A Dark Energy Timeline for the Universe

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Abstract

Dark energy is a form of vacuum energy that generates a negative pressure in space and is described by the cosmological constant, Λ. This vacuum pressure serves to generate a negative gravitational field pointed radially outward. The purpose of this project is to calculate and compare the relative effects on dynamical relaxation of large cosmological structures due to gravity and dark energy negative pressure. We find that the relative effects of dark energy and gravity are comparable to within a few orders of magnitude over long enough time frames and at large enough size scales with the relative contribution of dark energy increasing for more vast structures.

1. Introduction

Our universe is an unfathomably large and expanding place and is still in its infancy after its Big Bang birth. In only about its thirteen billionth year ($= 13{,}000{,}000{,}000 = 1.3 * 10^{10}$) of existence, the universe is expected to live on until the one hundredth cosmological decade (this is a form of standard shorthand used in cosmology to discuss time scales over many orders of magnitude. The "n^{th} cosmological decade" refers to the year 10^{n} so the one hundredth cosmological decade is the year 10^{100}) or even beyond where it will face its effective end by the Big Crunch or Cosmological Heat Death or the Universe Phase Transition. Suffice it to say, the lifespan of our universe is very long and there is a lot about our universe that we are only just beginning to understand, including a recently proposed form of energy called dark energy.

The current widely accepted timeline is broken up into five eras: the universe's birth in the Primordial Era, followed by the Stelliferous Era, Degenerate Era, Black Hole Era and its eventual demise in the Dark Era. The Primordial Era runs from the Big Bang through the first 100,000 years of the universe's existence and is characterized by a rapid expansion, heavy background radiation and the transition to a matter dominated background followed by the formation of light atoms. The Stelliferous Era is dominated by galaxies and stars and runs from the sixth to the fourteenth cosmological decades where it will eventually end when all stars have burned out and transitioned to dwarf or neutron stars or gone supernova. The Degenerate Era follows from the fifteenth to the thirty-ninth cosmological decades. This era is populated by low energy dwarf and neutron stars fueled by dark matter particles and ultimately ends in dark matter depletion and the more dire proton and neutron decay. By the beginning of the Black Hole Era in the fortieth cosmological decade, the only major astronomical structure left will be black holes which dimly light the universe by Hawking Radiation but will evaporate away by the hundredth cosmological decade. Beyond that time, in what we call the Dark Era, there is no significant structure astronomical structure in the universe which is now incredibly sparse, cold and dark. Rarely brightened by the rapid formation and decay of positronium atoms, the universe awaits its ultimate crunch, heat death or phase transition (Laughlin & Adams, 1999)

2. Background

The scope of this project covers the end of the Stelliferous era and the beginning of the Degenerate era, from about the 12^{th} to the 20^{th} cosmological decades. As the Stelliferous era draws to a close, star formation ceases and all existing stars, if they can avoid supernova, will transition to lower energy white/brown dwarfs.

2.1 Dynamical Relaxation

As stars and solar systems travel about the galaxy's center and as galaxies move through space in relation to each other, there is a miniscule chance that two stars will collide. Significantly more likely, however, is the chance that two stars will come within close enough range that their mutual gravitational attraction will significantly deflect them from their initial trajectories.

