Determining the Size of the Proton Through Statistical Analysis of Measured Form Factors

A thesis submitted in partial fulfillment of the requirement for the degree of Bachelor of Science in Physics from the College of William and Mary in Virginia,

by

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Abstract:

In 2010, a proton charge radius measurement using muonic hydrogen spectroscopy obtained a value smaller than the accepted CODATA value = 0.8751±0.0061 fm [1]. The CODATA value comes from from hydrogen spectroscopy and electron scattering measurements. This discrepancy is known as the proton radius puzzle [2]. The combined measured cross sections from electron-proton scattering experiments done over the past 60 years were reanalyzed to obtain a fit to the electric form factor G_E and the charge radius of the proton. Padé approximates were used as a model for G_E. The fit to the data is reasonable and the extracted radius R_E= .846±0.021 fm, which within uncertainties is to be in agreement with the muonic hydrogen value.

1. Introduction

For some time the charge radius of the proton was thought to be known to with reasonably small uncertainty from electron scattering and atomic spectroscopy measurements [1]. But when spectroscopy experiments were done using muons, the results were found to be 4% smaller with a much smaller uncertainty, R_E=0.84087±0.00039 fm [3]. Until the experiment using muons, determinations using scattering and spectroscopy agreed with each other.

The charge radius was first measured in the 1950's to be about 0.8 fm [4]; and throughout the years the measurements have become more accurate. The CODATA group quoted the radius to be R_E=0.8751±0.0061 fm [1]. The value remained fairly steady at 0.88 fm until the muon experiment was done in 2010, which yielded a value of R_E=0.84087±0.00039 fm, which is seven standard deviations away from the accepted value from the CODATA group [5].

The charge radius, R_E, can be found from the slope of the electric form factor, G_E, as a function of the four-momentum squared, Q^2. The slope at Q^2=0 is needed, but no measurement extends down to Q^2=0, so the data must be extrapolated from the smallest measured Q^2. The charge radius R_E can be extracted from the second term in the expansion of the electric form factor,

$$G_E(Q^2) = 1 - \frac{1}{6} R_E^2 Q^2 + c_2 Q^4 + \cdots \quad (1)$$

I analyzed measured elastic scattering cross sections data from experiments dating back to 1965 and combined them all into a global fit. There were nine experiments yielding a total of 373 data points at Q^2 ranging from 0.005057 to 5.3699 GeV^2 [6-14].

2 Formalism:

In electron scattering the 4-momentum transfer squared is

$$Q^2 = -q^2 = 4EE' sin^2 \frac{\theta}{2} \quad (2)$$

where E is the incident energy of the electron that scatters from the proton at rest, θ is the scattering angle, and E' is the scattered electron energy. Electric and magnetic form factors have traditionally been extracted from the cross sections using a so-called Rosenbluth separation. The Rosenbluth separation uses varying beam energies and scattering angles to look at variations as a function of the kinematic variable ε with Q^2 kept constant [14]. The scattering cross section, $\frac{d\sigma}{d\Omega}$, can then be written in terms of the electric and magnetic Sachs form factors, $G_E(Q^2)$ and $G_M(Q^2)$:

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \frac{1}{(1+\tau)} \left[G_E^2(Q^2) + \frac{\tau}{\varepsilon} G_M^2(Q^2)\right] \quad (3)$$

The Mott cross section (scattering from a point charge) is

$$\left(\frac{d\sigma}{d\Omega}\right)_{Mott} = \frac{4\alpha^2\cos^2\frac{\theta}{2}}{Q^4}\frac{E'^3}{E} \tag{4}$$

where α is the fine structure constant, which is approximately $1/137$,

$$\varepsilon = \left(1 + 2(1+\tau)\tan^2\frac{\theta}{2}\right)^{-1} \tag{5}$$

and

$$\tau = \frac{Q^2}{4{M_p}^2} \tag{6}$$

where M_p is the proton mass, 0.938272 GeV/c^2. ε is the virtual photon polarization. Dividing $\frac{d\sigma}{d\Omega}$ by the Mott cross section gives the reduced cross section. E' can be calculated using

$$E' = \frac{E}{1 + (2E/M)\sin^2\frac{\theta}{2}} \tag{7}$$

The form factors at Q^2=0 are normalized such that

$$G_E(0) = 1 \tag{8}$$

$$G_M(0) = \mu_p \approx 2.793 \tag{9}$$

Here, μ_p is the magnetic moment of the proton. For most values of Q^2, those less than 8 GeV/c^2, G_E can be calculated using a parameterization of the G_E/G_M ratio obtained from recoil polarization experiments [15]

$$\mu_p\frac{G_E}{G_M} \approx 1 - \frac{Q^2}{8.02\ \mathrm{GeV}^2} \tag{10}$$

The mean square of the charge radius is given by,

$$R_E^2 \equiv -6\frac{dG_E}{dQ^2}\bigg|_{Q^2=0} \tag{11}$$

3 Theory:

There are two types of experiment that have been used to find the charge radius of the proton: hydrogen spectroscopy and electron scattering. Hydrogen spectroscopy looks at energy transitions of hydroge.

3.1 Muonic Hydrogen

A muon is identical to the electron, but its mass is about 200 times greater than the mass of an electron. Normal hydrogen is a proton and an electron, so muonic hydrogen is a proton and a muon. Because a muon is more massive than the electron, it is much closer to the proton and much more likely to be found inside the proton. The probability of the muon being inside the proton affects the sensitivity of the energy levels of the muonic hydrogen. The probability of the electron or muon being inside the proton depends on the ratio of the proton and atomic volumes and is approximately given by [3]:

$$(\frac{R_p}{a_B})^3 = (\alpha m_r R_p)^3 \tag{12}$$

R_p is the proton radius and m_r is the reduced mass of the electron or muon. Because the muon is so much more massive than the electron it is approximately 10^7 times more likely to be found inside the proton than is the electron is. When the electron is inside the proton, the binding strength is reduced, which changes the Lamb shift. When the muon is inside the proton, it changes the Lamb shift by 2% [2].

