Electromagnetically Induced Transparency

A thesis submitted in partial fulfillment of the requirements for the degree of Bachelor of Science degree in Physics from the College of William and Mary

by

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Introduction:

For my research, I began by reading and learning about the density matrix, three level systems, and electromagnetically induced transparency. Once I had a basic understanding of density matrices, I started using the Atomic Density Matrix package in Mathematica. I completed through the tutorials that were included in the documentation. Once I felt comfortable with the package, I plotted the propagation of an optical field through a two level system. Then, I plotted the propagation of a probe field through a three level system and demonstrated electromagnetically induced transparency. The next step is to add a sinusoidal variation to the optical field, followed by a random variation to stimulate noise. The end goal is to model squeezing behavior in the system.

Electromagnetically Induced Transparency:

Electromagnetically induced transparency is a phenomenon in which a material becomes transparent to light when normally absorption would be at a maximum. It occurs as a result of matching between light fields and the quantum states of material. The system includes two optical fields, the "probe" field and the "pump" or "control" field. The pump field is much stronger than the probe field. Each field is set near resonance with a different transition level for the material. The material becomes transparent for a small range of frequencies near resonance.

A density matrix is a way to describe a certain quantum state and is useful when describing a mixed state that has multiple similar systems. The density matrix is defined by $\hat{\rho} = |\psi\rangle\langle\psi|$. [5] The elements of a density matrix are given by ρ_{ij}. The diagonal elements of the density matrix are the population of the corresponding states. The off diagonal elements of the density matrix are coherences. The density matrix allows us to find the expectation values for an observable A using this formula:

$$\langle A\rangle=\sum_j \rho_j \langle\Psi_j|A|\Psi_j\rangle=tr(\rho A) \tag{1}$$

The density matrix formalism is important because it allows us to represent mixed states and coherences.[2] This cannot be done with a normal wave function. This is helpful for studying electromagnetically induced transparency because it allows us to track the change of the system through time.

Rabi flopping is the process by which a system oscillates between the ground state and an excited state when interacting with a resonant optical field. The frequency at which it osciallates is called the Rabi frequency, denoted with Ω. The generalized Rabi frequency is the square root of the sum of the square of the detuning and the square of the Rabi frequency.[3] The detuning, Δ, is the difference between the light frequency and transition frequency. The oscillation is a result of alternating stimulated absorption and stimulated emission. [3]

Two Level System:

In order to model the propagation of optical fields through a rubidium cell, I started by modeling the propagation for a time independent two level system before moving onto the time independent three level system. I used Mathematica and the Atomic

Density Matrix package for my calculations.

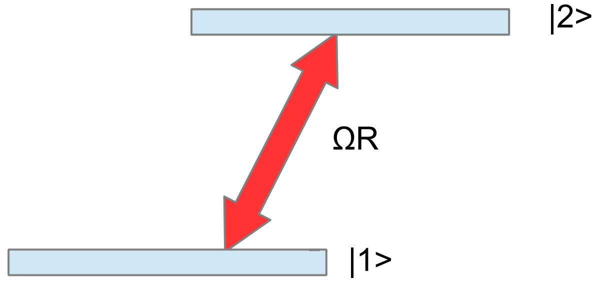

Figure 1: *The two level system with optical probe field, Ω_0.*

Since I wanted to find time independent solutions, I used the Atomic Density Matrix package to account for spontaneous decay from the excited state to the ground state and for ground state atoms entering the system. I used γ for the rate at which atoms leave the light beam, called transit relaxation. I used Γ for the intrinsic relaxation.[2] There are also repopulation mechanisms because atoms that decay from the excited state will go into the ground state and atoms in ground state can enter the light beam. The repopulation uses the same variables. The Hamiltonian for the system is:

$$\begin{pmatrix} 0 & -\Omega R \, \mathrm{Cos}[t\,\omega] \\ -\Omega R \, \mathrm{Cos}[t\,\omega] & \mathrm{Energy}[2] \end{pmatrix} \qquad \begin{pmatrix} 0 & -\frac{\Omega R}{2} \\ -\frac{\Omega R}{2} & -\Delta \end{pmatrix}$$

Figure 2: *The Hamiltonian of the system* Figure 3: *Hamiltonian after RWA applied*

