

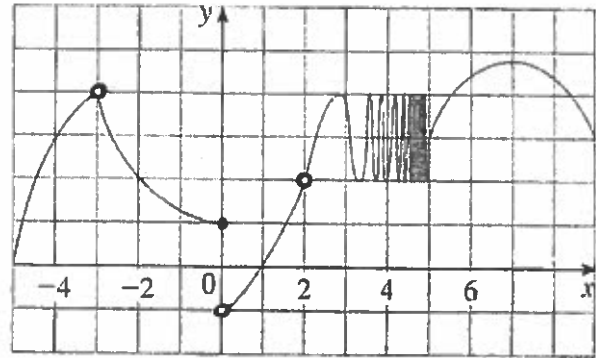
1. For the function h shown, state the value of each quantity. If it does not exist, state DNE and explain why.

(a) $\lim_{x \rightarrow -3} h(x) =$ _____

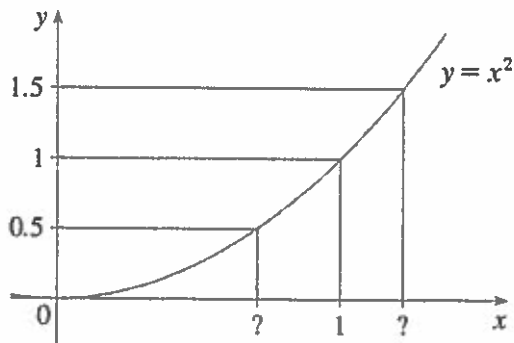
(b) $h(-3) =$ _____

(c) $\lim_{x \rightarrow 0} h(x) =$ _____

(d) $\lim_{x \rightarrow 5^-} h(x) =$ _____



2. Below is the graph of $f(x) = x^2$ with $\lim_{x \rightarrow 1} f(x) = 1$ clearly identified. Find the largest number δ such that if $|x - 1| < \delta$ then $|x^2 - 1| < \frac{1}{2}$.



3. Show the definition of continuity in finding the constants a and b so that $f(x)$ will be continuous for all x .

$$f(x) = \begin{cases} x^2 + bx + 2, & x < 5 \\ 8, & x = 5 \\ ax + 3, & x > 5 \end{cases}$$

4. (a – d) Find each limit, if it exists. If the limit does not exist, state DNE and explain why. If you use L'Hospital's rule, indicate this as well as the indeterminate form present. Justify any limit of $\pm\infty$.

$$(a) \lim_{x \rightarrow 4^-} \frac{x^2 + 3x}{x^2 - x - 12}$$

$$(b) \lim_{x \rightarrow -1} \frac{\sqrt{4-x^2} - \sqrt{3}}{x+1}$$

$$(c) \lim_{x \rightarrow +\infty} \frac{6x-3}{\sqrt{4x^2+3x}}$$

$$(d) \lim_{x \rightarrow 1} (1 + \ln x)^{\frac{1}{x-1}}$$

5. Use the limit definition of the derivative to find $f'(x)$ if $f(x) = \frac{x}{x-1}$.

6. (a – d) Find the derivative of each function. Simplify your final answer.

(a) $y = f(x) = (\cos(2x))^{1/x}$

(b) $g(t) = \frac{4+t}{te^t}$

(c) $y = \tan^{-1}\left(\frac{1}{x}\right)$

(d) $x^2 - 3xy + 2y^2 = -2$ (y is a function of x)

7. Show that the function $f(x) = x^3 + e^x$ has exactly one real root. (Hint: you will use the Intermediate Value Theorem and the Mean Value Theorem.)

8. Find the absolute maximum and absolute minimum value of $f(x) = x\sqrt{x-x^2}$ on the interval $[0,1]$.

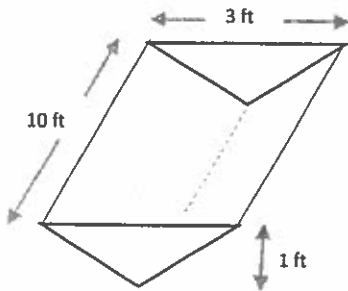
9. Use a linear approximation or differentials to estimate the value of $\tan\left(\frac{\pi}{4} + .02\right)$.

10. An unmitigated breakout of an unknown virus initially infects one person, but rapidly spreads, infecting 9 people in 5 days. It is assumed that the number of infections grows at a rate proportional to the population of infected people. Let $p(t)$ denote the number of infections after t days.

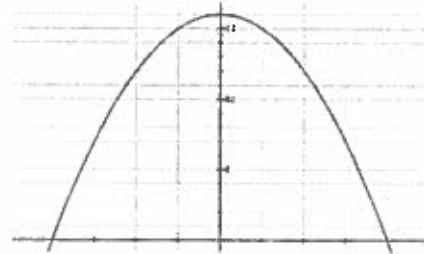
(a) Write the differential equation that $p(t)$ satisfies.

(b) Determine the relative growth rate of the virus.

11. A trough is 10 ft long and its ends have the shape of isosceles triangles that are 3 ft across at the top and have a height of 1 ft. If the trough is being filled with water at a rate of $12 \text{ ft}^3/\text{min}$, how fast is the water level changing when the water is 4 inches deep? Include proper units in your final answer.



12. A rectangle is constructed with its base on the x-axis and two of its vertices on the parabola $y = 16 - x^2$. What is the maximum area of such a rectangle?



13. Let $g(x) = \int_{-2007}^x f(t) dt$ where $f(t) = \int_1^{2t^2} \frac{\sqrt{1+u^3}}{u} du$. Find $g''(x)$.

14. Consider the function $f(x) = e^{2x-x^2}$. Find the following or state DNE if none exist.

(a) Interval(s) of increase _____ decrease _____

(b) Local maximum(s) _____ minimum(s) _____

Show supporting work here:

15. Consider the function $f(x) = x^4 - 4x^3 + 10$. Find the following or state DNE if none exist.

(a) Interval(s) where f is concave up _____ concave down _____

(b) Inflection point(s) _____

Show supporting work here:

16. Find the general antiderivative F of the function $f(x) = 5^x - \frac{3}{x}$.

17. A car is traveling at $\frac{220}{3}$ ft/s when the brakes are fully applied, producing a constant deceleration of 22 ft/s². What is the distance traveled before the car comes to a stop?

18. Evaluate the integral, $\int_{-1}^1 1 - \sqrt{1 - x^2} dx$, by interpreting it in terms of area.

19. Use the limit definition of the definite integral to evaluate $\int_{-2}^0 3x - 5 dx$. Note: $\sum_{i=1}^n i = \frac{n(n+1)}{2}$

20. Evaluate each integral.

(a) $\int \sqrt{x}(6 + 5x) dx$

(b) $\int_1^4 \frac{\sqrt{y} - y}{y^2} dy$

21. Write True if the statement is always true, otherwise write False and correct what is underlined.

(a) _____ $\lim_{x \rightarrow 0^+} \ln(\sin x) = \underline{1}$.

(b) _____ Estimating the area under $f(x) = \sin x$, from $x = 0$ to $x = \pi$ using $n = 3$ rectangles and left endpoints as sample points yields the value $\underline{\pi/3}$.

22. The graph of f' is shown in the figure. On the same coordinate plane, sketch the graph of f if f is continuous on $(0, 3)$ and $f(0) = -1$.

