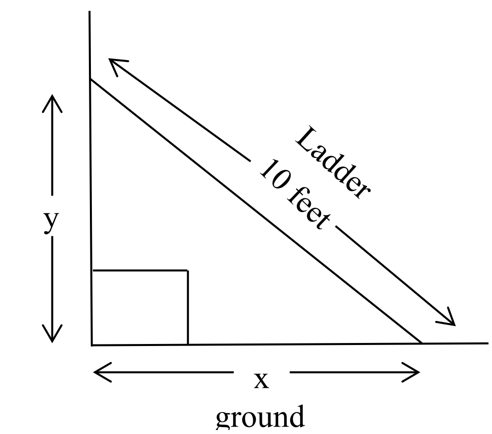
Related Rates

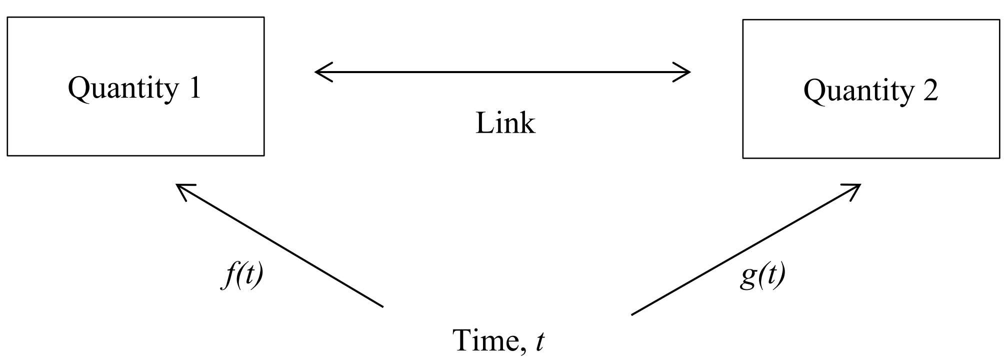
In each related rate problem there can be variations in the details. The problems, however, have the same general structure.

I. Relating Quantities: Independent of any connections with calculus, one sometimes encounters two measured quantities that are linked to one another by some relationship, frequently a geometric one.

Example 1. A ladder 10 feet long rests against a vertical wall. Let x measure the distance from the wall to the foot of that ladder and y measure the distance from the ground to the top of that ladder. Write a formula that links x and y.



II. Introducing Time: Now suppose a more dynamic situation is in process such that, each of the quantities above change with respect to a new variable, t, time. Now, both f and g are functions of time. So $f(t)$ and $g(t)$ provide values of Quantity 1 and Quantity 2, respectively.



We say the functions f and g are *link-consistent*, which means that for each time instant, t, the values of f and g satisfy the linking condition. If this sort of consistency is to hold, it is clear that the two functions, $f(t)$ and $g(t)$, cannot be chosen independently of one another.

We can easily include the linking condition that two consistent functions of the independent variable t must satisfy, *even though we do not know what those functions are*. Practice is helpful in acquiring the confidence to operate in this atmosphere of ignorance.

Example 2: Write the linking equation for the Example 1 ladder problem, using the independent variable t.

In related rate problems, we are interested in finding the rate of change (with respect to time) of a variable quantity as a dynamic situation unfolds.

III. The Plan of Attack: We employ the following steps in solving these related rate problems.

1. Sketch a diagram of the problem if possible and assign variables to quantities that will change with time.
2. Write the GIVEN information (constant values and known rates of change) using proper notation.
3. Write what the problem asks you to FIND using proper notation. Include "when" you want this.
4. Write an equation that relates the quantities in the problem.
   -there should be no derivatives in this equation
   -if a variable in your equation remains constant, substitute this value into the equation.
5. Differentiate each side of the equation *with respect to time*.
6. Substitute GIVEN information into resulting equation and solve for the unknown rate you wish to FIND. You might need to calculate a "GIVEN" value first (see example 1.).

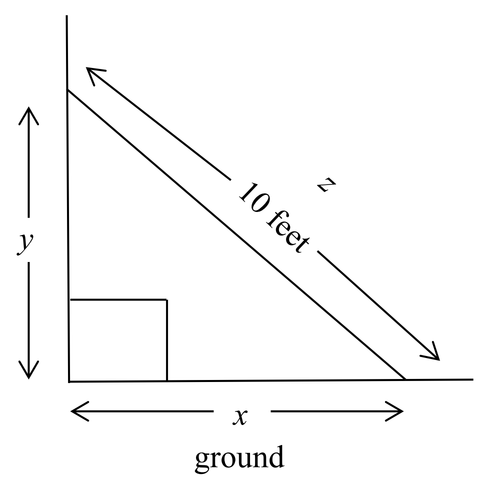
Example 3: The top of the ladder shown slides down the wall at the rate of 1 foot per second. How fast is the bottom of the ladder sliding away from the wall when the top of the ladder is 6 feet from the ground?

