



***Persistence and extinction dynamics in reaction-diffusion-advection stream population model with Allee effect growth***

**Yan Wang**

College of William & Mary, Department of Applied Science, 2019

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Advisor: Junping Shi, Professor of Mathematics

**Abstract**

The question how aquatic populations persist in rivers when individuals are constantly lost due to downstream drift has been termed the “drift paradox.” Reaction-diffusion-advection models have been used to describe the spatial-temporal dynamics of stream population and they provide some qualitative explanations to the paradox. Here random undirected movement of individuals in the environment is described by passive diffusion, and an advective term is used to describe the directed movement in a river caused by the flow. In this work, the effect of spatially varying Allee effect growth rate on the dynamics of reaction-diffusion-advection models for the stream population is studied.

In the first part, a reaction-diffusion-advection equation with strong Allee effect growth rate is proposed to model a single species stream population in a unidirectional flow. Under biologically reasonable boundary conditions, the existence of multiple positive steady states is shown when both the diffusion coefficient and the advection rate are small, which lead to different asymptotic behavior for different initial conditions. On the other hand, when the advection rate is large, the population becomes extinct regardless of initial condition under most boundary conditions. It is shown that the population persistence or extinction depends on Allee threshold, advection rate, diffusion coefficient and initial conditions, and there is also rich transient dynamical behavior before the eventual population persistence or extinction.

The dynamical behavior of a reaction-diffusion-advection model of a stream population with weak Allee effect type growth is studied in the second part. Under the open environment, it is shown that the persistence or extinction of population depends on the diffusion coefficient, advection rate and type of boundary condition, and the existence of multiple positive steady states is proved for intermediate advection rate using bifurcation theory. On the other hand, for closed environment, the stream population always persists for all diffusion coefficients and advection rates.

In the last part, a reaction-diffusion-advection benthic-drift model that links changes in the flow regime and habitat availability with population dynamics is established. In the model, the stream is divided into drift zone and benthic zone, and the population is divided into two interacting compartments, individuals residing in the benthic zone and individuals dispersing in the drift zone. The influence of flow speed, cross-sectional area, individual transfer rates between zones, and river heterogeneity on the population persistence and extinction is considered, and the criteria of population persistence or extinction are formulated and proved.

All results are proved rigorously using the theory of partial differential equation, dynamical systems, and various mathematical tools such as bifurcation methods, variational methods, and monotone methods are applied to show the existence of multiple steady state solutions of models.