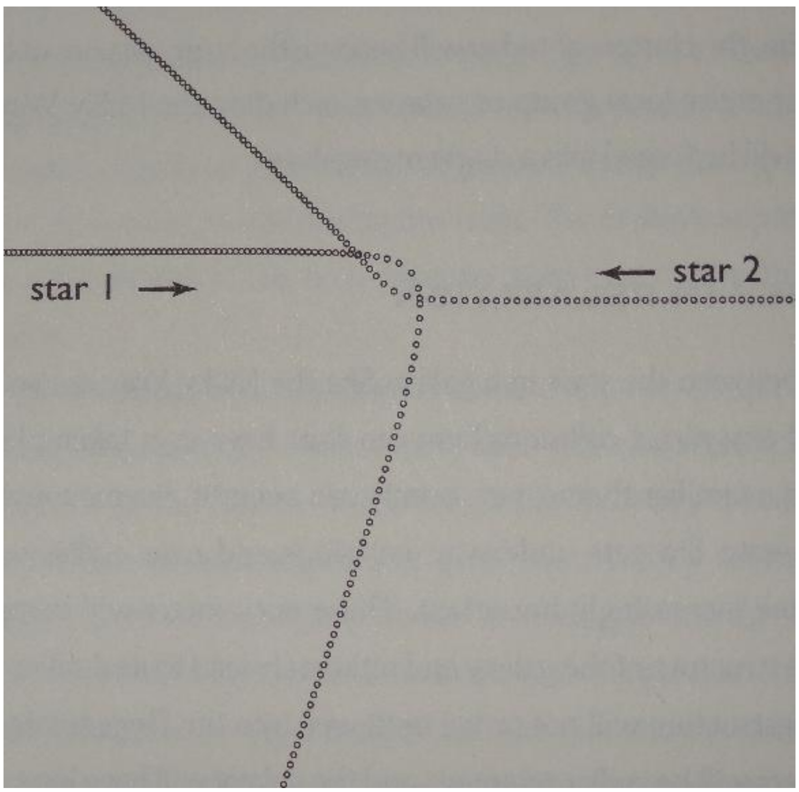

Figure 1 - An example diagram of the near-collision of two stars, typical of the process of dynamical relaxation (Laughlin & Adams, 1999).

This consistent pattern of near collisions can, over time, tear stars from their orbits and even send them hurtling out of galaxies into the intergalactic no-man's land. This process is call *dynamical relaxation* and, given enough time, enough stars can evaporate from the diminishing galaxy to effectively eliminate the galactic structure. The time scale associated with this process of dynamical relaxation is given by:

$$t_{relax} = \frac{R}{v}\frac{N}{8\ln(N)} \quad (1)$$

Where R is the radius of the galaxy, v is the average speed of constituent stars and N is the number of stars in the galaxy (Binney & Tremaine, 1987). Additionally, this equation will provide useful results for other similar structures including galaxy clusters and superclusters.

2.2 Dark Energy

Dark energy is a relatively recent cosmological discovery, talked about only within the past ten to fifteen years. It's considered an analog of vacuum energy and is thought to accelerate the expansion of our constantly growing universe (Ruiz-Lapuente, 2010). Another way to think about dark energy is to consider it as a sort of negative mass density generating a negative gravitational field of magnitude: $\frac{1}{3}\Lambda c^2 r$. Dark energy maintains a constant density in space, even as the universe continues to expand at an increasing rate. The value of dark energy is given by the cosmological constant:

$$\Omega_\Lambda = \frac{\rho_\Lambda}{\rho_c} = 0.73 \quad (2)$$

$$\Lambda = \frac{8\pi G\rho_\Lambda}{c^2} = 1.27 * 10^{-52} m^{-2} \quad (3)$$

Where (2) is the dimensionless parameter given as a ratio to critical density (Lahav & Liddle, 2011) and (3) represents a dimensionful value that's more useful for calculations (Carroll, 2001).

2.3 The Modified Schwarzschild Metric and Force of Cosmological Constant

The Schwarzschild metric represents a solution to Einstein's field equations for a uniformly rotating body in empty space. The cosmological constant modified metric is derived in section 1 of the appendix and is given as:

$$ds^2 = \left(1 - \frac{2m}{r} - \frac{1}{3}\Lambda r^2\right) dt^2 - \left(1 - \frac{2m}{r} - \frac{1}{3}\Lambda r^2\right)^{-1} dr^2 - r^2(d\theta^2 + \sin{(\theta)}^2 d\phi^2) \quad (4)$$

Where $m = GM/c^2$. The metric can alternately be written in its canonical form in terms of the potential Φ:

$$ds^2 = e^{2\Phi/c^2} c^2 dt^2 - dl^2 = \left(1 - \frac{2m}{r} - \frac{1}{3}\Lambda r^2\right) dt^2 - dl^2 \quad (5)$$

Where $e^{2\Phi/c^2}$ represents a general relativistic time dilation factor by which the standard time rate dt is adjusted. In a sufficiently weak static field (in a classical situation), we can approximate $e^{2\Phi/c^2} \approx 1 + 2\Phi/c^2$. Then we have:

$$1 + 2\Phi/c^2 = 1 - \frac{2m}{r} - \frac{1}{3}\Lambda r^2 \quad (6)$$

$$\Phi = -\frac{GM}{r} - \frac{1}{6}\Lambda c^2 r^2 \quad (7)$$

From this potential, we can calculate the force due to the gravity and dark energy:

$$F = -\nabla\Phi = -\frac{GmM}{r^2} + \frac{1}{3}\Lambda m c^2 r \quad (8)$$

It's important to note that the force of the gravitational field of the mass M and the force of the cosmological constant in space are pulling the mass m in opposite directions.