In the muon experiment, a beam of muons was shot into hydrogen gas. Some muons would replace electrons forming muonic hydrogen atoms in an excited state. The excited atom would emit a photon and go into lower energy states. The experiment only used atoms in the 2S state. When the muon entered the container with the hydrogen gas in it, a laser pulse was delivered, which if it was exactly at the resonant energy it would drive the atom up to the 2P state. Hydrogen only absorbs radiation at specific energies; if the tuned laser energy was right and the muonic atom could be driven into the 2P state [3]. A muon will never be found inside the proton in a 2P state because the wavefunction doesn't overlap with the proton. So measuring the difference betnween the 2S and 2P state it was possible to determine the energy shift resulting from the time the muons spends inside the proton [3].

3.2 Electron Scattering

Electron scattering experiments are done differently. When you shoot a beam of electrons at hydrogen gas, some of the electrons will collide with the protons and bounce off in another direction. When they collide, there is a transfer of momentum; if they barely collide, there is a small transfer of momentum, but if they collide head on, there is a larger transfer of momentum. This transfer of momentum occurs by the exchange of a virtual photon with a given wavelength. The higher momenta transfer, the higher the energy and the shorter the wavelength. In order to see the full proton using scattering, you need to have a very long wavelength, as long as possible to get the smallest momenta transfer. Still one needs to extrapolate down to zero [2].

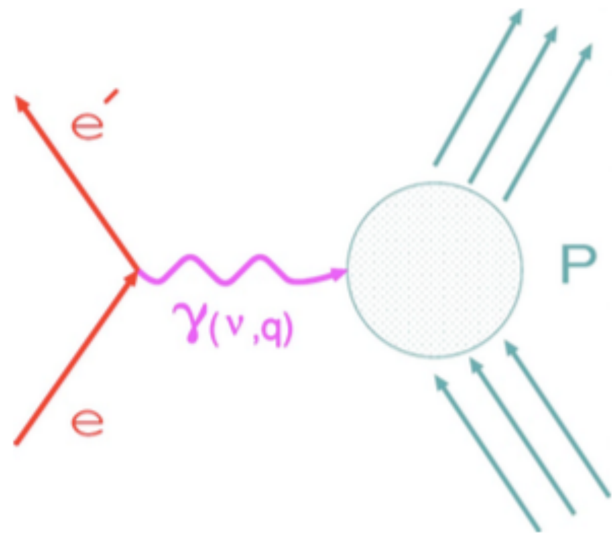


Figure 1: One-photon exchange diagram for electron-proton scattering [10].

3.3 Potential Explanations

There has to be some explanation for this discrepancy, since the proton cannot have two different charge radii. The electron and muon should interact with the proton in the same way because of the principle of lepton universality, which states that leptons-electrons, muons, and tau leptons, are identical except for mass [16]. A possible explanation for why this discrepancy exists is that there is some new physics that we haven't yet discovered. This could be a new particle that interacts preferentially with the muon and not the electron. Although this answer would be exciting, the chances of this are small [14]. Another explanation is that the experimental results are wrong. This seems unlikely because several electron-proton experiments have been done by different groups, and they agree with each other and with the hydrogen spectroscopy experiments using electrons. The muonic hydrogen experiment was done with such high precision but it could be for some reason inaccurate, for example if the Rydberg constant is incorrect; to get the same radius as the muonic hydrogen experiment, the constant would need to be changed by almost 7 standard deviations. It has also been proposed that the calculations of contributions to the Lamb shift are incorrect. But several groups independently checked these. There could be other effects like two-photon exchange in the scattering, which is hard to calculate and poorly known, which might explain the difference. But measurements of the two photon effect give rather small values with large error bars [15]. Two photon exchange effects are also of more importance at larger Q^2 [3].

3.4 Current and Future Experiments

More experiments are being done and have been done since the muonic hydrogen results appeared in 2010. The muonic hydrogen experiment was redone, and the same result was obtained. Results similar to those found in the proton experiment were found when measuring the deuteron, which is made of one neutron and one proton. In this experiment, the deuteron charge radius was measured to be about 7.5 standard deviations smaller than the CODATA value from 2010, but 2.7 times more accurate [3]. The possible effects of a two-photon exchange are also being investigated to determine if it has more of a contribution to the data than previously thought, which was less than 1% for low Q^2 data. A scattering experiment using muons is being developed at PSI in Switzerland [3].

Analysis of the Mainz data using a continued fraction fit and linear fit for very low Q^2 data has given a radius R_E= 0.840±.016 fm, which is consistent with the muonic hydrogen data [15]. From this it would appear that electron scattering data can agree with the muonic hydrogen results, while the hydrogen spectroscopy results cannot as of yet.

