The Classics of Classical Mechanics:

An Analysis of Isaac Newton's *Principia*

A thesis submitted in partial fulfillment of the requirement for the degree of Bachelor of Science in Physics from the College of William and Mary in Virginia,

by

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Williamsburg, VA
May 18, 2022

Contents

Acknowledgements

Many of you have heard this story, but a few years ago, my Modern Physics class and I were treated to a guest speaker named Josh Erlich. I recall that his lecture was good, but transparently, I don't remember much of the content that was discussed. What I *do* remember, though, is that while deriving some sort of relationship between a pair of variables, Josh notated 'cf' in his proof. A classmate (and favorite Sunday brunch partner) of mine, Michael Parker, asked for clarification, as he had never seen that term used before. Josh responded, saying that he did not know why, but when we compare sets of variables in physics, we write 'cf.'

I, a green, eager to impress, little-too-big-for-her-britches sophomore, interrupted the conversation. I shared with the class that 'cf' was an abbreviation for *conferatur,* a Latin word meaning "to be compared with." Conveniently, it was a vocab word in my Intro Latin class just 10 minutes prior. Josh thanked me for my (out of turn and uncalled for) contribution and asked me to see him after class. I panicked for the remainder of the period, certain that I was in big, big trouble for interrupting. Perhaps this is why I don't recall the content of the lecture. After class, rather than chastising me, Josh asked me if I knew any Latin. I told him that I knew a bit and he asked me if I wanted to join a little research project of his. Many years later, we find ourselves here, in a senior thesis culminating three years of work on that little research project. I've come to grow quite fond of it, and think of it as my brainchild.

And of course, like any child, it took a village. I would not be here, in this place, doing what I am doing, without the friends, family members, coaches, and educators who have loved on me and pushed me to grow. **I am who I am because of you all, and in this way, this is your success far more than it is mine.** I have a few special Thank You's for some characters of note...

Josh: Thank you for giving me this opportunity to wonder and a space to be curious. I do not have words in English or Latin or any form of shorthand to tell you how thankful I am to have worked with you. I am grateful for your guidance and support, your goofiness, and for the long leash you gave me. I got to explore every part of this project that I wanted to and got to ask a million questions. I'm glad we got a few answers. Apologies again for speaking out of turn! :-)

My mother, Cathleen; my father, Garrett; my sister, Audrey; my cousins, grandparents, aunts, and uncles: Even with a degree in physics, I will never be able to describe the sheer magnitude of love that I have been blessed with. I am glad that you guys are my people. I love you big time. Sending all my x's & o's.

Karen, Ashley, and Luke: I, like everybody else, am a reflection of the people closest to me. Oh, how *incredibly* lucky I am to mirror such wonderful humans. The best parts of me are borrowed from you — I've picked up your mannerisms and your jokes and I hear myself say the things that you guys say *all the time*. I am a mosaic of the people I love and I am deeply blessed that y'all are the biggest shards in me. My happiest memories are with you.

Brendon Eaton and Barbara Watson, my high school physics teachers: thank you for creating and nurturing all of this. I fell in love with physics in your classrooms. You started this show — this all comes back to you.

Eric Ewing: Thank you for teaching me that I can Do Hard Things, and inspiring me to do them. When I think of grit, dedication, and courage, I think of you. I strive to emulate the effortless way that you love your people.

Professors Mitch Brown, Jessica Stephens, and Lily Panoussi: *Gratias ago!* I came to William & Mary looking to check off a language requirement and left with a lifelong passion for History and Latin. Your classes were my very, very favorite. I will miss them dearly.

To my peers in this department — with special note to Mika, Noah, Seth, and Jack — I give my greatest thanks. I am so incredibly grateful for our community. With your help, I have accomplished things that I could not have even fathomed four years ago. I am thankful for the passion for science and the spirit of collaboration that we all share. I have never been afraid of feeling dumb when I ask a question — I know that you are all rooting for my success as much as I am rooting for yours — that is unique to William & Mary and something that I hold very, very, very dear to my heart. Thanks for investing in me. I will still be celebrating your achievements, even from afar!

