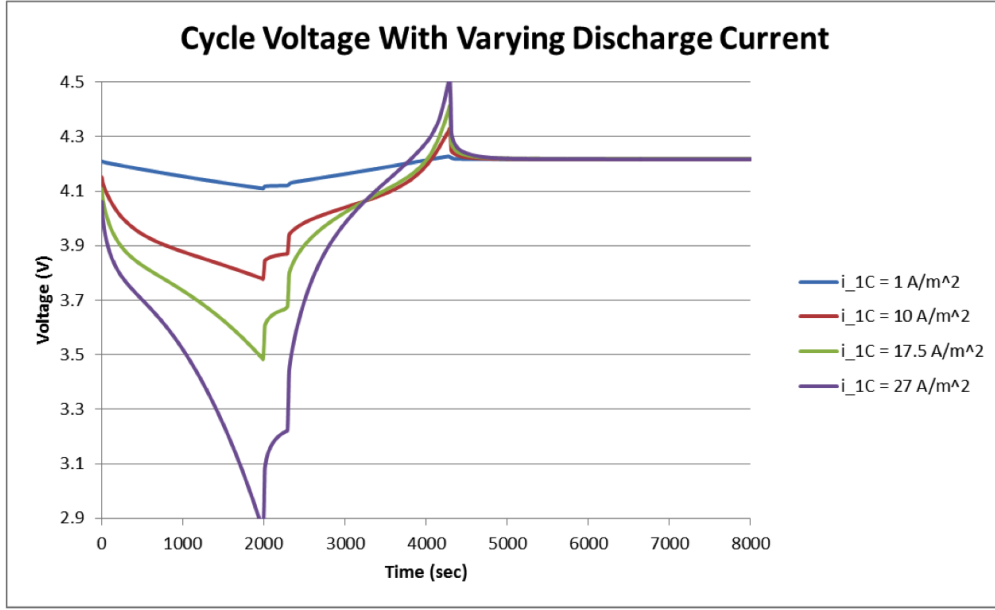
Figure 3.7: Effects of discharge current on output voltage

Discharge current was also tested to investigate any significant changes in discharge curves. No significant changes were found, so it can be said that discharge curves do not respond to changes in discharge current.
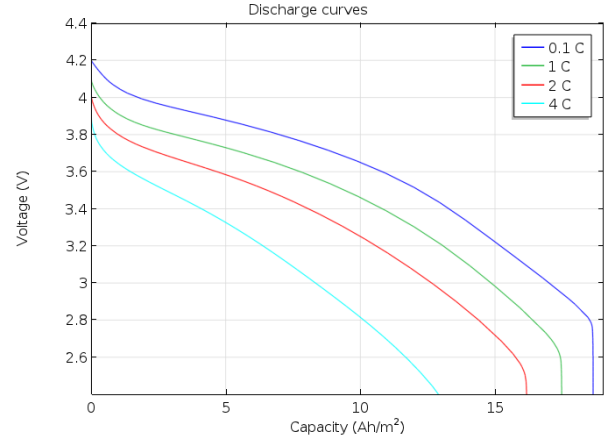
3.4   Diffusion Coefficient (D_l)

The last parameter tested on concentration was the diffusion coefficient of the electrode. After variation in diffusion values, no significant difference in concentration profiles was detected in concentration gradient or output voltage. However, the diffusion coefficient did have a significant effect on discharge curves. The discharge curves measure the voltage discharged with respect to the capacity left in the battery. In the graphs provided below, there are four curves in each graph. These curves correspond to battery discharge when the current in the battery is a multiple of the discharge current. The discharge current was kept at its default value of 17.5 A/m^2. So, 1C means that there is a discharge current of 17.5

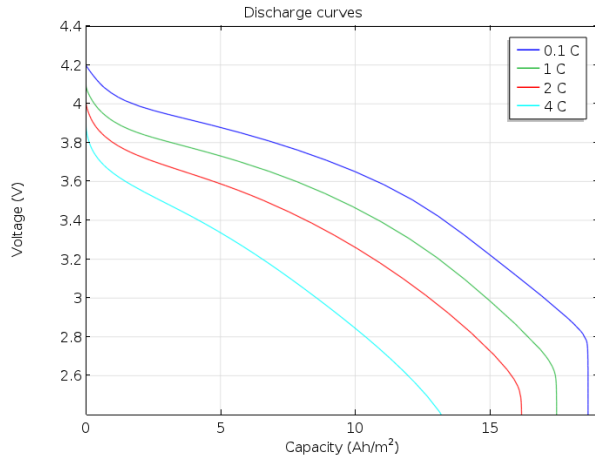A/m^2, 2C represents a discharge current of 35 A/m^2, and so on.