We use the rotating wave approximation (RWA) to simplify the Hamiltonian. The rotating wave approximation is used to neglect rapidly oscillating terms in the Hamiltonian. This can be done when the frequency of the light field is near resonance. It works because the rapid oscillations will average out over time. The relaxation and repopulation parameters are combined with the Hamiltonian for the system in the Liouville equation to approximate the time evolution of the system. The Liouvulle equation is:

$$\frac{\partial \rho}{\partial t} = \frac{1}{i\hbar}[H, \rho] \qquad (2)$$

This gives equations for the evolution of the system:

$$0 = \gamma - \gamma\,\rho_{1,1} - i\left(\frac{1}{2}\,\Omega R\,\rho_{1,2} - \frac{1}{2}\,\Omega R\,\rho_{2,1}\right) + \Gamma\,\rho_{2,2}$$
$$0 = \frac{1}{2}\left(-\gamma\,\rho_{1,2} - (\gamma + \Gamma)\,\rho_{1,2}\right) - i\left(\frac{1}{2}\,\Omega R\,\rho_{1,1} + \Delta\,\rho_{1,2} - \frac{1}{2}\,\Omega R\,\rho_{2,2}\right)$$
$$0 = \frac{1}{2}\left(-\gamma\,\rho_{2,1} - (\gamma + \Gamma)\,\rho_{2,1}\right) - i\left(-\frac{1}{2}\,\Omega R\,\rho_{1,1} - \Delta\,\rho_{2,1} + \frac{1}{2}\,\Omega R\,\rho_{2,2}\right)$$
$$0 = -i\left(-\frac{1}{2}\,\Omega R\,\rho_{1,2} + \frac{1}{2}\,\Omega R\,\rho_{2,1}\right) - (\gamma + \Gamma)\,\rho_{2,2}$$

Figure 4: *The evolution equations for the system.*

For my initial conditions, the all atoms were in the ground state. I then solved for the steady state solution of the density matrix in terms of Δ, Γ, γ, and Ω. From these solutions, I neglected any higher order Ω terms because their contribution is small. At this point I had

$$\rho_{1,2} = (i\,\Omega)/(2\,\gamma + \Gamma + 2\,i\,\Delta) \qquad (3)$$

My goal was to solve the equation:

$$d\Omega\,/dz = i * N * \kappa * \rho_{1,2} \qquad (4)$$

where κ is a constant equal to $3\,\Gamma\,\lambda^2/8\pi$ and N is the number of atoms in the system. Then, I plotted the real and imaginary parts of the result. I used the parameter values that were default for the Atomic Density Matrix package, setting Γ to 1 and γ to .01. The imaginary part is proportional to the dispersion of the optical field and is correctly antisymmetric. The dispersion is 0 on resonance.

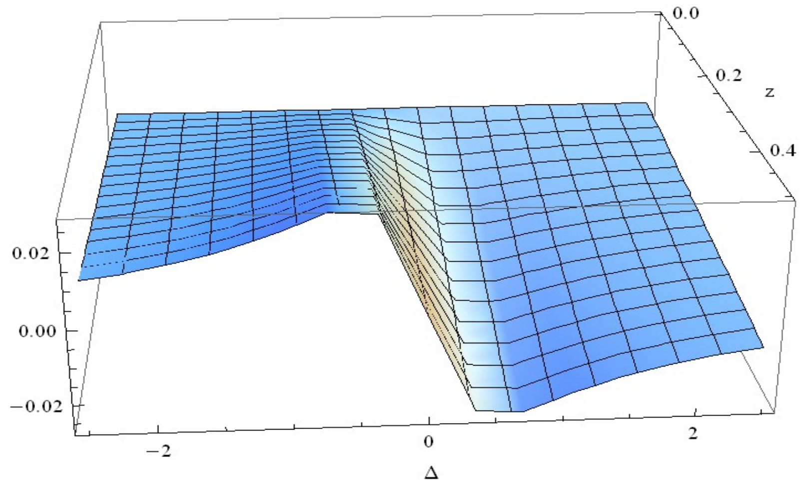

Figure 5: *A plot of the change in phase for light in a two level system.*

The real part of the equation shows the transmission of the optical field because the Rabi frequency is proportional to the energy.[3] The graph is correctly symmetric. The two level system obviously does not exhibit electromagnetically induced transparency.

