The Logarithmic Method and the Solution to the TP2-Completion Problem

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Field: Applied Mathematics, Degree: Ph.D.

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Abstract

A matrix is called TP2 if all 1-by-1 and 2-by-2 minors are positive. A partial matrix is one with some of its entries specified, while the remaining, unspecified, entries are free to be chosen. A TP2-completion, of a partial matrix T, is a choice of values for the unspecified entries of T so that the resulting matrix is TP2. The TP2-completion problem asks which partial matrices have a TP2-completion. A complete solution is given here. It is shown that the Bruhat partial order on permutations is the same as a certain natural partial order induced by TP2 matrices and that a positive matrix is TP2 if and only if it satisfies certain inequalities induced by the Bruhat order. The Bruhat order on permutations is generalized to a partial order, GBr, on nonnegative matrices, and the concept of majorization is generalized to a partial order, DM, on nonnegative matrices. It is shown that these two partial orders are the same on the set of nonnegative matrices. Using this equality and the Hadamard exponential transform on nonnegative matrices, explicit conditions for TP2-completability of a given partial matrix are given. It is shown that the set of matrices used in the exponents of the inequalities forms a finitely generated cone with integral generators. This gives finitely many polynomial inequalities on the specified entries of a partial matrix of given pattern as conditions for TP2-completability. A computational scheme for explicitly finding the generators is given and the combinatorial structure of TP2-completable pattern is investigated.