3. Calculation of Dynamical Relaxation Time Scales

Now that we have values for the force on astronomical bodies by both the gravity of a mass M object and by the dark energy value Λ, we can determine the characteristic time scale, t_{relax}, at which each force affects dynamic relaxation and the magnitude of deflection, $\Delta v_{\perp}$, each can affect a body's original trajectory. We consider two stars on an initially-straight-line, near-collision path separated by a time-dependent distance $r(t)$, a time-dependent parallel distance $x(t)$ and a perpendicular separation of b such that $r(t)^2 = x(t)^2 + b^2$. Section 2 of the appendix shows calculations of the perpendicular deflection of the velocity from the straight-line trajectory. We determined the deflection attributed to both gravity and the cosmological constant to be, respectively:

$$\left|\delta v_{\perp,gravity}\right| \cong -\frac{2GM}{bv} \tag{9}$$

$$\left|\delta v_{\perp,\Lambda}\right| \cong +\frac{1}{3}\,\Lambda c^2 bt \tag{10}$$

We'll treat each contributing factor separately at this point to determine t_{relax} and $\Delta v_{\perp}$ as they are calculated individually from each factor. The change in velocity from the gravity factor: $-2GM/bv$, occurs

$$\delta n = \frac{N}{\pi R^2} 2\pi b db = \frac{2N}{R^2} b db \tag{11}$$

many times from as many different impact parameters within the range $b + db$, with the sum total equaling zero but the squared sum equaling:

$$\delta {v_{\perp}}^2 = \left(\frac{2GM}{bv}\right)^2 \frac{2N}{R^2} b db \tag{12}$$

Integrating over the entire radius of the galaxy:

$$\Delta v_{\perp,gravity}{}^{2} = \int_{b_{min}}^{R} \left(\frac{2GM}{bv}\right)^{2} \frac{2N}{R^{2}} bdb = 8N \left(\frac{GM}{Rv}\right)^{2} ln\left(\frac{R}{b_{min}}\right) \tag{13}$$

And understanding

$$v^{2} \approx GNM/R \tag{14}$$

$$n_{relax} = v^{2}/\Delta v_{\perp}{}^{2} \tag{15}$$

$$t_{relax} = n_{relax} * t_{cross} = n_{relax} * R/v \tag{16}$$

We conclude that:

$$t_{relax,gravity} = \frac{R}{v} \frac{N}{8ln(N)} \tag{17}$$

Where $R/b_{min} \approx N$.

Alternately, the calculation of t_{relax} from

$$|\delta v_{\perp}| \cong \frac{1}{3} \Lambda c^{2} bt \tag{18}$$

is more direct since the squared cumulative contribution is not affected by n different parameters. We have:

$$\delta v_{\perp,\Lambda}{}^{2} = \Delta v_{\perp}{}^{2} = \left(\frac{1}{3} \Lambda c^{2} bt\right)^{2} \tag{19}$$

$$n_{relax} = \left(\frac{\Delta v_{\perp}{}^{2}}{v^{2}}\right)^{-1} = \left(\frac{1}{9} \frac{\Lambda^{2} c^{4} b^{2} t^{2} R}{GNM}\right)^{-1} \tag{20}$$

$$t_{relax,\Lambda} = \frac{9GNM}{\Lambda^{2} c^{4} b^{2} t^{2} v} \tag{21}$$

4. Numerical results

The table below lists the numerical results for both the characteristic relaxation value t_{relax} and the cumulative deflection effect $\Delta v_{\perp}$ for the gravitational and cosmological constant forces. These values are listed for stellar effects at two different size scales, galaxy and galaxy cluster and galactic effects on the supercluster scale. Also listed in the table are the values for each variable used in the calculation.