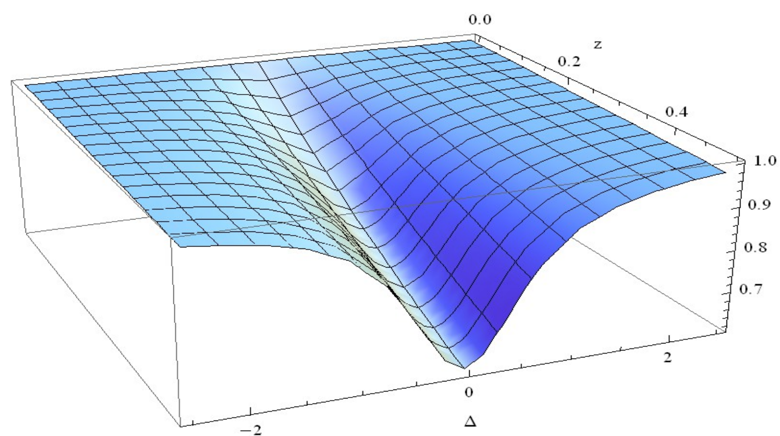

Figure 6: *A plot of the transmission of light through a two level system.*

Three Level System:

The three level system that we want to study is an example of a mixed state. We set it up in a Λ configuration where there are two ground states, |1> and |2> and one excited state, |3>. There are two optical fields, Ωa and Ωb. Ωa is the probe field and Ωb is the pump field. Ωa links |1> and |3> and Ωb links |2> and |3>. No ideal three level system occurs naturally. In the labratory, we have to replicate it by using alkali metals that can form a similar Λ configuration. The hyperfine splitting of the ground state allows us to use it as states |1> and |2>. [4]

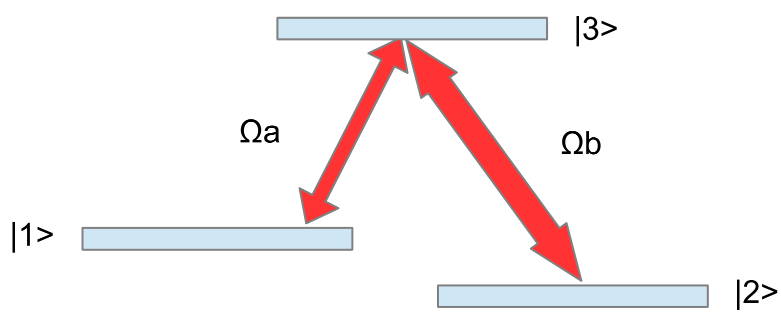

Figure 7: *Ωa is the probe field and Ωb is the pump field.*

If all the atoms remain in the ground state, no energy will be taken from the fields and there will be no absorption. The condition in which this happens is called two photon resonance because the difference in frequency between the two optical fields are the same as the difference in frequency between state |1> and |2>.[1] From this, we can get a so-called dark state that does not interact with the field:

$$|D\rangle = (\Omega_b e^{-i\omega 1t}|1\rangle - \Omega_a e^{-i\omega 2t}|2\rangle)/(\Omega_{12}+\Omega_{22})^{1/2} \qquad (5)$$

The orthogonal counterpart to the dark state is the bright state.[1] All of the atoms in the bright state will eventually end up in the dark state because atoms in the bright state will be excited and decay until they end up in the dark state. Atoms in the dark state will remain there as long as the state exists. This is electromagnetically induced transparency. The medium becomes transparent where it would ordinarily be most opaque. The relative phase between the two optical fields must be kept constant for the dark state to exist. Detuning is the difference between the light frequency and transition frequency. A detuning may be allowed, but must remain small, $\Delta\tau \approx\leq 1$. The dark state does not have an infinite lifetime. The characteristic lifetime given by τ. [1]

I started modeling the three level system in the same way as the two level system. I used the same assumptions for relaxation and repopulation, except that the excited state decayed evenly into both ground states.

$$\begin{pmatrix} \gamma & 0 & 0 \\ 0 & \gamma & 0 \\ 0 & 0 & \gamma + \Gamma \end{pmatrix}$$

Figure 9: *The relaxation matrix*

$$\begin{pmatrix} \frac{\gamma}{2} + \frac{1}{2}\Gamma\rho_{3,3} & 0 & 0 \\ 0 & \frac{\gamma}{2} + \frac{1}{2}\Gamma\rho_{3,3} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Figure 10: *The repopulation matrix*

I built the Hamiltonian and applied the rotating wave approximation.

$$\begin{pmatrix} \Delta a & 0 & -\frac{\Omega Ra}{2} \\ 0 & \Delta b & -\frac{\Omega Rb}{2} \\ -\frac{\Omega Ra}{2} & -\frac{\Omega Rb}{2} & 0 \end{pmatrix}$$

Figure 8: *The Hamiltonian with RWA for the three level system*

I used these matrices and the Liouville equation to form evolution equations for the system.

