

Maximum and Minimum Values

(4.1)

Prelab: Read page 276 which includes definitions 1 & 2. Read the definition of a critical number (p. 280). Read Fermat's Theorem (p. 279).

I. Derivatives are used to find maximum and/or minimum values (also called extreme values or extrema) of a function. There are local (or relative) extrema as well as global (or absolute) extrema.

Example 1: Use the given graph and the proper definitions to determine the absolute and local maximum and minimum value(s) of the given function.

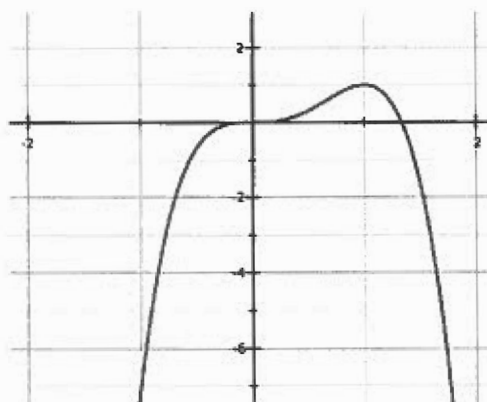
(a) $f(x) = 4x^3 - 3x^4 ; (-\infty, \infty)$

Absolute maximum:

Absolute minimum:

Local maximum:

Local minimum:



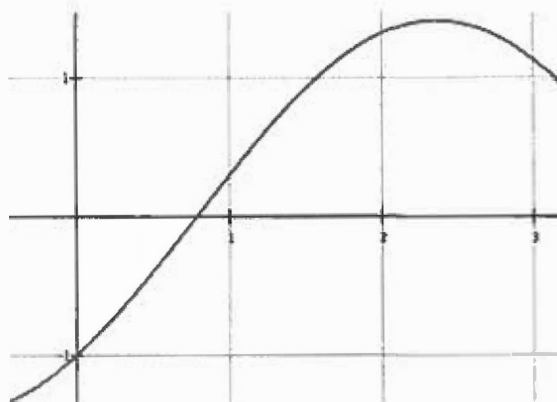
(b) $g(x) = \sin x - \cos x ; [0, \pi]$

Absolute maximum:

Absolute minimum:

Local maximum:

Local minimum:



*Notice an endpoint cannot satisfy the definition of a *local* maximum or minimum.

Question: If the interval in example 2(b) above had been $(0, \pi]$, what extreme value for $g(x)$ changes and why?

Using only the graph of a function, we can just *approximate* the exact location of absolute and local extrema. Calculus methods are needed to find the *exact* location(s).

II. Fermat's Theorem gives us information regarding extrema. It states if the derivative of a function exists at a point where there is a local maximum or minimum, then that derivative will equal zero.

Example 2: Verify Fermat's Theorem for each function in the previous example.

(a) $f(x) = 4x^3 - 3x^4; (-\infty, \infty)$:

*Notice, the converse of Fermat's Theorem is not necessarily true:

(b) $g(x) = \sin x - \cos x; [0, \pi]$:

III. **Closed Interval Method:** Used to find absolute max. and min. of a continuous function on a closed interval.

- 1.) Find the values of the function at the endpoints.
- 2.) Find the critical numbers of the function and the function values at these critical numbers.
- 3.) Compare these function values; the largest is the absolute max. and the smallest is the absolute min.

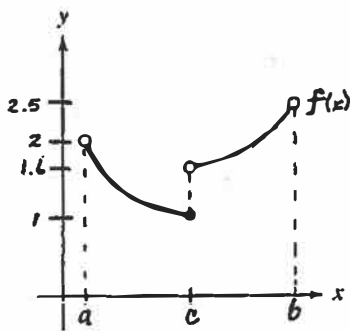
Example 3: Find the absolute maximum(s) and the absolute minimum(s) of $g(x) = \sin x - \cos x; [0, \pi]$.

Example 4: Find the absolute maximum(s) and the absolute minimum(s) of $f(x) = e^{-x^2}; [-2, 1]$.

Work each problem showing all supporting work ON THIS PAPER. You may use your textbook, lab and notes. Students may work cooperatively but each submits his/her own set of Lab Exercises. No calculator.

1. Use the given graph and the proper definitions to determine the absolute and local maximum and minimum value(s) of the given function. For extrema that do exist, your answer must include the location as a coordinate pair and use of the proper definition as in the Example 1. For extrema that do not exist, state none.

(a)



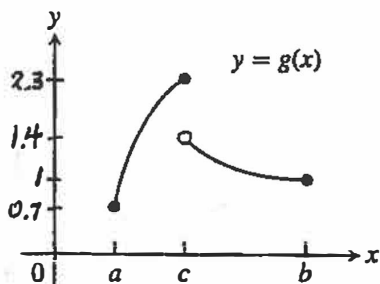
Absolute maximum:

Absolute minimum:

Local maximum:

Local minimum:

(b)



Absolute maximum:

Absolute minimum:

Local maximum:

Local minimum:

2. Review the definition of a critical number. For each function below, find any critical number(s).

(a) $f(\theta) = 2 \sec \theta + \tan \theta$, $0 < \theta < 2\pi$ (fractional answer(s) in terms of π)

(cont.) (b) $f(x) = x\sqrt{4-x}$, $x \leq 4$

3. Use the closed interval method to find the absolute maximum and absolute minimum values of each function. Final answers should include the coordinate pair(s).

(a) $f(x) = \frac{x^2}{x^2+3}$, $[-1, 1]$

(b) $g(x) = x\sqrt{4-x^2}$, $[-2, 2]$