Matrix Results and Techniques in Quantum Information Science and Related Topics

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Abstract

Several matrix-related problems and results motivated by quantum information theory is presented in this dissertation. In the first problem we look at 2^n-by-2^n unitary matrices, which describe operations on closed a n-qubit system. We define a set of simple quantum gates, called controlled single qubit gates, and their associated operational cost. We then present a recurrence scheme to decompose a general 2^n-by-2^nunitary matrix to the product of no more than $2^{(n-1)}(2^n-1)$ single qubit gates with small number of controls. In the second problem Next, we address the problem of finding a specific element P among a given set of quantum channels S that will produce the optimal value of a scalar function $D(r_1,P(r_2))$, on two fixed quantum states r_1 and r_2. Some of the functions we considered for D (-,-) are the trace distance, quantum fidelity and quantum relative entropy. We discuss the optimal solution when S is the set of unitary quantum channels, the set of mixed unitary channels, the set of unital quantum channels, and the set of all quantum channels. In the third problem, we focus on the spectral properties of qubit-qudit bipartite states with amaximally mixed qudit subsystem. More specifically, given positive numbers $a_1>=...>=a_{2n}>=0$ we want to determine if there exist a 2n-by-2ndensity matrix r having eigenvalues $a_1,...,a_{2n}$ and satisfying $tr_1(r)=I_n/n$. This problem is a special case of the more general quantum marginal problem. We give the minimal necessary and sufficient conditions on $a_1,...,a_{2n}$ for $n<=6$and state some observations on general values of n. Next, we discuss projection methods and illustrate its usefulness in: (a) constructing a quantum channel P, if it exists, such that $P(r_1)=s_1$,..., $P(r_k)=s_k$ for given n-by-n density matricesr_1,...,r k and m-by-m density matrices $s_1,...,s_k$; (b) constructing a multipartite state r having a prescribed set of reduced states $r_1,...r_s$ on s of its subsystems; (c) constructing a multipartite state r having prescribed reduced states and additional properties such as having prescribed eigenvalues, prescribed rank or low von Neuman entropy; and (d) determining if a square matrix A can be written as a product of two positive semi definite contractions. Lastly, we examine the shape of the Minkowski product of convex subsets K_1,K_2 of the complex plane given by $K_1K_2=\{ab$: a is in K_1, b is in $K_2\}$ which has applications in the study of the product numerical range and quantum error-correction. In 2011, Karol et al, conjectured that K_1K_2 is star-shaped when K_1 and K_2 are convex. We give counterexamples to show that this conjecture does not hold in general but we show that the set K_1K_2 is star-shaped if K_1 is a line segment or a circular disk.