$$0 = \frac{\gamma}{2} - \gamma\,\rho_{1,1} - i\left(\frac{1}{2}\,\Omega Ra\,\rho_{1,3} - \frac{1}{2}\,\Omega Ra\,\rho_{3,1}\right) + \frac{1}{2}\,\Gamma\,\rho_{3,3}$$

$$0 = -\gamma\,\rho_{1,2} - i\left(\Delta a\,\rho_{1,2} - \Delta b\,\rho_{1,2} + \frac{1}{2}\,\Omega Rb\,\rho_{1,3} - \frac{1}{2}\,\Omega Ra\,\rho_{3,2}\right)$$

$$0 = \frac{1}{2}\,(-\gamma\,\rho_{1,3} - (\gamma + \Gamma)\,\rho_{1,3}) - i\left(\frac{1}{2}\,\Omega Ra\,\rho_{1,1} + \frac{1}{2}\,\Omega Rb\,\rho_{1,2} + \Delta a\,\rho_{1,3} - \frac{1}{2}\,\Omega Ra\,\rho_{3,3}\right)$$

$$0 = -\gamma\,\rho_{2,1} - i\left(-\Delta a\,\rho_{2,1} + \Delta b\,\rho_{2,1} + \frac{1}{2}\,\Omega Ra\,\rho_{2,3} - \frac{1}{2}\,\Omega Rb\,\rho_{3,1}\right)$$

$$0 = \frac{\gamma}{2} - \gamma\,\rho_{2,2} - i\left(\frac{1}{2}\,\Omega Rb\,\rho_{2,3} - \frac{1}{2}\,\Omega Rb\,\rho_{3,2}\right) + \frac{1}{2}\,\Gamma\,\rho_{3,3}$$

$$0 = \frac{1}{2}\,(-\gamma\,\rho_{2,3} - (\gamma + \Gamma)\,\rho_{2,3}) - i\left(\frac{1}{2}\,\Omega Ra\,\rho_{2,1} + \frac{1}{2}\,\Omega Rb\,\rho_{2,2} + \Delta b\,\rho_{2,3} - \frac{1}{2}\,\Omega Rb\,\rho_{3,3}\right)$$

$$0 = \frac{1}{2}\,(-\gamma\,\rho_{3,1} - (\gamma + \Gamma)\,\rho_{3,1}) - i\left(-\frac{1}{2}\,\Omega Ra\,\rho_{1,1} - \frac{1}{2}\,\Omega Rb\,\rho_{2,1} - \Delta a\,\rho_{3,1} + \frac{1}{2}\,\Omega Ra\,\rho_{3,3}\right)$$

$$0 = \frac{1}{2}\,(-\gamma\,\rho_{3,2} - (\gamma + \Gamma)\,\rho_{3,2}) - i\left(-\frac{1}{2}\,\Omega Ra\,\rho_{1,2} - \frac{1}{2}\,\Omega Rb\,\rho_{2,2} - \Delta b\,\rho_{3,2} + \frac{1}{2}\,\Omega Rb\,\rho_{3,3}\right)$$

$$0 = -i\left(-\frac{1}{2}\,\Omega Ra\,\rho_{1,3} - \frac{1}{2}\,\Omega Rb\,\rho_{2,3} + \frac{1}{2}\,\Omega Ra\,\rho_{3,1} + \frac{1}{2}\,\Omega Rb\,\rho_{3,2}\right) - (\gamma + \Gamma)\,\rho_{3,3}$$

Figure 11: *Evolution equations for the three level system*

In order to solve the system of equations, I had to include the density matrix property $\rho_{1,1}+\rho_{2,2}+\rho_{3,3}=1$. This equation is true for all density matrices because it only means that the whole population must be in one of the states at all times. [2] I neglected all higher order Ωa (probe field) terms because the probe field is weak. I am interested in the probe field propagation only. In order to achieve the correct EIT shape, I had to multiply the imaginary part by i and the real part by -i. I kept the pump field on resonance and so set Δb to 0. I set Ωb to .5. I set Γ to 1 and γ to .01 as in the two level system.