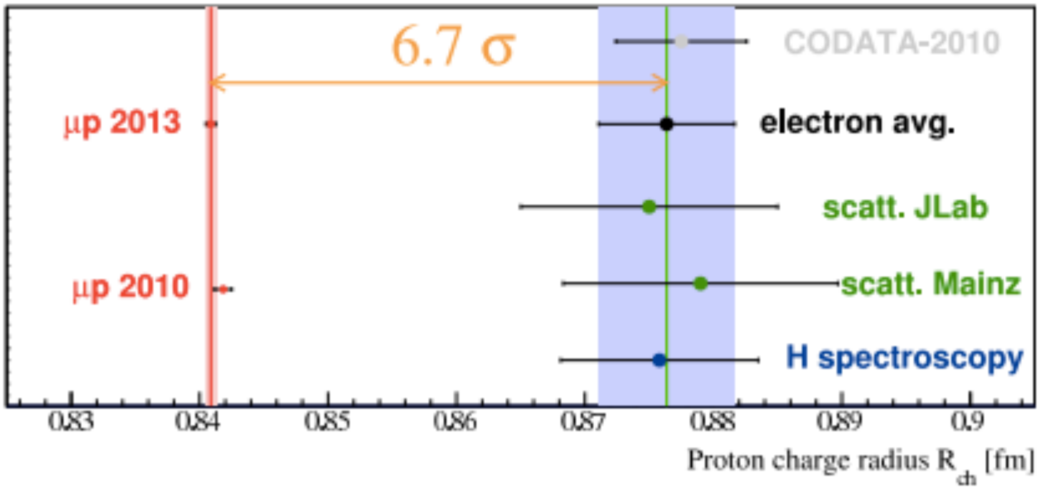


Figure 2: Charge radius values for muonic hydrogen (μp), hydrogen spectroscopy, the CODATA value, the electron scattering values from Jefferson Lab (J.Lab) and Mainz, and an average of earlier world data [17]. Note: this Mainz value is not the same as the one obtained from the analysis in [9].

4 Data Analysis:

Data from Q^2=0.005057 to 5.3699 were fit using Padé approximates. This model gave a good fit and allowed for the number of parameters to be kept fairly low. A Padé approximate can give a reasonable description of an unknown distribution using minimal parameters. It uses a ratio of polynomials of a given order to approximate a given function. With more and more parameters, the fit can begin to fit the systematic uncertainties and skew the fit and in turn, the radius extracted from the fit. Therefore it was important to keep the number of parameters as low as possible. The fits used 4 parameters. They were then used to extrapolate the slope at Q^2=0. From the slope, it is possible to calculate the charge radius using equation 11.

4.1 Fit Using $\mu_p \frac{G_E}{G_M} \approx 1 - \frac{Q^2}{8.02\ GeV^2}$

The Rosenbluth separation mentioned above was used to determine $G_E(Q^2)$. The fit used was,

$$G_E = .9934\left(\frac{1-0.16x}{1+3.045x+1.254x^2}\right) \tag{13}$$

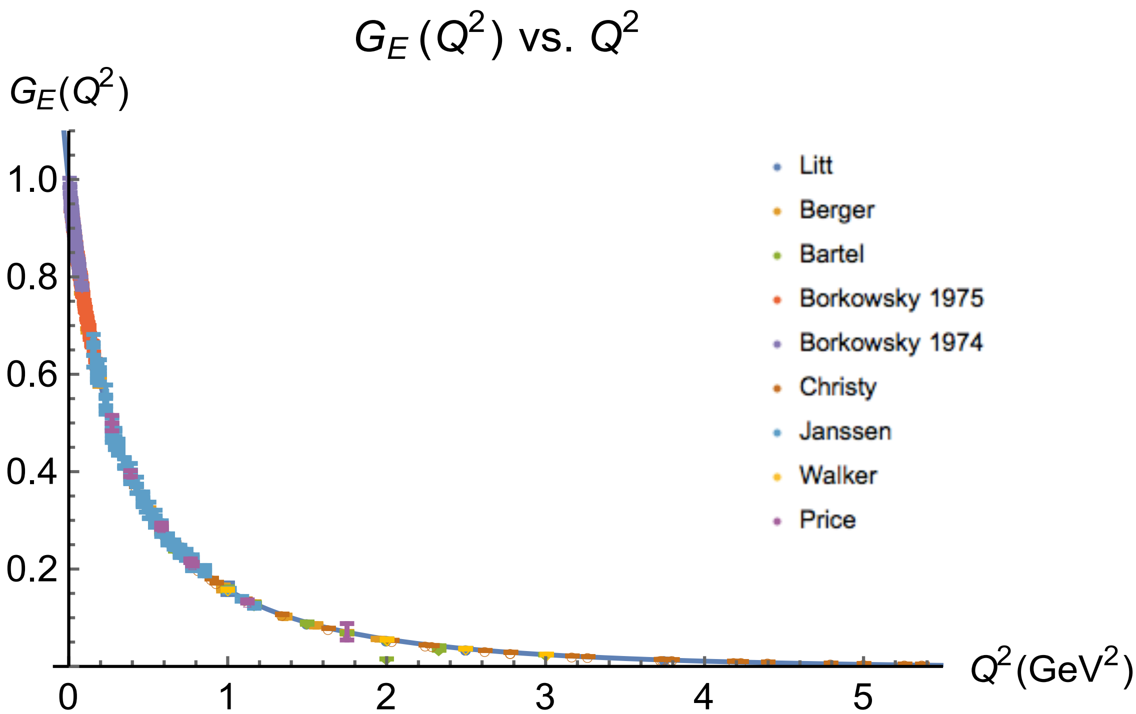


Figure 3: Plot of G_E vs. Q^2 with error bars and Padé Approximation.

From this fit, the slope at $Q^2=0$ could be extracted and was -3.1839, which gives a proton charge radius $R_E=0.862\pm.021$ fm. This value lies between the radius using muonic hydrogen and the CODATA radius. The χ^2 was 1 and the R^2 was .99968. Examining the error bars closely shows that a majority cross the fit line, implying that the fit is decent.

