Mathematics

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Requirements for Major

The study of mathematics is motivated by its wide applicability and its intrinsic beauty. Mathematical theories often grow out of problems that appear in the physical and biological sciences, engineering, economics, finance and the social sciences. Applications often draw on mathematics that was created for completely different purposes. The mathematics program at William and Mary allows students to design a major based on their own interests and career goals and prepares students for post-baccalaureate employment and for further study of mathematical sciences and related disciplines. There are three concentrations within the major – the Standard Concentration, the Applied Mathematics Concentration and the Pre-College Mathematics Teaching Concentration. Study options include applied and pure mathematics, operations research, statistics, and teaching at the elementary or secondary level. Students can also design elective programs needed for careers in actuarial science and industrial mathematics, for interdisciplinary work in fields such as economics, business and social sciences, or for graduate studies.

Information about the mathematics major, career choices and appropriate courses of study is available from the department's academic advisors and the Office of Career Services as well as informally from the mathematics faculty.

Major Writing and Computer Proficiency Requirements

A student in any Mathematics major concentration normally satisfies the upper-division mathematics writing requirement in one of the following ways:

1) completion of Math 300 with a grade of C- or better;
2) completion of Math 495-6 with a grade of C- or better;
3) submission of a paper on a mathematical topic that is judged by the department chair to show appropriate study of some advanced mathematical topic. For example, students who participate in the summer REU programs in mathematical sciences here or elsewhere often complete the upper-division writing requirement in this way, and papers written for upper-division mathematics courses here may also be suitable.

A student in any Mathematics major concentration satisfies the Mathematics Major Computer Proficiency requirement by showing proficiency in some high-level computer programming language at the level of CSCI 141. This is normally done by receiving a grade of at least C- in CSCI 141. Exceptions require the department chair's permission.

Enriching the Mathematics Major

The requirements described below are the minimal requirements for the mathematics major, and most mathematics majors take courses beyond that minimum. Students wishing to obtain a deeper understanding of mathematics (e.g., in preparation for graduate school) should take additional upper-division courses. Second courses to make year-long sequences in linear algebra, analysis, abstract algebra, numerical analysis, statistics, or operations research are particularly recommended.

The Standard Mathematics Concentration

This is the most flexible of the three concentrations, allowing the widest choice of electives. Students who are considering graduate study often pursue this concentration, as do some students aiming for pre-college teaching, but the flexible requirements of the concentration are also appropriate for students with other goals. The major requirements of the Standard Concentration are:

1) a core consisting of Math 111 or 131, 112 or 132, 211, 212 or 213, and 214;
2) completing the major writing requirement and computer proficiency requirement as described above;
3) Math 307 and 311 plus either
   (a) Math 495-6 and three other three-credit 400 level mathematics courses and one three-credit mathematics course at the 300-400 level (for a total of at least eight upper-division courses), or
   (b) (excluding Math 495-6) three three-credit mathematics courses at the 400-level, plus two other three-credit mathematics courses at the 300-400 level (for a total of at least seven upper-division courses).

With permission of the department chair, certain three-credit upper-division mathematical courses from other departments (e.g., Computer Science, Economics, or Physics) may be used as upper-division elective courses in this requirement.

The Applied Mathematics Concentration

This concentration is designed for students who want to pursue applications of mathematics or a double major in mathematics and another discipline. The major requirements of the Applied Mathematics Concentration are:

1) a core consisting of Math 111 or 131, 112 or 132, 211, 212 or 213, and 214;
2) completing the major writing requirement and computer proficiency requirement as described above;
3) at least one of Math 307 and 311 plus either
   (a) Math 495-6 plus at least five distinct three-credit courses at the 300-400 level chosen from the four applied areas listed below and meeting both the breadth and depth requirement (for a total of at least eight upper-division courses); or
   (b) excluding Math 495-6, at least six distinct three-credit courses at the 300-400 level with at least five being chosen from the four applied areas listed below and meeting the breadth and depth requirement (for a total of at least seven upper-division courses).

Breadth requirement: three distinct courses, one from three of the four applied areas listed below;

Depth requirement: three courses within one of the four areas below. One of these courses may be one of the courses satisfying the breadth requirement.