Lastly, a mention of honor to my grandfather, Barry O'Brian. Thank you for being curious with me. From ketchup bottles to dinosaurs to minerals and geodes, you have walked with me, always, on every step of my journey in science. The best gene that you passed down is your insatiable love of learning and the greatest gift that I have ever been given is the opportunity to share this love with you. You are brilliant, resilient, kind, and brave, and everything good that I aspire to be.

Abstract

Isaac Newton's *Philosophiae Naturalis Principia Mathematica (Principia)* is one of the most influential documents in the history of science. Revolutionary for its time, the *Principia* is a study of earthly and celestial forces, their behaviors, and the different types of motion that they create. William & Mary's Swem Library owns a first-edition copy of the *Principia*, unique for its extensive handwritten annotations, which we have identified partially as content from later editions. This paper investigates the history of physics, W&M's copy of *Principia*, and the annotations written inside, including the identity of the annotator and the meaning of the shorthand we identified written in the marginalia. By analyzing the changes between the first and subsequent editions of *Principia*, we also investigate how Newton created and developed classical mechanics. Our contemporary study of Newton's derivation of resistive force and experimentation with fluids serve to compare the methodology and standards of seventeenth-century proofs to those of modern day. Through analyzing William & Mary's copy of his *Principia*, we are able to explore Newton's *modus operandi* to examine the origin and evolution of classical mechanics.

Chapter One:
How Did We Get Here?

When we think about where discoveries happen, scientists tend to look towards the outskirts of human knowledge. We push forward on our individual quests to find the largest, smallest, hottest, coldest, fastest, and slowest. We build bigger machines, invent newer technology, innovate smarter techniques, all in the pursuit of the hidden truth of the world around us. In the search for new knowledge, we look *forward*.

I, however, think that there's something to be said about looking *backward*. What discoveries can we find by looking towards the past, rather than the future?

Historically, the greatest advancements in science occurred in two great intellectual periods, punctuated by the Middle Ages. The first was instigated by ancient Greek philosophers around 600 BCE-100 CE, investigating the fundamental essence of nature and the universe. The second was sparked by the imaginative thinkers of the Renaissance, who now could analyze the heavens with some mathematics in their toolbox. This second great intellectual period persists until modern day. But when we juxtapose today's science with what the Phonecians were studying, the contrast is black and white. How did the philosophical mutterings of old men in togas evolve into quantum chromodynamics, special relativity, and cosmology?

The purpose of this first chapter is to answer this question. My goal is to provide an overview of the history of physics, from the first inquiries in the philosophy of nature up until Newton. I want us to see the *Principia* in its proper historical context so that we may evaluate the science and mathematics without being obstructed by the reverence of Newton.

The story that follows is based largely on Julian Barbour's *The Discovery of Dynamics,*[1] along with my own knowledge of antiquity. But before we begin, I must acknowledge that my degree has constrained my knowledge of history to a specific period of time within the bounds of the Mediterranean. Advancements in math and physics were surely made elsewhere in the world, and the Eurocentric perspective that follows would certainly be enriched by the contributions of Asian, African, and American thinkers. The history I have learned while studying classical civilizations ends around 600 CE — past this, I must rely heavily on other sources. Furthermore, even my sources from antiquity are dubious. The writings from the pre-Alexandrian period (600-400 BCE) have nearly all been lost and our references to them come from the *testimonia* of later Greek authors, who are undoubtedly jaded from the passage of time. I also acknowledge that the history of physics is well-trodden ground; I have my own analysis and perspective of antiquity, but I do not claim to till new soil in the Renaissance. My garden blooms in Chapter III, with my scholarship on Thomas Staughton Savage and the marginalia in William & Mary's copy. With these declarations made, let us begin our journey with a quote Isaac Newton penned in a letter to Robert Hooke in 1675, "If I have seen further, it is by standing on the shoulder of giants.[2]" Let us meet the cast of giants bold enough to withstand the millennia:

I turn first to Thales of Miletus (*circa* 623-*circa* 545 BCE). In the sixth century BCE, Miletus was a thriving merchant town on the western coast of Anatolia, present day Turkey. Thales is credited with developing some of the most elementary propositions in geometry; for example, that circles are bisected by their diameter and that the sides of similar triangles are proportional.[3] Even more fundamental, Iamblichus of Chalcis (*circa* 325 CE) tells us, albeit almost a millennium later, that Thales brought the concept of *numbers* as a system of units from Egypt to Greece. While this may or may not be true, history can certainly look at Thales as a founder of mathematical science. While the statements and concepts he developed seem trivial and intuitive, their simplicity shows us that the Greeks are moving into the abstract. They have left the realm of assumptions

[1] Barbour, Julian B. *The Discovery of Dynamics*. Oxford University Press, 2001.
[2] Newton, Isaac, and Robert Hooke. *Isaac Newton Letter to Robert Hooke, 1675*.
[3] The Mathematics Teacher. "Thales: The First of the Seven Wise Men of Greece." *National Council of Teachers of Mathematics*, vol. 23, no. 2, 1930, pp. 84–86., https://doi.org/10.5951/mt.23.2.0084.

and crossed the threshold into the world of geometric proofs and deductive reasoning. Geometry before Thales was based on measurement and observations. The problems it could solve were limited to the tangible, like computing the height of a pyramid or the distance of a ship from shore. To apply to nonphysical problems, geometry was abstracted by philosophers through the application of logical thought progressions.

The most famous to do so was Thales's mentee, Pythagoras (*circa* 572-497 BCE), who founded the Pythagorean school of philosophy and mathematics that flourished over the next century. From Pythagoras, we see the discovery of irrational numbers and the elevation of mathematical harmony[4]. Pythagoras claimed that the universe was composed of numbers and that the relationship between those numbers (i.e. their *ratios*) underlies everything humanity can perceive. *Purity* and *harmon*y were the threads by which the great tapestry of the universe was woven.

Rival schools of philosophy began to develop across Greece, sponsoring the next generations of intellectual thought. A few miles north, Heraclitus of Ephesus (*circa* 536-470 BCE) flipped the existing philosophical script, declaring that nothing in the world actually persists. *Change* was the only reality: the world exists in constant flux. Heraclitus believed that these ceaseless changes, though, occurred in an ordered manner: a small statement with an enormous impact. This idea foreshadowed the concept of "laws" in nature that were powerful enough to govern even the universe. The reactions to such a proclamation spurred the formation of other hypotheses. The Eleatic school of thought carried Heraclitus' logic to the extreme, most notably Zeno of Elea's (*circa* 490-430 BCE) famous paradox "proving" that even motion itself was an illusion.

In Thrace, a couple of trailblazing philosophers, Leucippus and his student Democritus (*circa* 460-370 BCE), invented a new branch of philosophy that they called *atomism*. Atomism claims that the universe is broken up into infinitely many components, or atoms, made with different shapes and sizes. The unique combinations of shape and size

[4] You may note that I make no reference to his famous theorem. There is debate among scholars of whether Pythagoras should be credited with this theorem, as the rule was in explicit and widespread use in the Old Babylonian Empire over a thousand years before Pythagoras was born.

were what accounted for the diversity of observable bodies. Atomism argues that when you stretch out your hand and feel something, you know that the object is tangible and real. It defined the void as something non-tangible, but declared that the *lack* of sensation was still something real. Therefore, empty space, no-thing, was in its own way a kind of reality, too. This hypothesis was monumental for the concept of absolute space — an idea that Newton later formalized — which sparked the development of mechanics.