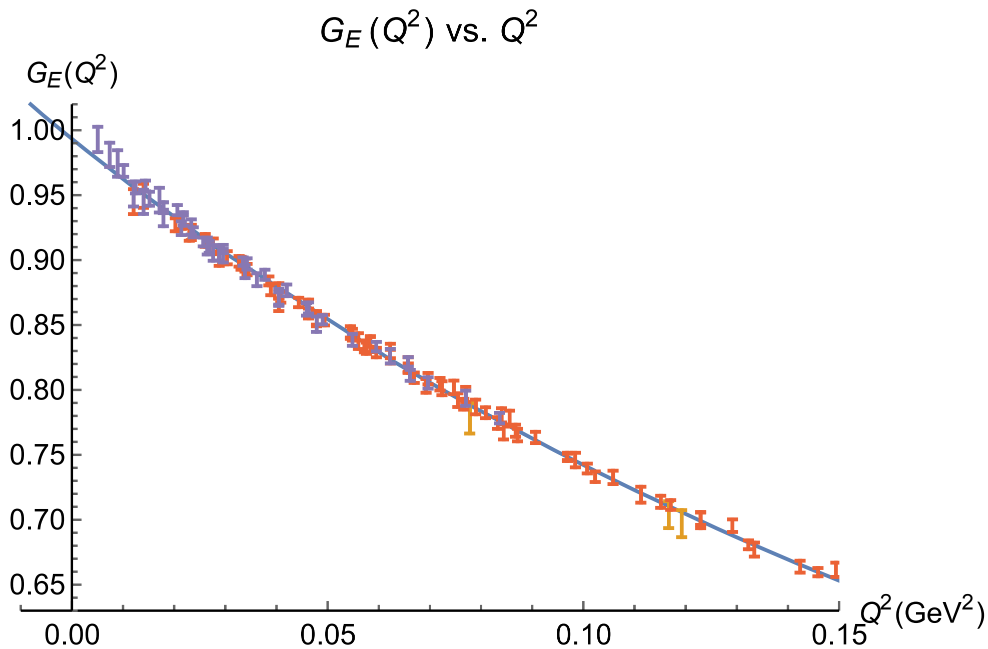


Figure 4: $G_E(Q^2)$ vs. Q^2 for Q^2=0 to Q^2 = 0.15 GeV^2.

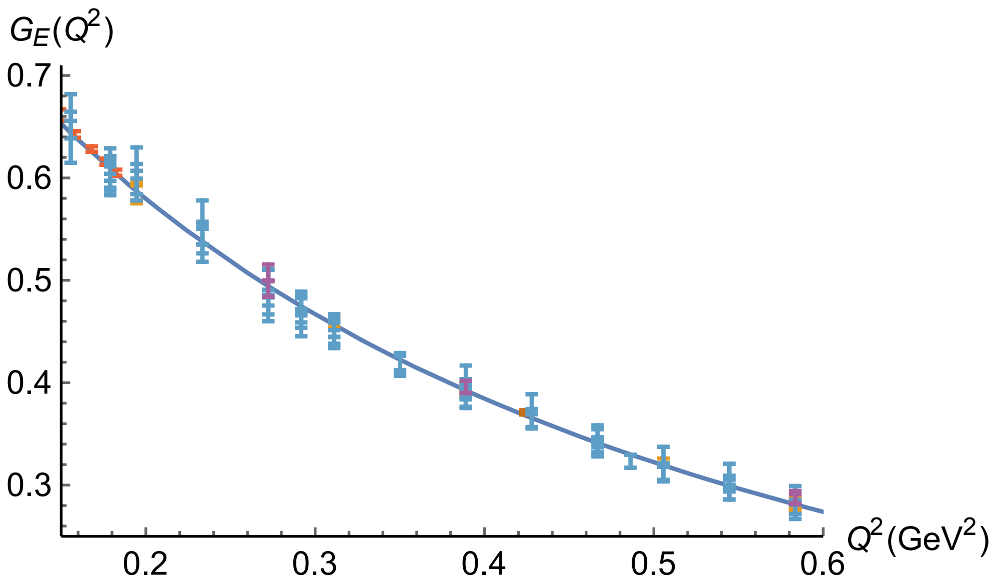


Figure 5: $G_E(Q^2)$ vs. Q^2 for Q^2=0.15 to Q^2 = 0.6 GeV^2.

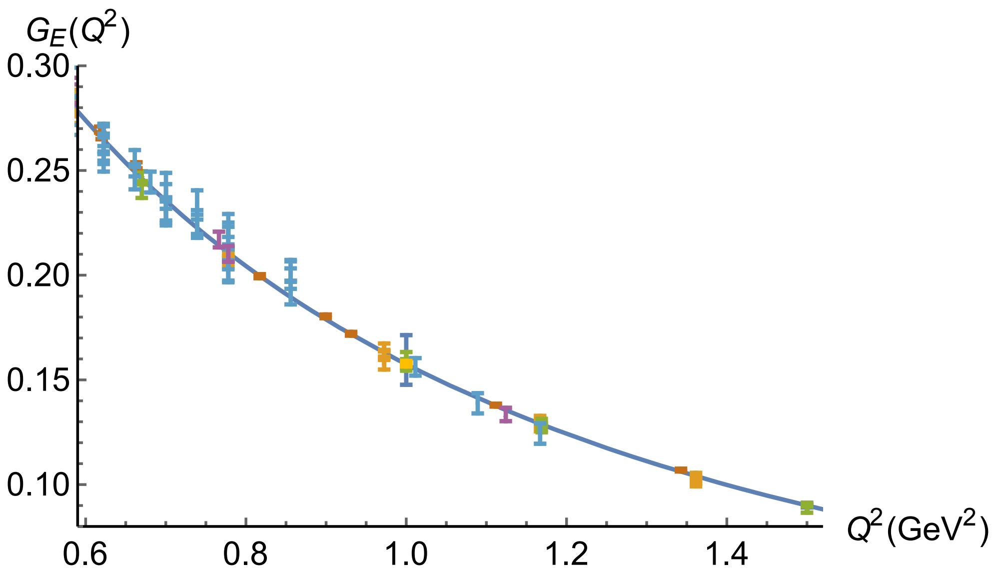


Figure 6: $G_E(Q^2)$ vs. Q^2 for Q^2=0.6 to Q^2 = 1.5 GeV^2.

