Optimization

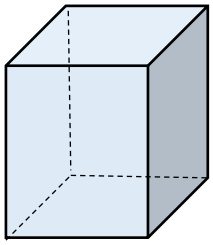
(4.7)

Prelab: In section 4.1, review the Closed Interval Method on page 281. In section 4.7, read the suggested problem-solving steps on pages 330 and 331. Review examples 1 - 5 and the First Derivative Test for Absolute Extreme Values.

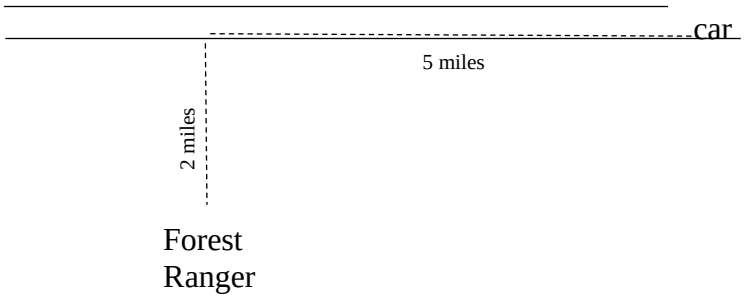
One of the most powerful applications of calculus is in optimization. One might want to know how to minimizing cost, the amount of materials used or the time it takes to travel from one point to another. Other optimization problems might be that of maximizing profits or the volume of a container. These modeling problems usually involve the same general steps:

1. Understand the problem…draw a sketch and assign values and variables.
2. Write the formula for the **objective function**.
3. Write the formula for the **constraint function** (if there is one). This is used to express the objective function in a single independent variable.
4. Determine the domain based on the physical problem and note if it is a closed interval.
5. Find the critical numbers of the objective function.
6. Use either the closed interval method or the first derivative test to find the optimal solution.

Example 1: A closed box is constructed out of two different types of metal. The top and bottom of the box are square and the metal costs \$1.00 ft^2. The metal for the sides costs \$2.00 ft^2. Find the dimensions that minimize the cost if the box has a volume of $20\ ft^3$.



Example 2: A forest ranger is in the forest 2 miles away from a straight road. His car is located 5 miles down the road as seen in the sketch. If he can walk 3 miles per hour in the forest and 4 miles per hour on the road, determine the route he should take to minimize the time it takes for him to reach his car.

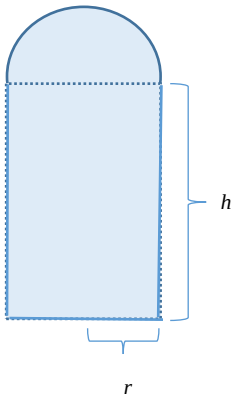


Math 111 S22 Lab 10 Exercises Name: _____ Section: _____ Score: _____

Work these problems ON THIS PAPER. You may use your textbook, lab and notes. Remember to include units where applicable. Students may work cooperatively but each submits his/her own set of Lab Exercises You may use a calculator. Unless the optimization method is indicated in the problem, clearly indicate your method. *Your supporting work must show use of either the closed interval method (if a closed interval exists) or the first derivative test.*

1. Find the coordinates of the point on the graph of $y = x^2$ that is closest to the point $(18, 0)$. Express your final answer in exact values, not decimal values.

2. A large decorative window has the shape of a rectangle with a semicircle attached to the top as shown. There is 600 feet of outer-edging material to create the opening for such a window. Create the proper objective function and use the first derivative test to find the dimensions (height of rectangle and width of rectangle (or radius of circle)) that maximize the area of the window. Decimal values accepted.



3. A rancher travels in his truck from his ranch to the nearest city (see diagram). He can travel across the open range of his ranch at a speed of 40 mph and along the highway at 50 mph. Determine the route he should take in order to reach the city in the shortest amount of time. Your supporting work must include all computations using the proper optimization method.

Decimal answers are acceptable.