Enter stage right, Socrates (*circa* 470-399 BCE), Plato (426-347 BCE), and Aristotle (384-322 BCE), the three great philosophers that dominated Classical Athens. Socrates was an eccentric[5] philosopher whose bizarre ideas attracted a lot of negative attention from the dominant aristocracy. In 399 BCE, Socrates was put on trial under rather unconventional charges — 'not honoring the Gods' and 'corrupting the youth of Athens.' These charges might not have truly been the reason Socrates was on trial[6], but his public denunciation of Zeus and the other Gods in favor of atheism made him inimical to Athenian society. He was consequently found guilty of his crimes and sentenced to death. While Socrates had brilliant ideas on ethics and morality, his untimely death prevented him from exploring the philosophy of nature. Thus, it was the scientific contributions of Plato and Aristotle that helped shape the future of natural philosophy.

Plato, a student of Socrates, picked up where the atomists left off. He viewed nature as an abstract and theoretical concept; he disliked philosophy's emphasis on the material world and thought that motion could never be comprehended. Plato and the atomists attempted to explain place and shape using three-dimensional geometry, but struggled to understand how bodies "switch" between movement and rest. Aristotle, Plato's

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[5] Socrates was notoriously ugly: In the *Symposium (v. 5-7)*, Socrates describes how he will win a beauty contest by defining his beauty as functionalism: "*Your eyes see only straight ahead, but mine see also to the side, since they project…your nostrils look to the ground, but mine flare so as to receive smells from all sides…my flat nose does not block my vision but allows my eyes to see whatever they wish…*"

[6] It is conceivable that the bizarre charges brought against Socrates were motivated less by the pursuit of justice and more from a desperate attempt to rid Athens of a public nuisance. Socrates was at first convicted as guilty of the charges in a relatively close vote (281:220). Socrates responded with his defense (in Greek, his *Apologia*, which, despite how the word looks, is not an apology) in which he proposed that rather than be punished, he instead should be *rewarded* for his "crimes." He audaciously demanded of the jury and prosecutors that he be provided with free meals in the *Prytaneion* — a public dining hall in the city center — at the expense of the state. After hearing his request, the jury condemned him to death by an overwhelming vote of 391:110.

student, cared less about the "switch" and focused instead on instances of movement versus instances of rest. Aristotle rejected the previous, continuous approach of Plato and asserted that motion was a phenomenon that had to be studied in subintervals, thus not reducible to three-dimensional geometry.

Aristotle believed that, with the exception of Plato, none of his predecessors had studied motion appropriately: they had not carefully enough considered concepts like *place* and *shape*. Writing before the development of trigonometry, Aristotle introduced a concept of space that was almost exclusively topological. He believed that position was not a point in Euclidean space, but a *place* — the place of wine within the bottle, the ship within the river, etc. Place, however and wherever it might be, was defined by its material container.

Proper place and *natural motion* were central ideas of Aristotelian doctrine. The overarching principle was that everything happened for a specific purpose; nature did nothing in vain. According to Aristotle, the proper place of the earth was at the center of the universe. Thus, the falling of a stone could be explained by the striving of earth, the predominant element of stone, to reach the center of the universe, at the center of our planet. Similarly, the proper place of fire was on the periphery of the planet, which explains why it strives to fly upwards. Aristotle defined his four elements by their characteristic motions:

> Let "the heavy" then be that whose nature it is to move towards the center, "the light" that whose nature it is to move away from the center, "heaviest" that which sinks below all other bodies whose motion is downwards, and "lightest" that which rises to the top of the bodies whose motion is upwards.[7]

Thus the ordering: earth, water, air, fire, and with them, the first emergence of a concept resembling mass.

Although his work was based on phenomena that appeared intuitive, Aristotle is evidence that infallible intuition can be difficult to develop. With hindsight, we can see

[7] *De Caelo* p. 19 (Book I, iii, 269b)

that Aristotle's greatest mistake was failing to appropriately appreciate the empirical aspects of motion. Plato and the atomists had detailed, moderately complete views on three-dimensional motion[8]. Aristotle was revolutionary for having four[9], but his ideas were fuzzy, qualitative, and at times, misinformed. The laws of dynamics and the relationships between force and acceleration were hidden from Aristotle's *mortalis visus*; it was not until Galileo, who insisted that the world could be conceived with as much precision in four dimensions as Plato had seen in three, that this veil was *eripit*[10].