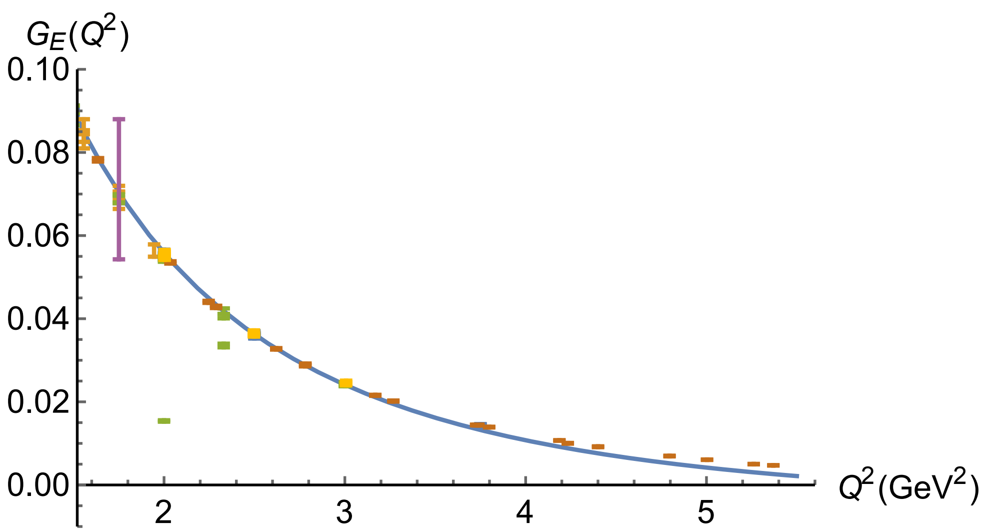
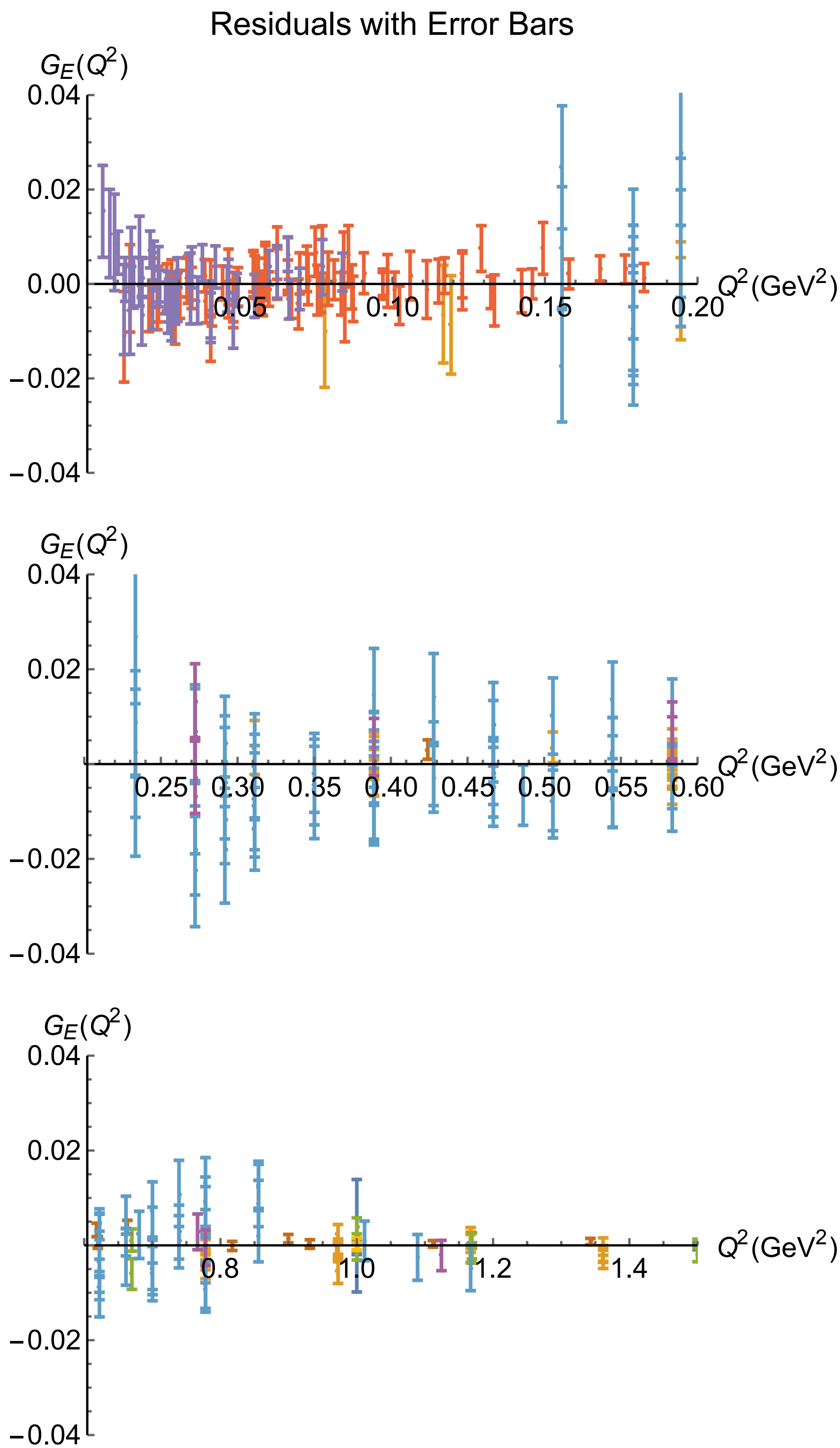


Figure 7: $G_E(Q^2)$ vs. Q^2 for Q^2=1.5 to Q^2 = 5.6 GeV^2.


