Departmental Narrative for Math 108

Math 108 is an introductory calculus course designed primarily for prospective School of Business students. It differs from Math 111, the standard Calculus I course, in four ways: Math 108 includes nothing on trigonometric functions; it does not prepare students for any subsequent mathematics course; it includes a wider array of topics than does Math 111; and, with only minor exceptions, its examples and applications are all related to business, economics, and finance.

Calculations: As in all freshman mathematics courses, every exam and every homework assignment in Math 108 involves a substantial number of calculations. Examples include:

a) At \$2 each, we can sell 10,000 hamburgers. If we raise our price to \$2.40, we can sell only 8,000. Assuming linear demand, find the slope and the equation of the demand curve.

b) Use tangent line approximation to estimate $\sqrt{99}$.

c) Given the demand curve $q = \sqrt{1000 - p^2}$, compute elasticity of demand at price level $p = 15$.

d My bank account has risen from \$1000 to \$1,100 in three years. How long will it take my money to triple?

e) Find the area bounded by the curves $y = 8 - x^2$ and $y = 2x$.

f) Assuming a continuous interest rate of 4%, compute the amount that I must give the College today if my goal is to provide the mathematics department with an income stream of \$1,000 per year for the next 20 years.

g) When you graph $y = \frac{\ln(x)}{x}$ on your calculator you will see that the curve has a local maximum value somewhere near $x = 3$. Use calculus techniques to locate that high point exactly, and perform a y'' calculation to verify that the point you found really is a local maximum.

Theoretical Explanations The lectures in Math 108 include proofs of theorems from mathematical economics and proofs of certain mathematical assertions when they are based on geometry. Examples include:

h) Use the limit definition of the derivative to prove that if $f(x) = x^2 - 3x + 5$, then $f'(x) = 2x - 3$.

i) Maximum profit occurs when marginal revenue equals marginal cost.

j) In any situation with a linear demand curve, if a firm's costs rise by $\$C$ per unit produced, then in order to maximize profit, the firm's optimal strategy is to increase its prices by $\$ \frac{C}{2}$.

k) Show that at the production level where average cost per unit produced is minimal, marginal cost equals average cost.

l) Suppose $f(x) \geq 0$ for all x in $[a, b]$ and that $A(x)$ is the cumulative area between the x-axis and the curve $y = f(x)$ between a and x. Then $A'(x) = f(x)$.

m) Given a demand curve $q = f(p)$ for our product, $R' > 0$ at price level p_0 if and only if demand elasticity $E = |\frac{pf'}{f}| < 1$ at price level p_0.

n) The present value of an income stream of length Y years and of size \$K per year is $\int_0^Y Ke^{-rt}\,dt$ where r is the decimalized continuous interest rate at the moment when the stream is created.

o) Properties of the derivative $f'(x)$ completely determine the shape of the graph of $f(x)$ – where $f(x)$ is increasing or decreasing, where $f(x)$ has high and low points, and where the graph of $f(x)$ is concave up or down.

Our goal in presenting the above proofs in Math 108 is to show students that mathematics can easily explain microeconomic ideas that are hard to justify without the precise language of mathematics, and to encourage students to see mathematics as more than a list of formulas. We do not ask Math 108 students to reproduce theoretical arguments on exams, with the following exceptions. The first hour exam and the final exam in Math 108 ask students to work through a proof as in item (h) above. Hour exams 1 and 2, as well as the final exam, ask students to work theoretical problems of type (o).

Applications: Applications of calculus to business, financial, and economic problems are at the heart of Math 108. Applications from physics are restricted to three or four falling body problems that are used to illustrate the idea that derivatives give rates of change (e.g., the derivative of position is velocity, and the derivative of velocity is acceleration). These examples are historically important and help students understand, later in the course, why Newton and Leibnitz cared about antiderivatives. A page called "High Points in Math 108" is passed out on the first day of the course so that students will know what Math 108 is trying to do. It presents seven sample problems that students will be able to solve by the end of the course.

1) In a large sports arena, history tells us that we can sell 10,000 hamburgers per day if we charge \$2 per burger, and that raising the price to \$2.40 caused daily sales to drop to 8,000. It costs us 60 cents to make each hamburger. What price should we charge in order to maximize our profits?

2) In the late 1930s, a University of Oregon anthropologist discovered sandals made of tree bark in an Oregon cave. Chemical analysis revealed that the sandals contained about one-third of the carbon-14 found in living tree bark. When were the sandals made? (This discovery led researchers to double their estimates about when humans settled the Pacific Northwest.)

3) Gizmos are a product distributed by only one business in a certain region. Demand for gizmos is given by $q = f(p) = \frac{200-p}{3}$ where q and p represent the number of gizmos demanded by the market and the unit price, respectively. Suppose that the cost of producing q gizmos is given by $C(q) = 75 + 80q - q^2$, for $0 \leq q \leq 40$. What price p generates maximum profit? Now suppose that a tax of \$4 per gizmo produced is imposed on the business. In order to maximize profits, how much of the tax should the business pass on to its customers?

4) How can one tell whether consumer demand for a product is elastic at a certain price level, and why is it true that in an elastic demand situation, price and revenue move in opposite directions?

5) Why is it true that in a community where mass media spread information, the spread of information about a product follows Newton's law of heating, while in a community where information is spread by word-of-mouth, the spread of product information follows the logistic growth curve from population biology?

6) You owe your credit card company \$1,000 and make a plan to (a) stop using the card immediately, and (b) pay off the debt at the rate of \$20 per month. The card charges a continuous interest rate of 1% per month. How long will it take for you to pay off the debt?

7) In 1980, the USSR wanted to build a natural gas pipeline from its Siberian gas fields to some western European country, from which its natural gas would be distributed to buyers in Europe. But the USSR did not have the money to build the pipeline so it invited various European nations to lend the needed construction funds to the USSR, with repayment (in natural gas) to begin in 1985 when the pipeline would be complete. Negotiations with West Germany were successful and, in broad terms, the agreement was as follows. The Germans would lend 20 billion Deutsch marks to the USSR in 1980. Once the pipeline was completed in 1985, the USSR would begin repaying the loan by allowing the Germans to take, at no cost, a constant annual amount of natural gas, forever, from the pipeline. In 1980, the price of natural gas was 0.10 DM per cubic meter and this price was expected to be generally stable over the foreseeable future. In addition, the prevailing continuous interest rate was 10%. The Germans' bid was "In return for the 20 billion DM that we will lend you to build the pipeline, we will take K million cubic meters of natural gas each year, forever." The question is: How could they determine K?

In some semesters, all seven problems are covered. In fall 2007, the first four problems have already been studied (as of October 30) but because Math 108 has fallen behind schedule somewhat, it is not certain that Problem (5) in that list will be studied. However, I expect to discuss all other problems by the end of the term.