In most accounts of the history of mathematics and science, Aristotle and his qualitative work on dynamics are viewed unfavorably. While Aristotelian physics bears little resemblance to modern physics, Carlo Rovelli, an Italian physicist, declares that Aristotelianism is subject to unfair press. He argues that it is misleading to dismiss Aristotelian physics: "…Contrary to common claims, Aristotle's physics is counterintuitive, based on observation, and correct in its domain of validity in the same sense in which Newtonian physics is correct in its domain.[11]" He proposes a comparison in relationships between Aristotle's and Newton's physics and between Newton's and Einstein's. From our modern perspective, Newton's model of gravity is only applicable to a certain set of conditions. Outside this set, his theories are, strictly speaking, wrong. Aristotle's observations were based on what he saw with his limited human eye and his arguments were rooted in a deeply flawed human perspective. Although almost all of his work was wrong, Aristotle's spirit of rational inquiry about the world around him along with the advancements in geometry laid the foundations for progress in statics by Archimedes (*circa* 287-212 BCE) and later, dynamics in the early seventeenth century with Galileo (1564-1642).

Aristotle's works were very influential to Newton, as they include the first full-scale discussion on the absolute vs. relative debate. Furthermore, prototypes of Newton's

[8] That is, two-dimensional motion + time
[9] Adding the third dimension of spae
[10] *Vergil*, Aeneid 2.605; Ancient literature often features a motif depicting a God — Venus, in the *Aeneid* — lifting the veil of mortal perception so that a hero receives divine insight or perspective.
[11] Rovelli, Carlo. "Aristotle's Physics: A Physicist's Look." *Journal of the American Philosophical Association*, v.2, 18 Aug. 2014, pp. 23–40. *2*, https://doi.org/10.1017/apa.2014.11.

three fundamental laws of motion appear prominently in Aristotle's major works. He is critical of the vague language the atomists used to describe motion:

> When therefore Leucippus and Democritus speak of the primary bodies as always moving in the infinite void, they ought to say with what motion they move and what is their natural motion. Each of the atoms may be forcibly moved by another, but each one must have some natural motion also, from which the enforced motion diverges. Moreover, the original movement cannot act by force, but only naturally. We shall go on to infinity if there is to be no first thing which imparts motion naturally, but always a prior one which moves because itself set in motion by force.[12]

This is a qualitative, yet solid, description of inertial motion. But while Newton has just one inertial motion, Aristotle has several natural motions, none of them corresponding exactly with Newton's. Nevertheless, Aristotle was heading in the right direction. Years later, as Alexander the Great conducted his military campaign, the ideas of Aristotle and the Greeks were spread throughout the Mediterranean. Codified through translation, Aristotle's works were accepted as truth and went largely unchallenged for centuries.

The extensive period of time between the two great intellectual periods suggests that a solid argument was not enough to bring about significant changes in thinking. Opposition to the paradigm was voiced by many thinkers in this time like John Philoponus (490 CE-570 CE), whose conception of space, inertia, and force aligned closely with Newton, but never with enough strength to dethrone mighty Aristotle. It was not until the invention of the telescope that astronomers could provide proof that Aristotle was fundamentally wrong. An empirical, quantitative study of the sky by Copernicus and Kepler finally bypassed the philosophy of Aristotle and championed the transition into evidence-backed science. The first notes of this waltz resounded after the fall of Romanized Constantinople to the Ottoman Empire in 1453. Greek scholars fled from the city to Italy, bringing the ancient works with them. It was the rediscovery of these works in Medieval Europe that sparked the second great intellectual period.