Residuals with Error Bars
GE(Q2)
0.04
0.02
0.00
−0.02
−0.04
0.05
0.10
0.15
0.20
Q2(GeV2)
0.25
0.30
0.35
0.40
0.45
0.50
0.55
0.60
0.8
1.0
1.2
1.4

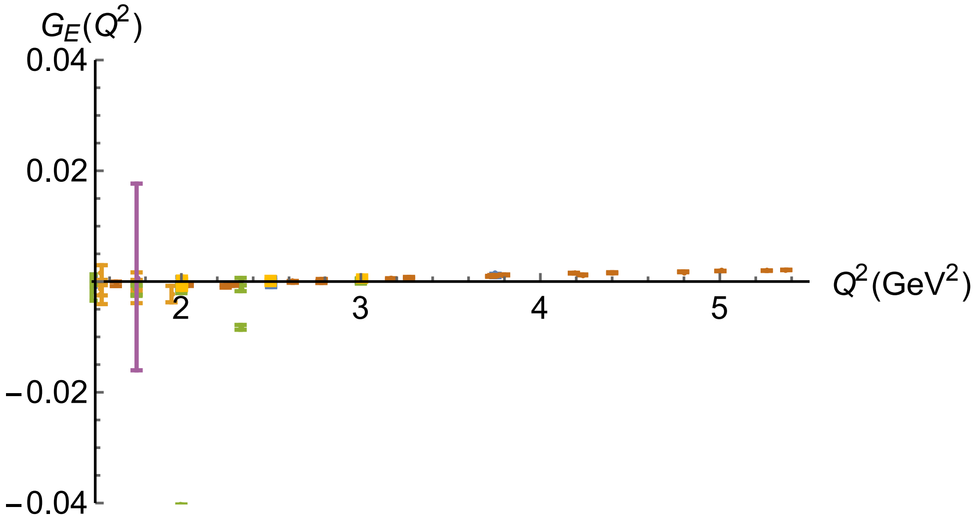


Figure 8: Residuals vs. Q^2 with error bars at different ranges of Q^2.

Residuals are the observed value minus the expected value, so they show how the data deviates from the fit line. For a perfect fit, there would be no residuals; the points would all be on the line. Realistically, this would not happen; so you must look at the error bars of the residuals and how many overlap with the fit line. The residuals overlap with the zero line for most of the data points, until about x>2, where they start to sit above the zero line. The residuals for lower Q^2 values are considerably larger because measurements at such low momentum transfer are harder to make. The residuals vs. order of the data set plot makes it easier to see which data points have residuals that overlap with the zero line and what patterns in residuals each individual data set may or may not have.

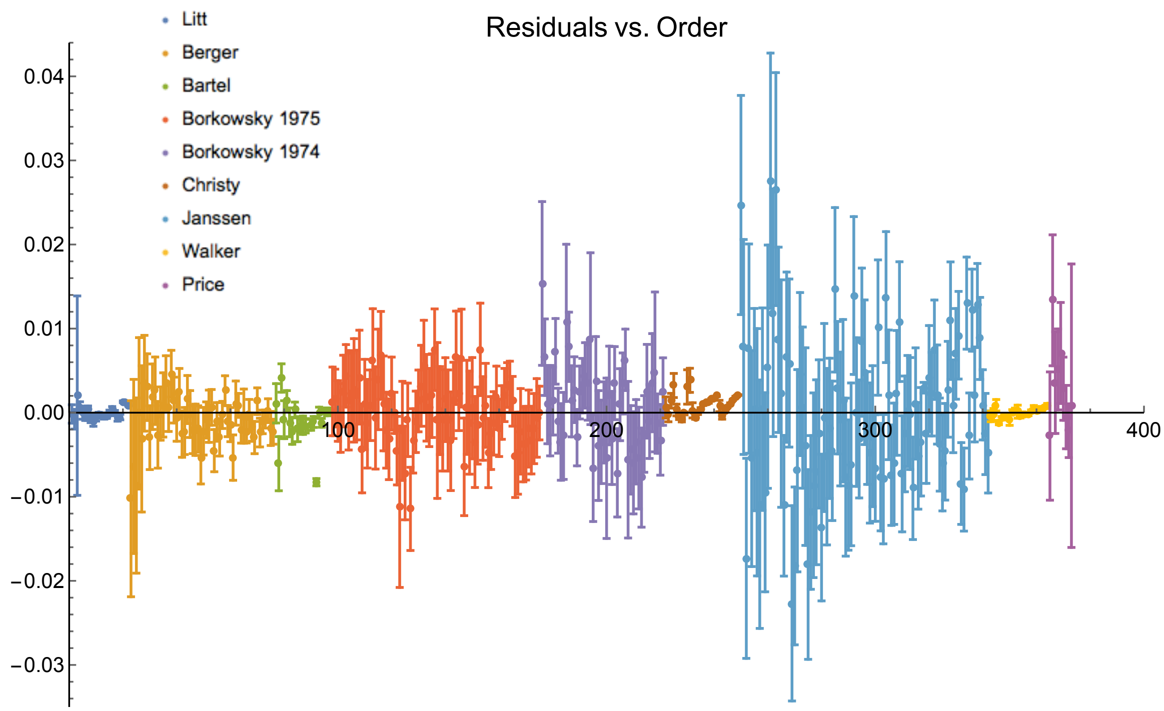


Figure 9: Residuals plotted against the order of the data. Each color shows a different data set.

