On Certain Sets of Matrices: Distance Matrices, Ray-nonsingular Matrices and Matrices Generated by Reflections

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Abstract

In this dissertation, we study three different sets of matrices. First, we consider the cone of Euclidean distance squared matrices. Given n points in Euclidean space, we can construct an n-by-n Euclidean distance matrix by assigning to each entry of the matrix the pairwise interpoint Euclidean distance. Originally studied by statisticians and psychometricians, the study of distance matrices is useful in such areas as computational chemistry and structural molecular biology. The purpose of the first part of the dissertation is to better understand this set and its different characterizations so that a number of open problems might be answered and known results improved. We look at geometrical properties of this set, investigate forms of linear maps that preserve this set, consider the uniqueness of completions to this set and look at subsets that form regular figures.

In the second part of this dissertation, we consider ray-pattern matrices. A ray-pattern matrix is a complex matrix with each nonzeroentry having modulus one. A ray-pattern A is said to be ray-nonsingular if any positive entry-wise scaling is nonsingular. An open problem concerning this set is whether or not an n-by-n ray-nonsingular matrix exists for different values of n. A full ray-pattern matrix has no zero entries. It is known that for $n > 5$, there are no full ray-nonsingular matrices but examples exist for $n<5$. We show that for $n=5$, there are no full ray-nonsingular matrices.

The last part of this dissertation studies certain of the finite reflection groups. A reflection is a linear endomorphism T on the Euclidean space V such that: $T(v)=v-2(v,u)u$ for all v in V. A reflection group is a group of invertible operators in the algebra of linear endomorphism on V that are generated by a set of reflections. One question that has recently been studied is the form of linear operators that preserve finite reflection groups. We first discuss known results about preservers of some finite reflection groups. We end by showing the forms of the remaining open cases.