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[12] *De Caelo* p. 273 (Book III, ii, 300b)

We still see precursors to Newton from this time period, though, notably Jean Buridan (1301-1358), born in Béthune, Paris, and his impetus theory. In *Questions on the Eight Books of the Physics of Aristotle*, Buridan shows a clear awareness of inertial motion and introduces his concept of impetus:

> Thus we can and ought to say that in the stone or other projectile there is impressed something which is the motive force (*virtus motiva*) of that projectile. And this is evidently better than falling back on the statement that the air continues to move that projectile. For the air appears rather to resist. Therefore, it seems to me that it ought to be said that the motor and moving a moving body impresses (in Latin, imprimit, or pressing into) in it a certain *impetus* or a certain motive force (*vis motiva*) of the moving body, [which impetus acts] in the direction toward which the mover was moving the moving body, either up or down, or laterally, or circularly. And by the amount the motor moves that moving body more swiftly, by the same amount, it will impress in at a stronger impetus. It is by that impetus that the stone is moved after the projector ceases to move. But that impetus is continually decreased (*remittitur*, be sent back by) by the resisting air and by the gravity of the stone, which inclines it in a direction contrary to that in which the impetus was naturally predisposed to move it. Does the movement of the stone continually become slower, and finally that impetus is so diminished or corrupted that the gravity of the stone wins out over it and moves the stone down to its natural place?[13]

This passage anticipates not only Newton's First and Second Laws of motion but also his identification of *momentum* as a fundamental concept of dynamics, which he defined as the product of a body's mass and its velocity. In the last sentence, Buridan denotes the strength of the impetus as proportional to the speed of the body that is thrown. These descriptions are remarkably similar and it is not until Newton himself that fundamental qualities of dynamics are formulated with such clarity and effectiveness. Thus, Buridan's chief contribution was overcoming the Aristotelian idea that no motion is possible without a constant "pusher." But even so, there was not yet a tie between the development of new physical concepts and experimental measurement. Nevertheless,

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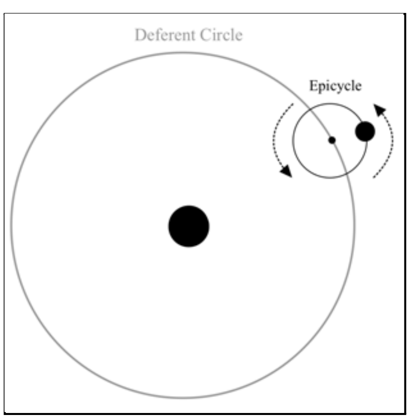
[13] Buridan, Jean, et al. *Quæstiones Super Octo Libros Physicorum Aristotelis*. Brill (2016.)

impetus theory gained widespread acceptance, and by 1600 had almost completely replaced Aristotelian theory.

The discovery of dynamics in antiquity hinged on a dozen or so insights that were all centered on mathematical descriptions of empirically observed motions. The Middle Ages in Europe failed to produce a single such insight. Buridan's argument lacked hard evidence that could be expressed in a mathematically elegant way. The ideas were there; the mathematics had been developed; but for some reason or another, they were not put together. Thus, when the thinkers of the Renaissance connected these dots, it revolutionized the world. Let us look now at their advancements.

Nicolaus Copernicus (1473-1543) was born in Toruñ in eastern Poland. At the age of 18, he studied in Cracow, where he acquired several astronomical treatises. Most notably, he read the Alphonsine Tables, which tallied calculations of the position of the planets at any given time based on Ptolemaic theory, which said that the path of the planets could be estimated by a series of circles. His uncle, a bishop of the Catholic church, sent Copernicus to be educated at the University of Bologna where he studied Greek, mathematics, law, and medicine. He returned to Poland in 1503 to work for his uncle until the bishop died in 1512. It is unclear when he had the idea that redefined cosmology, but sometime between 1510-1515, he began to carefully revise Ptolemy's work, thus embarking on a period of observations that lasted nearly the rest of his life. In the preface to his *De Revolutionibus*, Copernicus himself noted that he had worked on his idea "not merely until the ninth year, but by now, the fourth period of nine years."