The four applied areas within the applied concentration, and their associated courses, are:

- **Computational Mathematics:** Math 408, 413, 414, CSCI 426, and, with permission of the Mathematics department chair and the instructor, any other courses in the Computational Operations Research program, taken as independent study courses. In addition, CSCI 303 may be counted for the purpose of satisfying the depth requirement in computational mathematics.
- **Operations Research:** Math 323, 424, and (with permission of the Mathematics department chair and the instructor) any other courses in the Computational Operations Research program, taken as independent study courses. In addition, if a student elects to fulfill the depth requirement in Operations Research, then (and only then) Math 401

may be counted toward Operations Research rather than toward Probability and Statistics;

- Probability and Statistics: Math 351, 352, 401, 452, 459, Econ 408 and (with permission of the Mathematics department chair and the instructor) CSCI 616 and CSCI 680 taken as independent study courses;
- Scientific Applications: Math 302, 345, 405, 408, 417, 441, 442, Physics 475 and (with permission of the Mathematics department chair and the instructor) CSCI 616 and CSCI 680 taken as independent study courses.

The department chair may allow appropriate three-credit sections of Math 380 and Math 410 to count toward applied areas in this concentration.

In addition to the Computing Proficiency Requirement described above, students in the Applied Concentration must demonstrate proficiency in CSCI 241; this is normally done by taking and passing the course. Students who are considering graduate school in mathematics are strongly advised to take both Math 307 and Math 311.

The Pre-College Mathematics Teaching Concentration

This concentration is designed for students seeking certification as pre-college mathematics teachers. The major requirements of this concentration are:

1) a core consisting of Math 111 or 131, 112 or 132, 211, 212 or 213, and 214;
2) completion of the major writing requirement and computer proficiency requirement as described above;
3) Math 302, 307, 323, 351, 412, 416 and at least one additional three-credit upper-division mathematics course;
4) either EDUC 301 or EDUC 310

The department chair may authorize variations in the requirements for this concentration for individual students. In particular, Math 401-452 may replace Math 351. In addition, Math 495-496 may be substituted for Math 300 or 490.

Advanced Standing

Entering students may receive credit for mathematics courses through AP or IB and transfer credit. In each of the mathematics major concentrations, well-prepared students may begin their studies beyond Math 111 without receiving credit for earlier courses listed in the core requirements section of each concentration. Each skipped course for which the student does not receive credit must be replaced by an additional three-credit 300-400 level mathematics course.

Requirements for Minor

A minor in mathematics requires six Mathematics courses, each of at least three credits, distributed as follows: All of the courses must be numbered above 110, and two of the courses must be numbered above 300. Math 150 may not be counted toward a minor. A well-prepared student may elect to skip Math 111 or 131, or Math 111-112, or 131-132. No skipped course can count toward the requirement unless Advanced Placement credit, International Baccalaureate credit, or credit by examination has been received for that course.

Description of Courses

Note: A student cannot receive credit for any mathematics course that is a prerequisite for another mathematics course for which the student has already received credit. The department chair may authorize individual exceptions to this rule.

103. Pre-calculus Mathematics.

Fall (3)

A study of the real number system, sets, functions, graphs, equations, inequalities and systems of equations, followed by a study of the trigonometric functions and their properties. This course is designed only for students intending to take Math 108 or Math 111, and whose background is deficient in algebra and trigonometry. Juniors and seniors must obtain permission from the instructor to enroll. This course may not be applied toward either the minor or major in mathematics or the satisfaction of GER requirements. A student may not receive credit for this course after successfully completing a Mathematics course numbered above 107, with the exception of Math 150.

104. The Mathematics of Powered Flight.

(GER 1) Fall and Spring (3,3)

Applications of elementary mathematics to airplane flight. Wind and its effect on airport design and aircraft operation. Maps and map projections. Magnetic variation and compass navigation. Static air pressure: buoyancy and the altimeter. Use of a flight simulator will illustrate the mathematical analysis of certain aircraft instruments. Not open to students who have credit for a Mathematics course numbered higher than 104, with the exception of Math 150.

106. Elementary Probability and Statistics.

(GER 1) Fall and Spring (3,3)

Introduction to basic concepts and procedures of probability and statistics including descriptive statistics, probability, classical distributions, estimation, hypothesis testing, correlation and regression, in the context of practical applications to data analysis from other disciplines. Not open to students who have successfully completed a mathematics course numbered above 210.

108. Brief Calculus with Applications.

(GER 1) Fall and Spring (4,4)

An introduction to the calculus of polynomial, rational, exponential and logarithmic functions, including some multi-variable calculus, with applications in business, social and life sciences. Algebra proficiency required. Maple or Matlab may be used in the course. Students may not receive credit for more than one of Math 108, 111, and 131, and may not receive credit for Math 108 after receiving credit for any Mathematics course numbered higher than 108, with the exception of Math 150. To use Math 108 as a prerequisite for Math 112 or 132, students need approval of the department chair.