The default diffusion coefficient value was $7.5 \times 10^{-11} m^2/s$. The discharge curve at the default value and three other values can be found in Figure 3.8 below.
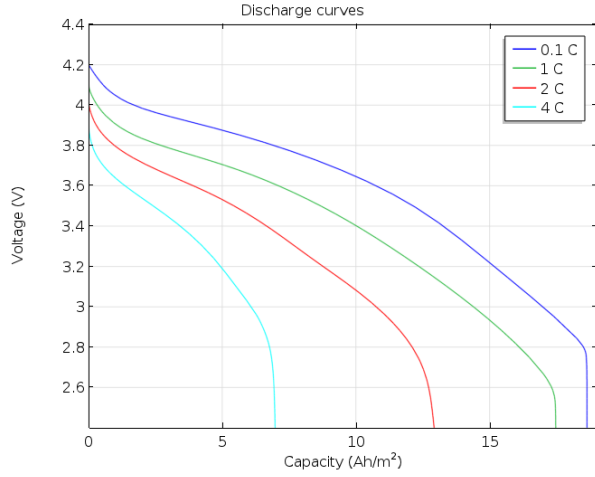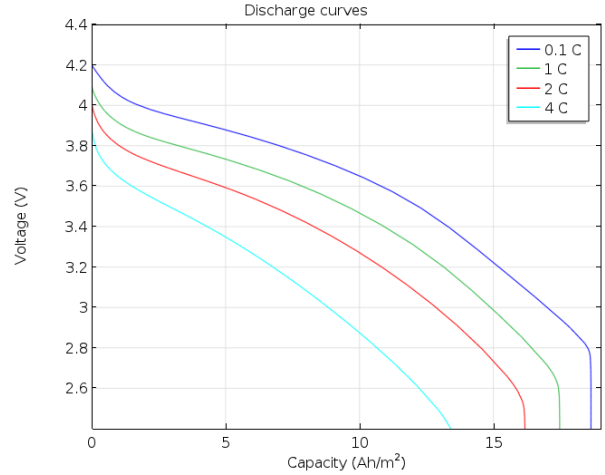

(a) Discharge Curves at D_l=3e-11 m^2/s

(b) Discharge Curves at D_l=7.5e-11 m^2/s

(c) Discharge Curves at D_l=8.5e-11 m^2/s

(d) Discharge Curves at D_l=1e-10 m^2/s

Figure 3.8: Effects of varying the Diffusion Coefficient, D_l, of the electrolyte on the Discharge Curves

From these graphs, one can see that a lower diffusion coefficient reduces discharge capacity. Higher diffusion coefficients do not have a significant effect on the discharge curves, but

when the diffusion coefficient was reduced to $3 \times 10^{-11} m^2/s$, the battery could not output as much voltage at higher currents. This could be explained as a result of less ions crossing the separator at low diffusion coefficients. With less ions crossing the separator, the battery is not able to output as much voltage at higher currents, and so the capacity of the battery is greatly drained.

Chapter 4

Conclusions

The results indicate that volumetric fraction, electrical conductivity of the positive electrode, and discharge current are most important in determining the concentration gradient and output voltage, while the diffusion coefficient most affects the discharge curves. Electrical conductivity of the negative electrode, did not have an effect on concentration, voltage, or discharge.

As volumetric fraction increased, there was a lower concentration in the electrodes and more output voltage. This can be explained as increasing volumetric fraction increases open space in the electrode, making it harder for charge to pass through, and thus reducing the potential difference across the battery. A reduced potential difference would decrease the concentration gradient values and the output voltage. As electrical conductivity decreased, irregularities appeared in the concentration gradient and output voltage graphs, which large conductivity values produced no significant differences. This can be explained as electrical conductivity decreases, it becomes harder for charge to move through the electrode, leading to a lower battery voltage output and irregularities in the graphs because the electrode is not conducting electricity like it should. However, when the electrode is very conductive, changes in conductivity do not produce significant changes in concentration gradient or output voltage because the electrode is conducting electricity like it should. For discharge

current, as i_{1C} increased, the amount of voltage required of the battery increased. This lead to large concentration gradients, as the electrolytic fluid had to be mostly consolidated on one side of the battery in order to output more voltage. It also lead to more extreme output voltage values, since the increase in current required more voltage to discharge. Lastly, for the diffusion coefficient, as D_l decreased, the discharge capacity reduced. This can be explained as a reduction in diffusivity causes less ions to flow through the separator, so the battery cannot discharge as much voltage at higher currents.

There are a few key conclusions from this project. One of the most important is that while having a high electric conductivity is important, it is not worth it to buy expensive materials in an attempt to increase the electrical conductivity, as changes to large electrical conductivity values do not impact the battery output. It is also important to ensure that the diffusion coefficient of the separator is not too low, or it will hinder discharge at high currents.

These results can be used to determine how different properties of a battery can impact its performance. It is the hope of this project to provide a stepping stone for further research in the area of battery construction and battery physics. One place to start would be designing the model to allow testing of more variables without the system crashing or error reports. This would allow a more full picture on what variables are most important when constructing and analyzing a Lithium-ion battery in the future.

4.1   Acknowledgements

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