What, exactly, did Copernicus propose? The basics are straightforward: Copernicus suggested that the earth simultaneously rotates about an axis and revolves around the sun. But it was not so much these motions that made Copernicus's proposed theory revolutionary. At the heart of his idea was a simple but nontrivial mathematical insight. Until now, the Greeks had represented the motion of the five planets known to them geometrically with an epicycle, a small circle whose center follows the circumference of a larger circle, as illustrated to the left, with the observer (Earth) at the center of the larger circle. Copernicus's first great insight was that, if a body was far enough away, its motion through the sky as a result of the epicycle-deferent configuration would be identical to the motion that would be observed if both the earth and the body moved in circular orbits of different radii about a common center. But, the ratio between the radii of the epicycle to the deferent circle must be the same as the radius of the earth's circular orbit compared to the body's.

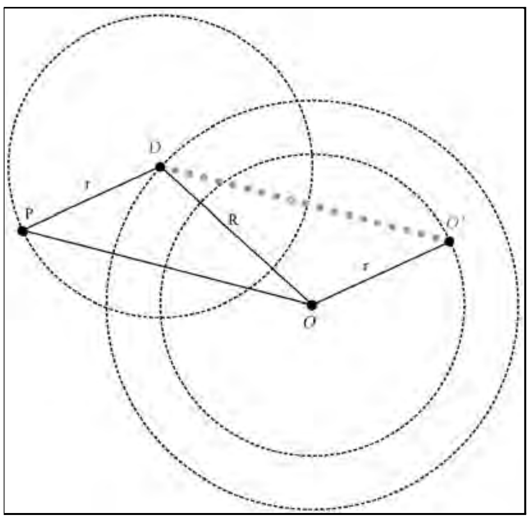


Figure 1: Geometric proof of Copernicus's work.

In the Ptolemaic configuration for an outer planet, the observer is stationed at O, the center of the deferent guide at D, and the planet at P. The sightline between the planet and the observer is along OP. In the Copernican configuration, the earth's orbit is centered on O, the earth is at O', and the planet is at D. The sightline between the planet and the observer is along O'D, which is parallel to OP. The observed phenomena, thus, are the same.

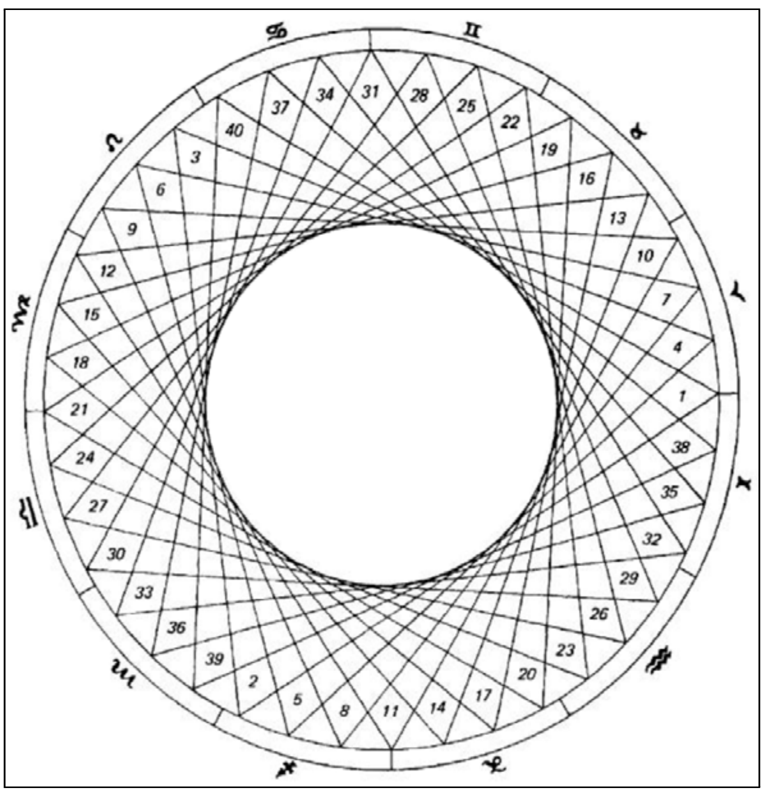
Following this note, Copernicus's second insight was that if the motion of the earth around the sun was a component of the observed motion of the planets, the radii of the other planet's orbits could be deduced from the observed motion and expressed in terms of the radius of earth's circular orbit. Immediately from this, it followed that the orbits of Mars, Jupiter, and Saturn must lie *outside* ours, and Mercury and Venus, for which it was necessary to invert the epicycle and deferent, must have orbits *inside* the earth's.