110. Topics in Mathematics.

Fall and Spring (3,3)

An introduction to mathematical thought with topics not routinely covered in existing courses. Material may be chosen from calculus, probability, statistics and various other areas of pure and applied mathematics.

111. Calculus I.

(GER 1) Fall and Spring (4,4)

Standard functions (linear, polynomial, trigonometric, exponential, logarithmic) and their graphs. Tangents, derivatives, the definite integral and the fundamental theorem. Formulas for differentiation. Applications to physics, chemistry, geometry and economics. Requires graphing calculator. Concurrent enrollment in Math 111 calculus lab required. Students may not receive credit for more than one of Math 108, 111, and 131.

112. Calculus II.

(GER 1) Fall and Spring (4,4) Prerequisite: MATH 111 or MATH 131.

Methods of integration. Applications of the integral to geometry, chemistry, physics and economics. Slope fields and the qualitative behavior of solutions to differential equations. Approximations: sequences, series, and Taylor series. Concurrent enrollment in Math 112 Maple or Matlab calculus lab required. Students may not receive credits for more than one of Math 112 and 132.

131. Calculus I for Life Sciences.

(GER 1) Fall (4)

Mathematical topics parallel to those in Math 111. Applications in Math 131 focus on issues of importance in the Life Sciences, e.g.,

mathematical models of population dynamics, ecology, physiology, genetics, neurology. Students may not receive credit for more than one of Math 108, 111, and 131.

132. Calculus II for Life Sciences.

(GER 1) Spring (4) Prerequisite: MATH 111 or MATH 131.

Mathematical topics parallel those in Math112. Applications in this course focus on issues of importance in the Life Sciences, mathematical models of population dynamics, ecology, physiology, and epidemiology. Students may not receive credit for both Math 112 and Math 132.

150W. Freshman Seminar: Topics in Mathematics.

Fall and Spring (4,4)

Each seminar is devoted to a specific mathematical topic. Writing about mathematics is emphasized. Normally only available to first-year students.

211. Linear Algebra.

Fall and Spring (3,3) Prerequisite: MATH 112 or MATH 132.

Linear equations, matrices, determinants, vector spaces, linear transformations, eigenvalues, orthogonality. Optional topics include least squares problems, matrix factorization, applications. A computer lab using the software package Matlab may accompany the class.

212. Introduction to Multivariable Calculus.

Fall and Spring (3,3) Prerequisite: MATH 112 or MATH 132.

Functions of several variables, surfaces in three-space, vectors, techniques of partial differentiation and multiple integration with applications. MAPLE or Matlab will be used in this course. Students may not receive credit for both Math 212 and 213.

213. Multivariable Calculus for Science and Mathematics.

Fall and Spring (4,4) Prerequisite: MATH 112 or MATH 132.

Covers all Math 212 material plus other vector calculus topics (including Gauss' and Stokes' theorems). Students may not receive credit for both Math 212 and MATH 213. Math 213 may replace Math 212 as a prerequisite and is particularly recommended for science and mathematics students.

214. Foundations of Mathematics.

Fall and Spring (3,3) Prerequisite: MATH 112 or MATH 132.

Fundamentals of advanced mathematics: Propositional logic, quantifiers and methods of proof; naive set theory including mathematical induction, relations, orders, functions, and countability.

300. Writing in the Mathematical Sciences.

Fall and Spring (1,1) Prerequisite: MATH 214.

Students will develop their mathematical writing skills in a term writing project. Sources for topics include the history of mathematics, research conducted by the student, or topics from an upper division course that the student has taken or is currently taking. Fulfills the major writing requirement.

302. Ordinary Differential Equations.

Fall and Spring (3,3) Prerequisite: MATH 211 and MATH 212 or 213 or consent of instructor.

First-order separable, linear, and nonlinear differential equations. First-order systems and forced second-order linear equations. Systems of linear equations and linearization. Numerical methods, bifurcations, and qualitative analysis. Applications to biology, chemistry, economics, physics, and social sciences.

307. Abstract Algebra.

Fall and Spring (3,3) Prerequisites: MATH 211, MATH 214 or consent of instructor.

Groups, rings, fields, isomorphisms; polynomials. Additional topics chosen from group theory and ring theory, as time permits.

309. Intermediate Linear Algebra.

Spring (3) Prerequisites: MATH 211, MATH 214.

Complex numbers; inner product spaces; adjoints of linear transformations; projections; unitary transformations. Spectral theorem for normal, Hermitian and unitary transformations. Polar and singular-value decompositions. Eigenvalues and eigenvectors; the Jordan canonical form. Bilinear and quadratic forms; Sylvester's Law of Inertia. Dual spaces: linear functionals, biorthogonal systems, annihilators. Tensors and tensor products.

