1. Evaluate $\int x \sin(5x) \, dx$

2. Evaluate $\int \frac{x^4 + x^2 - 1}{x^3 + x} \, dx$ (Hint: you will use long division.)
3. Evaluate \( \int \frac{1}{x^2 \sqrt{x^2 + 9}} \, dx \)

4. Evaluate \( \int_{0}^{\pi/4} \sin^3(\theta) \cos^3(\theta) \, d\theta \)
5. Evaluate \( \int_0^\pi \frac{\sqrt{\arctan x}}{1 + x^2} \, dx \)

6. Either find the exact value of, or show the following improper integral diverges;
\( \int_{-\infty}^{\infty} xe^{-x^2} \, dx \)
7. Consider the curves $y^2 = x + 5$ and $x = 3 - y^2$, shown below. Set up and solve a single definite integral to find the area between the two curves.

![Graph of curves](image)

8. Set up and solve a single integral to find the volume of the solid obtained by rotating the region enclosed by $y = \sin(x)$, $x = \pi$, and the $x$-axis, around the $x$-axis. Sketch the region and a slice of the solid.
9. A force of \( f(x) = \frac{x}{\sqrt{x + 1}} \) is applied in moving an object from a position of \( x = 0 \) meters to \( x = 20 \) meters in a straight line. Calculate the work done during the last 10 meters of the journey. Include the proper work units in your final answer.

10. Find the length of the curve \( y = \ln(\sec x) \) from \( x = 0 \) to \( x = \frac{\pi}{4} \).
11. (a) Approximate \( \int_1^4 \frac{1}{x} \, dx \) using the Midpoint Rule with \( n = 3 \).

(b) Estimate the error in the above approximation given the error bound formula, \( |E_M| \leq \frac{K(b-a)^3}{24n^2} \).

Remember to show your work in finding the value of \( K \).

12. A plate has the shape of an isosceles triangle with two sides measuring \( \sqrt{2} \) feet and the base measuring 2 feet. The plate is submerged 2 feet below the surface of a fluid with density \( \delta = 90 \text{ lb/ft}^3 \). Set up and solve a definite integral to find the hydrostatic force acting on one side of the plate. Remember to include the proper unit for force in your final answer.
13. Find the solution to the differential equation that satisfies the given initial condition.

\[ (1 - i) \frac{dy}{dt} - y = 0, \quad y(2) = -4 \]

14. Suppose some population \( P = P(t) \) follows the logistic model where \( \frac{dP}{dt} = 0.8P(1 - \frac{P}{1000}) \).
   (a) What is the carrying capacity of the population?
   (b) Without finding the explicit solution to the equation, draw a rough sketch of the following two solution curves; \( P(0) = 1100 \) and \( P(0) = 250 \). Label the carrying capacity and the initial values for each curve on your sketch.
15. Determine if the infinite sequence \( a_n = \frac{\ln n}{n^2 + 1} \) converges or diverges. If it converges, find the limit.

16. Determine if the series \( \sum_{n=1}^{\infty} \frac{2n+1}{(n^2 + 1)^2} \) converges or diverges. If it converges, find the sum. Identify the series/test(s) used and the conditions that justify each step.
17-19. Determine if the series is *absolutely convergent*, *conditionally convergent* or *divergent* (state this). Identify the series/test(s) used and the conditions that justify each step.

17. \( \sum_{n=1}^{\infty} (-1)^n \frac{3^n - 2}{5^n + 3} \)

18. \( \sum_{n=1}^{\infty} (-1)^n \frac{n^{1/2}}{\sqrt{n^2 + 2}} \)
19. $\sum_{n=1}^{\infty} n^3$

20. Find the power series representation for $f(x) = \frac{x^3}{1+x^3}$. Find the radius of convergence.
21. Determine the radius and interval of convergence of the power series \( \sum_{n=0}^{\infty} \frac{(-2)^n(x - 3)^n}{n^2} \). Remember to show your work in testing the endpoints for the interval of convergence.

22. Find the Taylor series for \( f(x) = e^{-3x} \) at \( a = 2 \).