Calculating a Limit
(2.2, 2.3)

Prelab: Read sections 2.2 and 2.3 in your textbook. Review: Example 9 and Figure 17 (page 91), Example 3 (page 98), Example 6 (page 99), and Example 8 (page 121).

I. In calculating a limit, we attempt to answer the question, "what happens to $f(x)$ as x gets closer and closer to a?" In this lab, we investigate some limits and important related concepts.

Example 1. Consider $f(x) = \dfrac{x^4 - 1}{x^3 - 1}$. In order to evaluate $\lim\limits_{x \to 1} f(x)$ without a graph or a table of values, we use the limit laws and direct substitution property (2.3) as well as some algebra.

*Important concept 1: What does "direct substitution" lead to? _____

This result is an **indeterminate form of a limit.** This is another way of saying "you can't use direct substitution so you'll need to think of some clever algebra (or something else) in order to **determine** the limit." So, we do the following:

$$\lim_{x \to 1} f(x) = \lim_{x \to 1} \frac{x^4 - 1}{x^3 - 1} =$$

Two other indeterminate forms are $\dfrac{\pm \infty}{\pm \infty}$ and $\infty - \infty$. You will learn more indeterminate forms later.

*Important concept 2: The *functions* $f(x) = \dfrac{x^4 - 1}{x^3 - 1}$ and $f(x) = \dfrac{(x^2 + 1)(x + 1)}{x^2 + x + 1}$ are not equal but the *limit of each function* (as $x \to 1$) yields the same value so the *limits* are equal.

II. One-Sided Limits

The limit found in Example 1, $\lim\limits_{x\to 1}\dfrac{x^4-1}{x^3-1}=\dfrac{4}{3}$, indicates that as values of x approach 1 from both the left (values below 1) and right (values above 1), the value of $f(x)$ approaches $\dfrac{4}{3}$. This satisfies the theorem:

$$\lim\limits_{x\to a} f(x)= L \text{ if and only if both } \lim\limits_{x\to a^-} f(x)= L \text{ and } \lim\limits_{x\to a^+} f(x)= L.$$

Example 2: Let $f(x)=\begin{cases} 2x, & x<-1 \\ x^2+1, & -1\le x<1 \\ (x-2)^3, & x\ge 1 \end{cases}$. Find the following:

(a) $\lim\limits_{x\to 1^-} f(x)$

(b) $\lim\limits_{x\to 1^+} f(x)$

(c) $\lim\limits_{x\to 1} f(x)$

III. Infinite Limits

Remember the value "L" in the intuitive definition of a limit in your textbook? What if $f(x)$ does not approach a number, L, but instead gets infinitely large (or infinitely small)?

Example 3: Evaluate $\lim\limits_{x\to 5^+}\dfrac{x+1}{x-5}$. (What does direct substitution lead to?) Justify any limit of $\pm\infty$ and state DNE where applicable.

Example 4: Evaluate $\lim\limits_{x\to 0^+}\left(\dfrac{1}{x}-\ln x\right)$ (What does direct substitution lead to?) Justify any limit of $\pm\infty$ and state DNE where applicable.

FYI: The forms $\infty+\infty$ and $-\infty-\infty$, are determinate but the form, $\infty-\infty$, **is indeterminate.**
* Important Concept 3: A "limit" of infinity does not mean a limit exists. It is one way in which a limit fails to exist so include "DNE" (does not exist) along with either $\pm\infty$ when finding these "limits".
* Important Concept 4: Identifying an infinite limit gives important information about the behavior of a function.
* Important Concept 5: In both Example 3 and Example 4 above, the limit **does not exist**. This does not mean the limit is **indeterminate**. There is a difference!

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For each limit below: (a) State the indeterminate form present *if any,* otherwise, state "NONE". (b) Calculate each limit or justify why it does not exist (DNE, state this) showing intermediate steps and use of the limit laws. Final answers will either be a numerical value, ∞, or $-\infty$. Do not use L'Hospital's Rule (section 4.4).

You may use your textbook, lab, notes and peer collaboration (each student must submit their own assignment, however). **Do not use a calculator**, **table of values or graph unless otherwise instructed.** Include proper intermediate steps for full credit.

1. $\lim\limits_{x \to 3^-} \dfrac{x-4}{x^2-9}$ (Work must include the limit of both the numerator and denominator.)

(a)

(b)

2. $\lim\limits_{h \to 7} \dfrac{h-7}{3-\sqrt{h+2}}$

(a)

(b)

3. $\displaystyle\lim_{x\to10}\dfrac{\dfrac{1}{10}-\dfrac{1}{x}}{x-10}$

(a)

(b)

4. $\displaystyle\lim_{x\to4}f(x)$, where $f(x)=\begin{cases}2-3e^{4-x}, & x>4\\[2mm] 2-\dfrac{3}{4}x, & x<4\end{cases}$ (You must show use of the proper theorem.)

(a)

(b)

5. $\displaystyle\lim_{x\to0^{+}}\left(\ln x-\dfrac{1}{x}\right)$ (Hint: find the limit of each term separately. You may also wish to use the graph $y=\ln x$. Remember, no calculator.)

(a)

(b)