311. Elementary Analysis.

Fall and Spring (3,3) Prerequisites: MATH 212 or MATH 213, MATH 214 or consent of instructor.

An introduction to the theory of real variables, the topology of the real line, convergence and uniform convergence, limits and continuity, differentiation, Riemann integration and the Fundamental Theorem of Calculus.

323. Operations Research: Deterministic Models.

Fall (3) Prerequisite: MATH 211.

An introduction to deterministic Operations Research techniques and applications. Topics include search algorithms, simplex search for linear programs, duality and sensitivity analysis for linear programs, shortest path problems, network models and discrete optimization.

345. Introduction to Mathematical Biology.

Fall (3) Prerequisite: MATH 112 or 132 or consent of instructor.

An introduction to developing, simulating, and analyzing models to answer biological questions. Mathematical topics may include matrix models, non-linear difference and differential equations, and stochastic models. Biological topics may include ecology, epidemiology, evolution, molecular biology, and physiology.

351. Applied Statistics.

Spring and Fall (3,3) Prerequisite: MATH 112 or MATH 132 or consent of instructor.

Basic concepts of statistical inference. Topics include: 1-sample and 2-sample location problems, analysis of variance, linear regression, applications of probability models and statistical methods to practical situations and/or actual data sets. No previous knowledge of probability is assumed. This course is recommended for students who wish to take a single, self-contained statistics course that emphasizes analysis of experimental data. Mathematics concentrators with an interest in applications are also encouraged to take this course followed by the more theoretical Math 401 and Math 452.

352. Data Analysis.

Spring (3,3) Prerequisite: MATH 351 or consent of instructor.

Case studies are used to provide in-depth exposure to the practice of statistics. Topics include: experimental design, data collection, data management, statistical analysis (beyond Math 351), statistical software, interpreting and reporting results.

380. Topics in Mathematics.

Fall and Spring (1-3) Prerequisites: MATH 211, MATH 212 or MATH 213, or consent of instructor.

A study of 300-level mathematical topics not covered by existing courses. Topics may be pure or applied. Course may be repeated for credit with permission of instructor.

401. Probability.

Fall and Spring (3,3) Prerequisites: MATH 211, MATH 212 or MATH 213, MATH 214 or consent of instructor.

Topics include: combinatorial analysis, discrete and continuous probability distributions and characteristics of distributions, sampling distributions.

403. Intermediate Analysis.

Spring (3) Prerequisite: MATH 311.

Sequences and series of functions; analysis in metric spaces and normed linear spaces; general integration and differentiation theory.

405. Complex Analysis.

Fall (3) Prerequisite: MATH 311 or consent of instructor.

The complex plane, analytic functions, Cauchy Integral Theorem and the calculus of residues. Taylor and Laurent series, analytic continuation.

408. Advanced Linear Algebra.

Fall (3) Prerequisites: MATH 211, MATH 214 or consent of instructor.

Eigenvalues, singular values, matrix factorizations, canonical forms, vector and matrix norms; positive definite, hermitian, unitary and nonnegative matrices.

410. Special Topics in Mathematics.

Fall and Spring (1-3, 1-3)

A treatment of topics of interest not routinely covered by existing courses. Material may be chosen from topology, algebra, differential equations and various other areas of pure and applied mathematics. This course may be repeated for credit with permission of the instructor.

412. Introduction to Number Theory.

Fall (3) Prerequisite: MATH 214 or consent of instructor.

An elementary course in the theory of integers, divisibility and prime numbers, a study of Diophantine equations, congruences, number-theoretic functions, decimal expansion of rational numbers and quadratic residues.

413. Introduction to Numerical Analysis I.

Fall (3) Prerequisites: MATH 211 and MATH 212 or MATH 213, CSCI 141, MATH 214 or consent of instructor.

A discussion of the mathematical theory underlying selected numerical methods and the application of those methods to problems of practical importance. Computer programs are used to facilitate calculations and illustrate analytical results. The topics covered are: linear systems of equations, sensitivity analysis, least-squares problems, the singular value decomposition, and eigenvalue problems. Students planning to take 414 are encouraged to take 413 first.

414. Introduction to Numerical Analysis II.

Spring (3) Prerequisites: MATH 211 and MATH 212 or MATH 213, CSCI 141, MATH 214 or consent of instructor.