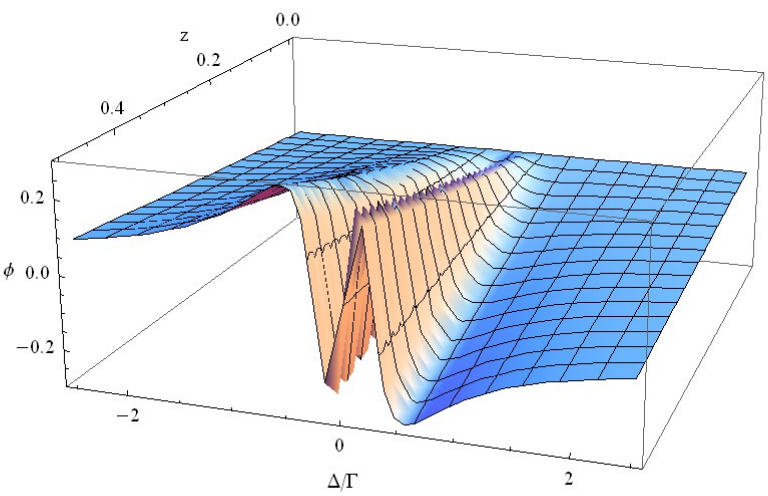

Figure 12: *A plot of the change in phase for light in a three level system.*

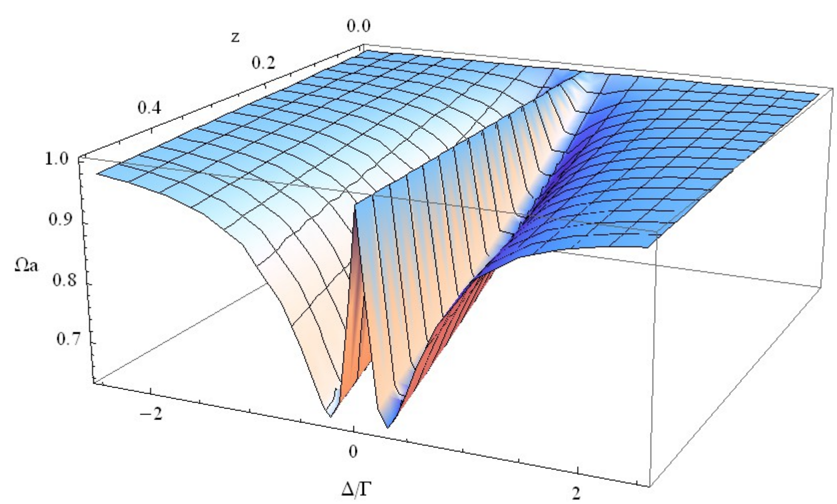

Figure 13: *A plot of the transmission of light through a three level system. Notice the peak in middle demonstrating electromagnetically induced transparency.*

Electromagnetically induced transparency is clearly visible. There is a peak in the middle where in the two level system transmission had been at a minimum.

Conclusion and Future Research:

The attempted next step was to add a sinusoidal variation to the input optical field. Then, this time varying addition could be changed to a random function to simulate noise. Unfortunately, when trying to solve with the sinusoidal addition, Mathematica could not solve the system of evolution equations and initial conditions for the density matrix elements. There has not been enough time to find the source of the error. Future research would include looking into squeezed light.

Squeezed states of light are special states in which the quantum noise in amplitude or phase is decreased below that of an ordinary coherent state. All light has noise as a result of its quantum nature. For a coherent state, this uncertainty is at a minimum, called the "shot noise." All measurements are limited by this level. For a coherent state, $(\Delta a_1 \Delta a_2) = \frac{1}{2}$, where Δa is the uncertainty. A laser will generally create a beam close to a coherent state, exhibiting shot noise. A squeezed state decreases the noise in one dimension below the shot noise. As a result of this decrease, the noise in the other dimension must increase. This gives the image of the uncertainty being squeezed in one direction and expanding in the other, giving it its name. Squeezed light has applications wherever extremely low noise is desirable. Measurements can be made more precise. Optical communications can be done below the shot noise limit.

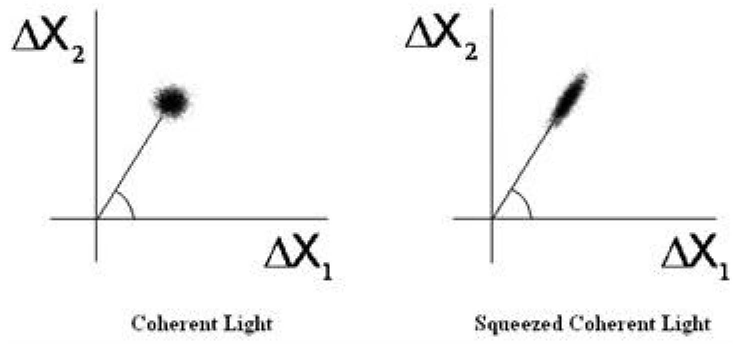

Figure 14: *A representation of the uncertainty in two dimensions. From http://physics.wm.edu/~inovikova/psrsqueezing.html*

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