Reviewing Derivatives

In this lab we review some rules of differentiation as we use the derivative in application. The general forms of the rules for differentiation are on Reference page 5 in the textbook.

I. **Slopes, Tangents and Normals**:

Example 1: Given $y = \frac{x}{2} + \frac{1}{2x-4}$, determine the points (if any) where the slope is $-\frac{3}{2}$.

Example 2: Find the equation of the line that is tangent to the curve, $x^2y^2 - 2x = 4 - 4y$, at (2, -2).

What change is made to find the equation of the line that is normal to the curve at (2, -2)?

II. **Rate of Change:**

Example 3: van der Waals' equation for a specific gas is $\left(P + \frac{5}{V^2}\right)(V - 0.03) = 9.7.$

Find the rate of change in volume with respect to pressure at the point (5, 1).

III. **Velocity and Linear Movement**: Imagine an object is located at the origin and can only travel along a horizontal axis. Its position along that line is given by a function, $s(t)$, where t represents time and $s(t)$ gives the horizontal position of the object at time t. Since the object is moving, it has a velocity, $v(t)$.

The velocity of the object is the first derivative of the position function, $s(t)$. Velocity is positive when the object is moving to the right, zero in the instant when it pauses to change direction, and negative when it moves to the left.

Position/Displacement	*s(t)*	*Example:* $s(t) = t^2$ (units are feet)
Velocity	$v(t) = s'(t)$	$v(t) = 2t$ (units are ft/sec)
Acceleration	$a(t) = v'(t) = s''(t)$	$a(t) = 2$ (units are ft per sec per sec)

Example 4: An object moves along a line according to a law of motion $s = f(t)$, where t is measured in seconds and s in feet. Let $f(t) = t^3 - 9t^2 + 15t + 10$.

a) When is the object at rest?

b) When is the object moving in the positive direction ("forward" or to the right)?

c) Sketch the motion of the object along the line.

Math 111 S22 Lab 5 Exercises Name: _____ Section: _____ Score: _____

Work each problem showing all supporting work ON THIS PAPER. You may use your textbook, lab and notes. Students may work cooperatively but each submits his/her own set of Lab Exercises. No calculator, use exact values.

1. Find an equation of the tangent line to the curve $xy + x^2y^2 = 6$ at (2, 1).

2. Find an equation of the line that is normal to the curve $y = 3\sin x \cos x$ at the point where $x = \frac{\pi}{3}$.

3. The output of an economic system *Q*, subject to two inputs, such as labor *L* and capital *K*, is sometimes modeled by the Cobb-Douglas production function as shown:

$Q = cL^a K^b$

Let $Q = 960$, $a = \frac{1}{3}$, $b = \frac{2}{3}$, and $c = 40$. Find the rate of change of capital with respect to labor, $\frac{dK}{dL}$.

4. An object moves along a line according to a law of motion $s = f(t)$, where *t* is measured in seconds and *s* in feet. Let $f(t) = t^4 - 8t^2$.

a) At what time(s) is the object at rest?

b) When is the object moving in a negative direction ("backwards" or to the left)?

c) Sketch the motion of the object along the line (as in Example 4). Identify the important locations such as the start position and where the object changes directions.