



The Definite Integral

(5.2)

Prelab: Read Definition 2 (p. 378), Theorem 4 (p. 380) and section on Properties of Definite Integrals (p. 385). Review Example 2 (p. 381) and Example 4 (p. 384).

I. Recall, in section 2.7, we learned the limit definition of the derivative. Fortunately, we know there are rules for differentiation that allow us to find the derivative more quickly. One of the topics covered in section 5.2 is the limit definition of the integral. Once again, we will find this isn't necessarily a preferred method of evaluating an integral. This limit definition, however, is fundamental to understanding the definite integral and its many applications.

In order to understand and use the definite integral/limit equality,

$$\int_a^b f(x)dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i^*) \Delta x,$$

we must define Δx and x_i , then set up and solve the resulting Riemann sum limit. Recall,

$$\Delta x = \frac{b-a}{n} \quad \text{and} \quad x_i = a + i\Delta x$$

Example 1: For the definite integral $\int_1^4 (x^2 - 4x + 2) dx$, define Δx and x_i , then set up (but do not solve) the limit of the Riemann sum that defines this integral.

Notice the Riemann sum is expressed in terms of both n and i . The limit is all about $n \rightarrow \infty$ (not i).

To evaluate the limit of this Riemann sum, we use combinations of the following rules:

$$1. \sum_{i=1}^n i = \frac{n(n+1)}{2}$$

$$4. \sum_{i=1}^n c = cn$$

$$2. \sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$5. \sum_{i=1}^n ca_i = c \sum_{i=1}^n a_i$$

$$3. \sum_{i=1}^n i^3 = \left[\frac{n(n+1)}{2} \right]^2$$

$$6. \sum_{i=1}^n (a_i \pm b_i) = \sum_{i=1}^n a_i \pm \sum_{i=1}^n b_i$$

Example 2: Use the limit definition to evaluate the integral in Example 1, $\int_1^4 (x^2 - 4x + 2) dx$.

II. Area

The properties of definite integrals help us evaluate them in terms of area. Some definite integrals define the area of simple geometric shapes.

Example 3: For each definite integral, sketch the area interpretation then evaluate in terms of areas.

(a) $\int_2^{10} \left(7 - \frac{1}{2}x\right) dx$

(b) $\int_{-1}^1 (1 - \sqrt{1-x^2}) dx$

Math 111 S22 Lab 11 Exercises Name: _____ Section: _____ Score: _____

Work each problem showing all supporting work ON THIS PAPER. You may use your textbook, lab and notes. Students may work cooperatively but each submits his/her own set of Lab Exercises. No calculator.

1. For each definite integral, (i) define Δx and x_i , (ii) set up the limit of the Riemann sum that defines this integral and (iii) solve the integral *using the limit definition*. Your supporting work must include proper set up and evaluation of the limit.

(a) $\int_0^2 (5 - x^2) dx$

(b) $\int_0^1 x^3 dx$

(c) $\int_1^4 (x+2) dx$

2. (a) Sketch the area defined by the integral $\int_0^4 \left(\frac{x}{3} + \sqrt{16-x^2} \right) dx$ and (b) evaluate the integral in terms of area. Hint: you can use property 2 on page 385 and first split this integral into the sum of two integrals.