A discussion of the mathematical theory underlying selected numerical methods and the application of those methods to problems of practical importance. Computer programs are used to facilitate calculations and illustrate analytical results. The topics covered are: nonlinear equations, interpolation and approximation, numerical integration, and numerical methods for the solution of ordinary and partial differential equations. Students planning to take 414 are encouraged to take 413 first.

416. Topics in Geometry.

Fall of even-numbered years (3) Prerequisites: MATH 211, MATH 212 or MATH 213, MATH 214 or consent of instructor.

A treatment of topics selected from Euclidean geometry, non-Euclidean geometry, projective geometry, finite geometry, differential geometry or algebraic geometry.

417. Vector Calculus for Scientists.

Spring (3) Prerequisites: MATH 211, MATH 212 or MATH 213, and MATH 302 or consent of instructor.

Directional derivatives, differential forms and the Poincaré lemma, chain rule: Jacobians, change of variable and application to Lagrangian mechanics; path integrals and the deformation theorem, surface integrals and Stokes' theorem. Additional topics will be covered if time permits.

424. Operations Research: Stochastic Models.

Spring (3) Prerequisite: MATH 401.

A survey of probabilistic operations research models and applications. Topics include stochastic processes, Markov chains, queueing theory and applications, Markovian decision processes, inventory theory and decision analysis.

426. Topology.

Fall of odd-numbered years (3) Prerequisite: MATH 311 or consent of instructor.

A study of topological spaces, metric spaces, continuity, product spaces, compactness, connectedness and convergence. As time permits, additional topics may be chosen from homotopy theory, covering spaces, manifolds and surfaces, or other topics in algebraic or set theoretic topology.

428. Functional Analysis.

Spring of odd-numbered years (3) Prerequisite: MATH 311.

Introduction to the geometry of Hilbert spaces, bounded linear operators, compact operators, spectral theory of compact self-adjoint operators, integral operators and other applications.

430. Abstract Algebra II.

Spring of odd-numbered years (3) Prerequisite: MATH 307.

The theory of groups, rings, and fields. Topics may include the fundamental theorem of Abelian groups, Sylow's theorem, field extensions, and Galois theory.

432. Combinatorics.

Spring of even-numbered years (3) Prerequisites: MATH 211, MATH 214 or consent of instructor.

A study of combinatorial theory and applications to practical problems. Topics include: graph theory, graphical algorithms, enumeration principles, inclusion-exclusion principle, recurrence relations, and generating functions. Optional topics: Polya counting principle, combinatorial designs, coding, Boolean algebra, and switching functions.

441. Ordinary Differential Equations II.

Fall (3) Prerequisite: MATH 302.

Linear systems of ODEs. Nonlinear systems; dynamical systems, existence/uniqueness of solutions; phase plane analysis; bifurcation; Poincare-Bendixson theory. Applications in biology, circuit theory, and mechanics. Discrete dynamical systems.

442. Partial Differential Equations.

Spring (3) Prerequisite: MATH 302.

An introduction to partial differential equations. Waves, diffusion, and boundary value problems; Fourier analysis; harmonic functions; Green's function and Green's identity. Introduction to numerical methods for approximating solutions.

452. Mathematical Statistics.

Spring (3) Prerequisite: MATH 401 or consent of instructor. MATH 351 recommended.

The mathematical theory of statistical inference. Possible topics include: maximum likelihood, least squares, linear models, methods for estimation and hypothesis testing. (Formerly MATH 402)

459. Topics in Statistics.

Fall and Spring (1-3, 1-3) Prerequisite: Consent of instructor.

Statistical topics not covered in other courses. Possible topics include: linear models, nonparametrics, multivariable analysis, computationally intensive methods. This course may be repeated for credit as topics change.

†490. Seminar.

Fall and Spring (3,3) Prerequisite: MATH 214.

Sections of this course will treat a single narrow topic. Possible areas of interest include linear algebra, operator theory, applied analysis, com-binatorial theory, operations research, statistics, history of mathematics, mathematical pedagogy and computational mathematics. Students will present written and oral work for discussion in class. May be repeated with permission.

†495-496. Honors.

Fall, Spring (3,3)

Students admitted to Honors study in mathematics will be enrolled in this course during both semesters of their senior year. The course comprises:

(a) supervised research in the student's special area of interest;

(b) presentation by April 15 of an Honors thesis; and

(c) satisfactory performance in a comprehensive oral examination in the field of the student's major interest. For College provisions governing the Admission to Honors, see catalog section titled Honors and Special Programs.

Graduate Program

See the Computational Operations Research Concentration description in the Department of Computer Science and the Applied Mathematics program in the Applied Science Department.