Additionally:

$\Lambda = 1.27 * 10^{-52} m^{-2}$

$c = 3 * 10^{8} m/s$

$G = 6.67 * 10^{-11} m^{3} kg^{-1} s^{-2}$

$M = 1.99 * 10^{30} kg$ (not valid for Virgo Supercluster calculations)

Further, we treat the variable $t = t_{relax}$. Then we have a factor of t^3 on the left side of equation (21) and, solving then for t_{relax} we can go back and plug the value into equation (19) to solve for $\Delta v_{\perp}$.

Table 1: Numerical results for characteristic relaxation time and perpendicular velocity deflection at three different size scales (Milky Way Galaxy, Local Group galaxy cluster and Virgo Super Cluster).

Structure	Variables[1]	Result	Gravity	Vacuum energy
Milky Way Galaxy	$N = 10^{11}$ $R = 5.203 * 10^{20} m$ $b = 1.5 * 10^{15} m$ $v = 2.2 * 10^{5} m/s$	$t_{relax}(s)$	$1.2 * 10^{24}$	$1.2 * 10^{22}$
		$\Delta v_{\perp}(\frac{m}{s})$	7.2	70.0
Local Group Galaxy	$N = 1.3 * 10^{12}$	$t_{relax}(s)$	$2.3 * 10^{27}$	$4.2 * 10^{21}$

[1] Sources for variables, respectively: (Binney & Tremaine, 1987), (Wethington, 2008), (Chaisson & McMillan, 1993), estimate based on (Karachentsev & Kashibadze, 2006), (Frommert & Kronberg, 2012), (Powell, 2006),

cluster	$R = 4.7 * 10^{22} m$ $b = 3.7 * 10^{16} m$ $v = 1.2 * 10^{5} m/s$	$\Delta v_{\perp}(\frac{m}{s})$	0.8	587.8
Virgo Supercluster[2]	$N = 5 * 10^{4}$ $R = 4.7 * 10^{23} m$ $b = 3.7 * 10^{21} m$ $v = 7.5 * 10^{5} m/s$ $M = 4 * 10^{40} kg$	$t_{relax}(s)$	$3.6 * 10^{20}$	$1.5 * 10^{19}$
		$\Delta v_{\perp}(\frac{m}{s})$	$2.2 * 10^{4}$	$1.1 * 10^{5}$

An interesting pattern of results is apparent. The time to relaxation is between 1 and 6 orders of magnitude greater for the gravity calculation than for the cosmological constant calculation. And further, as expected then, the cumulative effect of the velocity deflection was 1 to 2 orders of magnitude greater for the cosmological constant than for gravity.

5. Discussion and Conclusion

Examining the effects of both the gravitational field and vacuum energy density independently, we can see sizable effects on the characteristic relaxation time and the trajectory deflection—especially at greater size scales. In fact, as the size scale increases, the characteristic time scale for the cosmological constant decreases and the deflection value increases. It appears that the larger the cosmological structure's radius and the greater the stellar/galactic separation, the less of an effect gravity can play on near-collision encounters and the greater the role dark energy takes. At large enough scales, we see that even a tiny factor like $\Lambda = 1.27 * 10^{-52} m^{-2}$ can majorly affect the evolution of large cosmological structures.

[2] Applying these calculations to *galaxies* within the Virgo Supercluster rather than stars

There are several possible sources of error in these calculations that should be acknowledged. First, we assumed a uniform distribution of stars throughout the galaxies and galaxy groups and clusters. Typically stars are concentrated more towards the center of the structure and so our assumption will have introduced some error into the calculation. It might also be fair to question the choice of variable values for the numerical calculations, particularly b and v for each of the three size scales/structures. For v, we used the calculated $v^2 \approx GNM/R$ for an average-mass star/galaxy at $R/2$ from the center of the structure. In our determination of b, we assumed, again, a uniform distribution of stars/galaxies within the structure and valued b at $1/2$ the average separation between stars. These values may not provide an appropriate average value for use in calculations across an entire system. Finally, it is likely some error was introduced through the calculations of $\Delta v_{\perp}$ and t_{relax} across such drastically different size scales. The dynamics of stars within a galaxy, stars within a galaxy group and galaxies within a super cluster vary with some amount of significance and cannot all behave perfectly similarly by the same set of equations. Even if the equations are good approximations for all three size scales, there will be a non-negligible amount of error introduced.