4.2 Fit Using $\frac{\mu_p G_E}{G_M} = \frac{1}{\left(1+0.119Q^2+0.026\left(Q^4\right)\right)}$

There is another fit of $\mu_p G_E/G_M$ that can be used when extracting $G_E(Q^2)$ [19],

$$\frac{\mu_p G_E}{G_M} = \frac{1}{\left(1 + 0.119Q^2 + 0.026(Q^4)\right)} \tag{14}$$

This can be used in lieu of equation 10. Using the same fitting technique as above, a Padé approximant gives the best fit,

$$G_E = \frac{(.9934 - 0.2354x)}{(1 + 2.8503x + 0.8761x^2)} \tag{15}$$

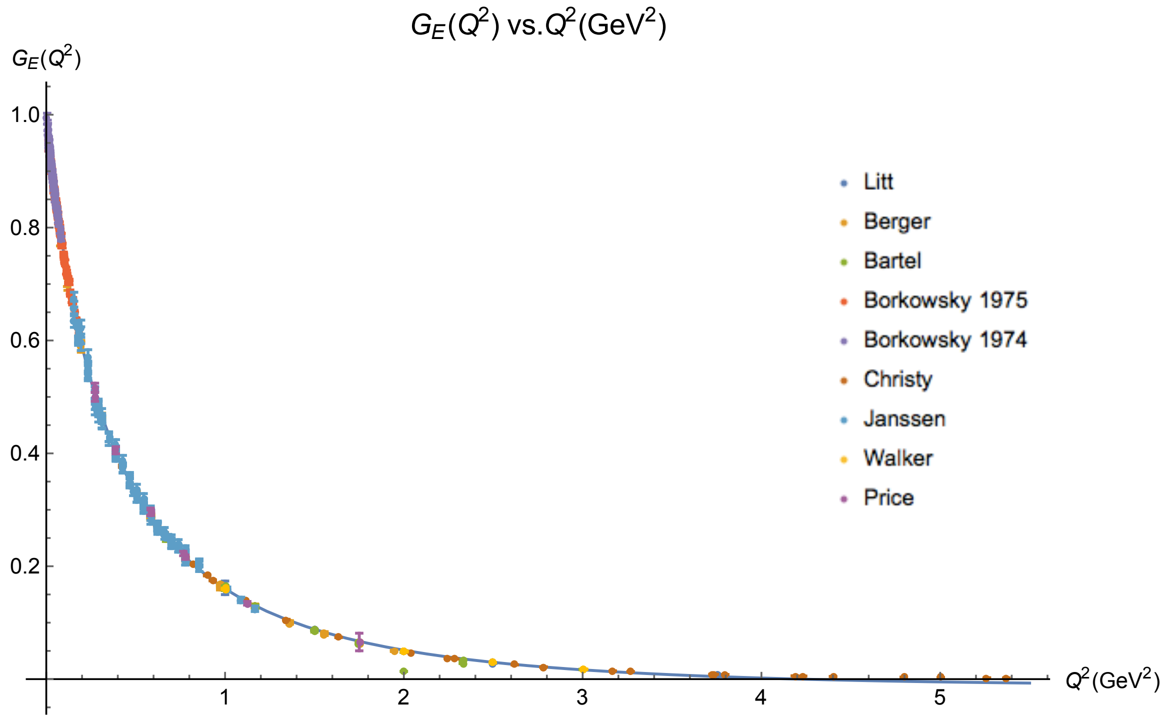
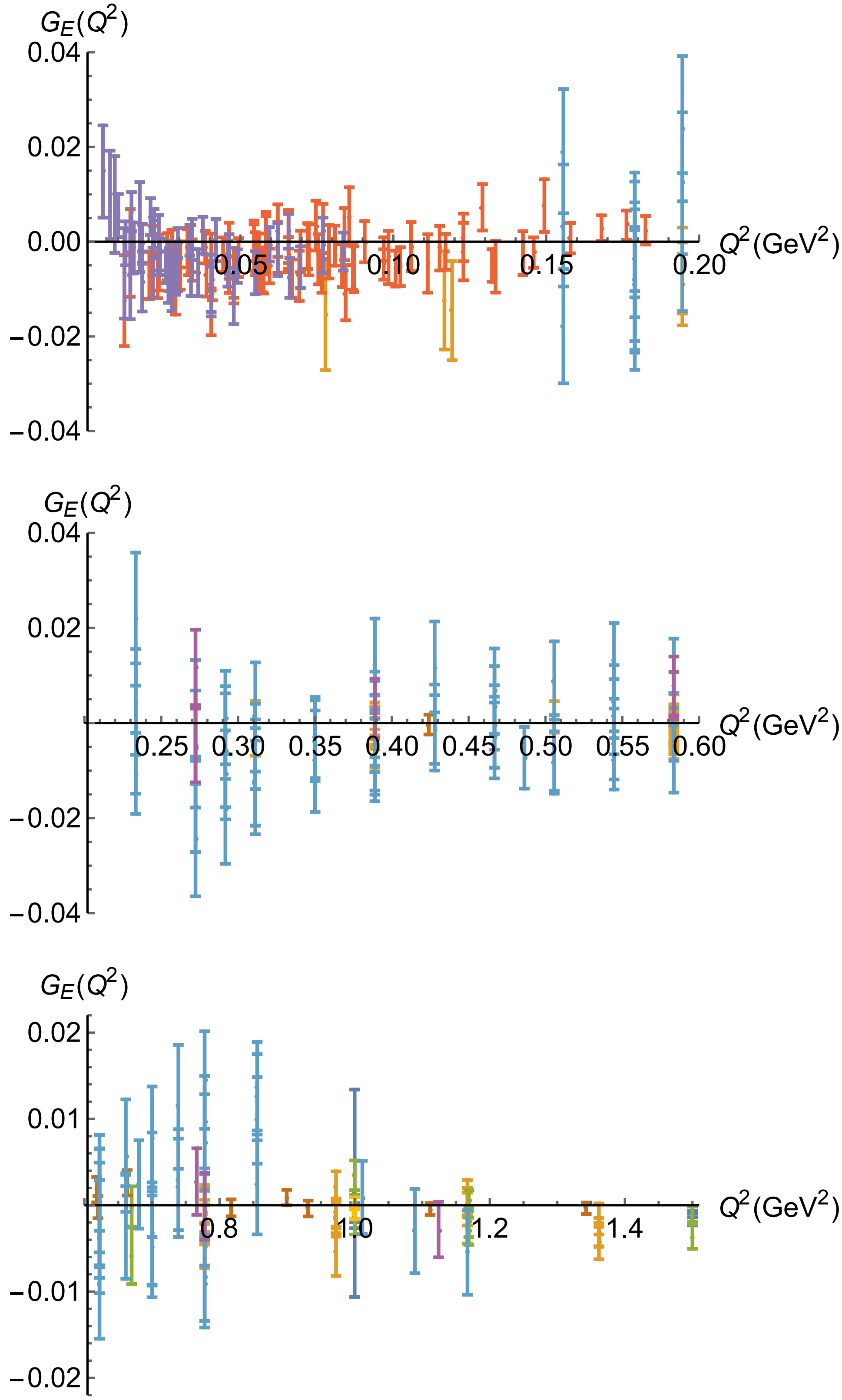


