Precise Definition of a Limit

(2.4)

Prelab: Read definition 1 on page 83. Review Figures 3 – 6 on page 107. Read Example 2 on page 108 as well as the three paragraphs before this example.

In previous sections you were working with the "intuitive" definition of a limit. Using the "precise" definition, we can quantify how close x must be to a in order for $f(x)$ to be within some specified distance from L.

Precise Definition of a Limit: Let f be a function defined on some open interval that contains the number a, except possibly at a. We say that the limit of $f(x)$ as x approaches a is L, and we write

$$\lim_{x \to a} f(x) = L$$

if for every number $\varepsilon > 0$ there is a number $\delta > 0$ such that

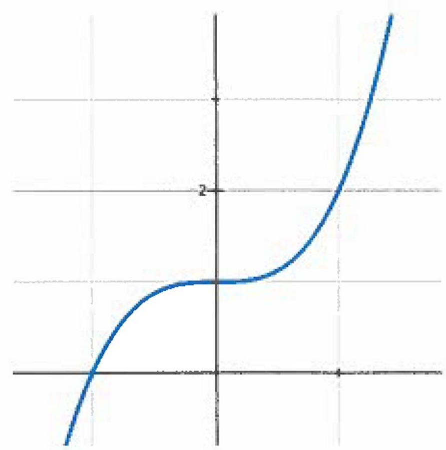
To understand the definition above, a visual approach can be helpful.

Example 1: The graph of $f(x) = x^3 + 1$ is shown.

(a) Illustrate the above definition as it applies to the limit equation, $\lim_{x \to 1} f(x) = 2$.

(b) On the graph provided, label $a, L,$ and ε, where $\varepsilon = 0.5$.

(c) Calculate the value of δ (this requires a calculator). That is, determine how close to 1 we must take x in order for $f(x)$ to be within 0.5 of 2.



The example above shows how the precise definition of a limit is used to find a specific δ, given a specific ε. One example is not enough to *prove* the limit written in 1(a). The *proof* of this limit must hold for **any** ε. The proof involves two parts:

1.

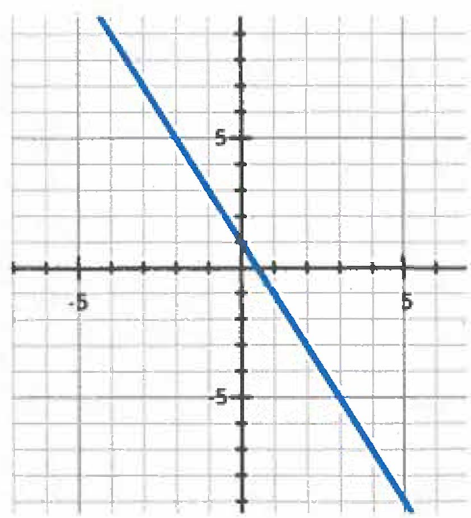
2.

Example 2: (a) Prove $\lim_{x \to 4}(1-2x) = -7$ using the ε, δ definition (precise definition) of a limit.

1.

2.

(b) Illustrate the precise definition and label $a, L, \varepsilon,$ and δ.



Math 111 S25 Lab 2 Exercises Name: ______________________ Section: _____ Score: ____

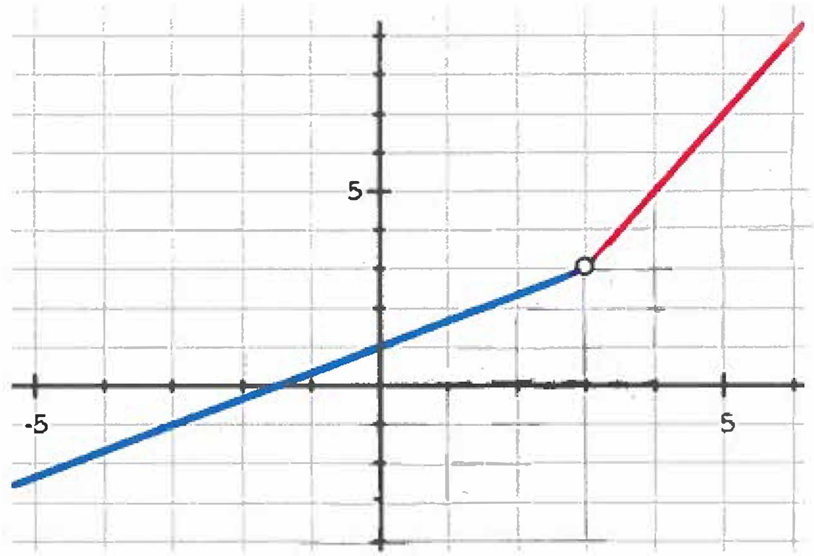
Work each problem showing all supporting work. You may use your textbook, lab and notes. Students may work cooperatively but each submits his/her own set of Lab Exercises.

1. (a) Use the graph below to estimate the following:

$\lim_{x \to 3} f(x) =$ _______

$\delta =$ ______ when $\varepsilon = 2$

(b) Label a, L, ε and δ on the graph as in Exercises 1 and 2.



2. (a) Complete the precise definition of a limit : We say $\lim_{x \to a} f(x) = L$, if for every $\varepsilon > 0$ there exists a $\delta > 0$ such that ______________________ whenever ___________________________ .

(b)Prove $\lim_{x \to 3}(5 - 2x) = -1$ using the ε, δ definition (precise definition) of a limit.

 Name: ______________________

3. (a) The formal limit definition, "for every $\varepsilon > 0$, there exists a $\delta > 0$ such that,

$\left|\sqrt{13-x}-2\right| < \varepsilon$ whenever $\left|x-9\right| < \delta$", defines the limit equation______________________.

(b) Find δ, when $\varepsilon = 1$. Show the steps of computation below.

(c) Illustrate the precise definition on the graph of $f(x)$ below and label the symbol and value for $a, L, \varepsilon,$ and δ.