1. The sketch and assigned variables:

2. GIVEN:

3. FIND:

4. We found the equation that relates the variables in Example 1. In Example 2, we introduced the dynamic situation of time. (Note, we normally do not express the variables as functions of time but treat them as such.)

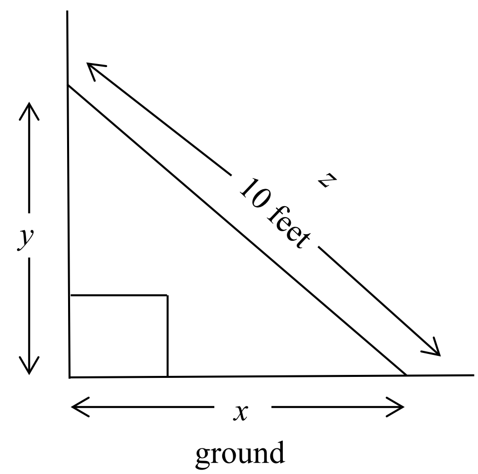


5. To differentiate with respect to time, we use the chain rule and implicit differentiation.

In the next example, we use the same physical situation described above but here, we find the value of a different quantity.

Example 4: A ladder 10 feet long rests against a vertical wall. Let x measure the distance from the wall to the foot of that ladder and y measure the distance from the ground to the top of that ladder. The top of the ladder shown slides down the wall at the rate of 1 foot per second. How fast is the angle between the ladder and the ground changing when the top of the ladder is 6 feet from the ground?

Use the steps provided on the previous page to solve the problem.

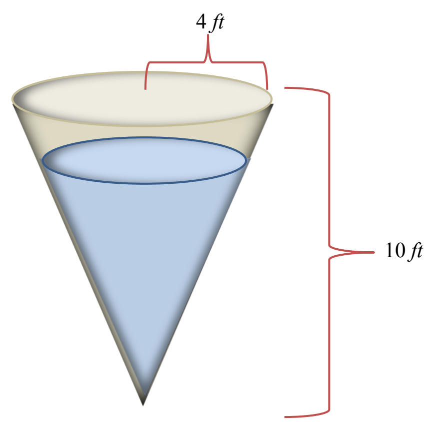


Math 111 S24 Lab 6 Exercises Name: ______________________________ Section: _____ Score: ______

Work each problem showing all supporting work ON THIS PAPER. You may use your textbook, lab and notes. Students may work cooperatively but submit individual Lab Exercises. Final answers should be in exact values and include units where applicable. No calculator unless noted otherwise.

1. Water drains from the conical tank shown at a rate of $3\frac{ft^3}{min}$. Use r for radius, h for height and t for time. Complete the steps below to find how fast the water level is changing when $h = 2\,ft$.

(a) GIVEN:



(b) FIND:

(c) Write the equation that relates volume and height.

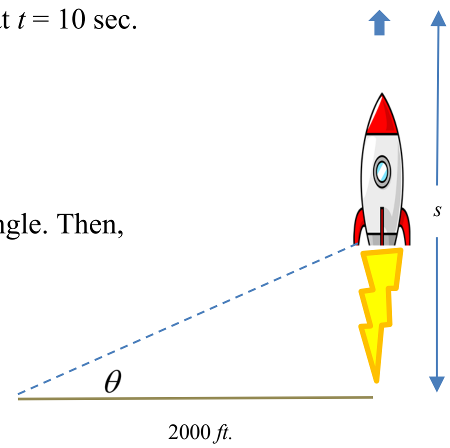
(d) Differentiate with respect to time then find how fast the water level is changing with $h = 2\,ft$.

2. An expanding plate has the shape of an equilateral triangle. Each side is increasing at a constant rate of $2\frac{cm}{h}$. Find the rate at which the area is increasing when a side is 4 cm.

Math 111 S24 Lab 6 Exercises (cont.) Name: ______________________________

3. A rocket has taken off and is rising vertically according to the position equation, $s = 50t^2$ (use this to find the "given" rate of change). It is being tracked by a device located 2000 *ft* from the point of lift-off. The angle θ (in radians) is the angle between the tracking line and the ground. Find the rate of change in the angle θ, 10 seconds after lift-off.

(a) GIVEN: Use the position function to find the given rate of change at $t = 10$ sec.



(b) FIND: Write the derivative you want to find.

(c) Write an equation relating the variable θ and two sides of the triangle. Then, differentiate with respect to time.

(d) Solve for the unknown derivative in (b) after substituting in known values. Note: first you will need to solve for both s and the length of the hypotenuse at $t = 10$ sec.. Use a calculator for your final computations.

4. Boat A and Boat B depart at noon from their common anchored location. Boat A travels straight north at a rate of 10 knots (nautical miles per hour or nm/hr). Boat B travels straight east at a rate of 15 knots. Determine the rate of change in the distance between the boats at 2pm.