Figure 10: Plot of G_E vs. Q^2 with Padé approximate fit line and error bars.

At the higher Q^2 values the fit dips slightly below zero, which is not possible experimentally. The R^2 value for this fit is .99965, which is slightly lower than that for the fit using the other form of $\mu_p G_E/G_M$. The chi squared statistic for this fit is also 1 when using the points that do not go negative. For this fit, the slope at x=0 is -3.067. Using the same extraction of the radius used above the charge radius is .846±.021 fm, which is quite close to the muonic hydrogen value of R_E=.84087±0.00039 fm. The agreement between the two values indicates that perhaps it is the hydrogen spectroscopy using electrons experimental results that are outliers, not the muonic hydrogen results.

Residuals with Error Bars



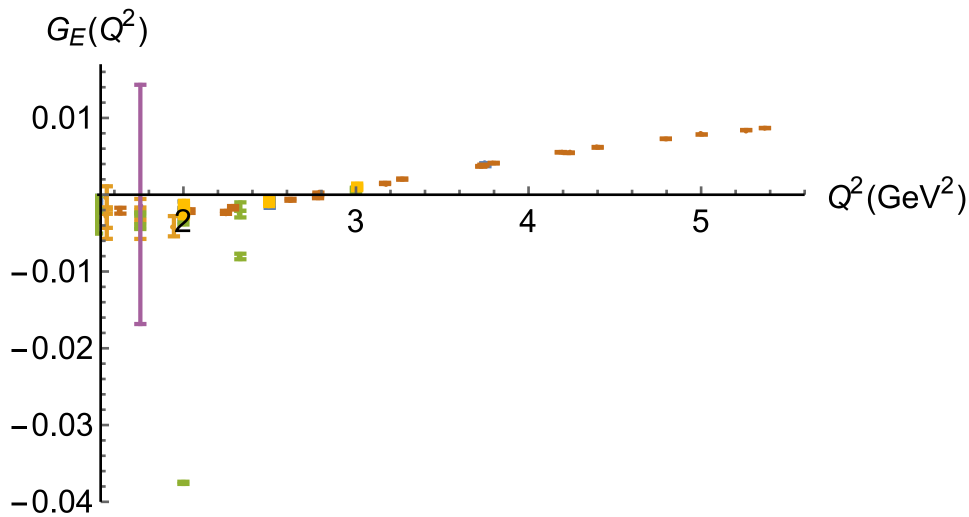


Figure 11: Residuals vs. Q^2 for the fit for the new form for $\mu_p G_E/G_M$.

The residuals for this fit are extremely similar to those for the previous fit, except for higher values of Q^2, $Q^2>3$. For the previous fit, the points past $Q^2=2$ are mostly within .004 from the zero line. For this fit, the residuals begin to grow up to .01 for the final point. The differences between the residuals of both fits become most apparent at about $Q^2=3.8$, which is where the fit line crosses the x-axis.

5 Conclusion

Measurements of the proton charge radius using muonic hydrogen have led to what is called the proton radius puzzle. The results from the muonic hydrogen experiment disagree with the accepted value but almost 7 standard deviations, but are much more precise than the results from previous experiments. The results from the analysis of previous electron scattering data going back to the 60's show that electron-scattering data could be in agreement with the muonic hydrogen results; the muonic hydrogen results, R_E=.84087±0.00039 fm, are within the uncertainty of the charge radius calculated in this paper, R_E=.846±.021 fm. Further experiments will need to be done, including more using muons, in order to solve the proton radius puzzle.

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