If, at worst, the error introduced affects the numerical results by several orders of magnitude, one still cannot ignore the effects of dark energy on the dynamical relaxation of galaxies through the late teens and early twenties cosmological decades. Given enough time and a large enough size scale to act on, dark energy can contribute just as greatly or more than gravitational fields.

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Appendix of Calculations

1. Confirming the cosmological constant Schwarzschild Metric[3]

Generic Schwarzschild equation:

$$ds^2 = g_{tt}dt^2 - g_{rr}dr^2 - g_{\theta\theta}d\theta^2 - g_{\phi\phi}d\phi^2$$
$$= e^{A(r)}dt^2 - e^{B(r)}dr^2 - r^2(d\theta^2 + \sin(\theta)^2 d\phi^2)$$

Curvature tensor components:

$$R_{\mu\nu} = g_{\mu\nu}\Lambda \rightarrow R_{tt} = g_{tt}\Lambda, R_{rr} = g_{rr}\Lambda, R_{\theta\theta} = g_{\theta\theta}\Lambda, R_{\phi\phi} = g_{\phi\phi}\Lambda$$

$$R_{\theta\theta} = g_{\theta\theta}\Lambda = -\Lambda r^2$$

Curvature tensor value, (Rindler, 2001)

$$R_{\theta\theta} = e^A(1 + rA') - 1 = -\Lambda r^2$$

$$e^A(1 + rA') = 1 - \Lambda r^2$$

with $e^A = \alpha$:

$$\alpha + r\alpha' = (r\alpha)' = 1 - \Lambda r^2$$

$$\alpha = e^A = e^{-B} = 1 - \frac{2m}{r} - \frac{1}{3}\Lambda r^2$$

Plugging back into the Schwarzschild metric:

$$ds^2 = \left(1 - \frac{2m}{r} - \frac{1}{3}\Lambda r^2\right)dt^2 - \left(1 - \frac{2m}{r} - \frac{1}{3}\Lambda r^2\right)^{-1} dr^2 - r^2(d\theta^2 + \sin(\theta)^2 d\phi^2)$$

[3] All equations carried out with $c = G = 1$

2. Calculating the characteristic dynamical relaxation time scale

Consider two stars on a slightly offset path of collision. We use a right triangle setup where r is the distance between two passing stars, b is the closest incidental distance between the two stars and x stands as the third side of the triangle such that $r^2 = x^2 + b^2$ (Binney & Tremaine, 1987). Then we have:

$$F_{\perp} = m\dot{v}_{\perp} = \left(-\frac{GmM}{r^2} + \frac{1}{3}\Lambda mc^2 r\right)\cos\theta = \left(-\frac{GmM}{x^2+b^2} + \frac{1}{3}\Lambda mc^2\sqrt{x^2+b^2}\right)\cos\theta$$

$$m\dot{v}_{\perp} = \left(-\frac{GmM}{x^2+b^2} + \frac{1}{3}\Lambda mc^2\sqrt{x^2+b^2}\right)\frac{b}{\sqrt{x^2+b^2}}$$

$$\dot{v}_{\perp} = -\frac{GMb}{(x^2+b^2)^{3/2}} + \frac{1}{3}\Lambda bc^2 = -\frac{GM}{b^2}\left(1+\left(\frac{vt}{b}\right)^2\right)^{-3/2} + \frac{1}{3}\Lambda c^2 b$$

We integrate this expression with respect to time to determine the cumulative effects of the gravitational field and vacuum energy density:

$$|\delta v_{\perp}| \cong \int \left[-\frac{GM}{b^2}\left(1+\left(\frac{vt}{b}\right)^2\right)^{-3/2} + \frac{1}{3}\Lambda c^2 b\right] dt$$

In the first half of the integral, for ease of calculation, we use $s = vt/b$ and $dt = ds * b/v$. Then:

$$|\delta v_{\perp}| \cong -\frac{GM}{bv}\int_{-\infty}^{\infty}(1+s^2)^{-3/2}ds + \frac{1}{3}\Lambda c^2 b\int_0^t dt = -\frac{2GM}{bv} + \frac{1}{3}\Lambda c^2 bt$$

$$\left|\delta v_{\perp,gravity}\right| \cong -\frac{2GM}{bv}$$

$$\left|\delta v_{\perp,\Lambda}\right| \cong +\frac{1}{3}\Lambda c^2 bt$$