Derivatives and Graphing
(2.2, 2.6, 4.3 and 4.5)

Prelab: Review the definitions of vertical asymptotes (p. 90) and horizontal asymptotes (p.128). Review Example 7 (p.298) and Example 1 (p. 317).

I. The first derivative tells us about slope and the second derivative tells us about concavity.

II. Many functions are a combination of different slopes and curvatures.
Recall that points on the graph where either \( f'(x) = 0 \) or \( f'(x) \) does not exist, are called “critical points” (CP). An inflection point (IP) on a graph is where \( f''(x) \) changes sign.

Example 1: On the graph below, mark slope and curvature using the proper derivative inequality (as seen on the graph above). Label the critical and inflection points.

III. Derivative Functions: The previous example gave us a general qualitative description of a graph. As you know, the derivatives of a function are themselves functions, and have their own graphs.

Example 2: Use the information in Example 1 and the graph of the function to sketch graphs of the first and second derivatives.
Example 3: Here is the graph of the derivative $f'(x)$ of a continuous function $f(x)$.

(a) On what intervals is $f$ increasing? Decreasing?

(b) Where does $f$ have a local maximum? Minimum?

(c) On what intervals is $f$ concave upward? Concave downward?

(d) State where $f$ has (an) inflection point(s).

IV. We use the first and second derivatives to locate important graphical features of a function.

Example 4: Consider the function $f(x) = x - 3x^{2/3}$. Find (a) intervals of increase/decrease (b) location(s) of maximum(s)/minimum(s) (c) intervals of concavity and (d) location(s) of any IP(s).

* We use this information along with the location of roots and the existence of horizontal and vertical asymptotes to graph a function.
Work each problem showing all supporting work ON THIS PAPER. You may use your textbook, lab and notes. Students may work cooperatively but each submits his/her own set of Lab Exercises. No calculators.

1. On the graph below, mark slope and curvature using the proper derivative inequalities (as seen on page 7-1 and Example 1). Label the critical and inflection points.

2. The graph of the derivative \( f'(x) \) of a continuous function \( f(x) \) is shown. Find the following or state “none”.

   (a) On what intervals is \( f \) increasing? Decreasing?

   (b) Locate the \( x \)-value(s) where \( f \) has a local maximum? Minimum?

   (c) On what intervals is \( f \) concave upward? Concave downward?

   (d) Locate the \( x \)-value(s) where \( f \) has an inflection point.
3. For the function \( f(x) = x - 4\sqrt{x} \), find the following if they exist or state “none”. 
(a) the zeroes (roots) 
(b) intervals of increase/decrease 
(c) location(s) \((x, y)\) of local maximum(s)/minimum(s) 
(d) intervals of concavity and 
(e) location(s) \((x, y)\) of any point(s) of inflection. You do not need to graph the function.