The impact of the Copernican revolution is immense; *finally*, the paradigm of Aristotelian terrestrial motion was debunked. Interestingly, there was no major observational discovery that instigated his theory: it was based purely on existing, predeveloped ideas, reinterpreting what was already known. In the history of physics, we see only one other comparable event — Einstein's creation of special relativity. Like Copernicus, Einstein's theories consisted of pure reinterpretation, but of Maxwell rather than Ptolemy.

What persisted from the Greeks, though, is the fascination with order and beauty. Tycho Brahe (1546-1601), an elite Danish astronomer, was greatly influenced by Copernicus, but he could not accept the Copernican cosmology. Perhaps because of the authority of his faith, perhaps because of his disbelief that earth could rotate without human awareness, but most likely, Brahe invented his own Tychonic system because of the sheer size of the universe that the Copernican theory implied. His own observations, made with unprecedented accuracy, implied huge distances between the earth and the stars. Brahe's observational golden age was between the years 1576 and 1597 on the island of Hven, just off the southwest coast of Sweden. King Frederick II of Denmark sponsored the construction and outfitting of an observatory, Uraniborg, which allowed Brahe to develop his passion for accuracy. Over the years, Brahe trained several assistants and made substantial contributions to the field of astronomy. He went to great lengths to measure the position of stars with minimal error, which he used to find planetary and lunar positions. After the death of his royal patron in 1588, Brahe found a new sponsor in 1599 in Rudolph II, the Holy Roman Emperor, who appointed him Imperial Astronomer and invited him to Prague. Benatky Castle was put at his disposal, but little significant work was completed in the last two years of his life. That is, except for the training of his new assistant named Johannes Kepler.

Johannes Kepler (1571-1630) was born in southern Germany in the village of Weil der Stadt. He was educated at the expense of the Duke of Württemberg and in 1589, he studied under the influence of Michael Mästlin, an excellent astronomer that introduced Kepler to Copernican astronomy. In 1594, Kepler began to teach mathematics and astronomy and started ruminating on cosmological questions. He sought to explain three things — the number of planets in our solar system, the diameters of their orbits, and the speed of their revolutions. In these questions, we can see the profound impact of the Copernican revolution, before which, these ideas could hardly be asked. In Kepler's writings, we can see the deep influence of *De Revolutionibus* and Copenicus's exaltation of the sun as the center of the universe. Even more so, Kepler was affected by Copernicus and the Greek's celebration of harmony. To this day, Kepler's Third Law is referred to as the *harmonic law*, acknowledging the beautiful relationship between the positions and speeds of the planets.

Indeed, Kepler was utterly convinced that God had created a harmonious, ordered world with the purpose of stimulating the senses of man. While modern minds might not consider this attitude to be aligned with science, Kepler's approach drove him to find harmony in mathematical relationships. He believed that numbers and geometry were the remnants of God's handiwork that allowed humanity to explore His masterpiece. This perspective aligned Kepler with the search of Plato and Pythagoras to find archetypal, harmonious relationships between the planets.



In July of 1595, Kepler was teaching his students about how the conjunction of Jupiter and Saturn is always separated by eight zodiac signs, passing gradually from one triangle to another. While drawing the figure above[14], he realized that the triangles inscribed in the larger circle circumscribed a smaller circle and that the ratio of the radii of these circles was approximately equal to the ratio of the radii of Jupiter and Saturn's orbits. He concluded that circumscribing and inscribing circles in geometrical figures held the key to unlocking planetary geometry, rather than numerical relationships. Drawing a square between Mars and Jupiter lent a reasonable fit to their actual orbits, and so did a hexagon between Mars and the Earth. The problem was that he could find no Godly reason for six planets, rather than 20 or 100. He decided that the five regular Platonic solids[15], rather than two-dimensional plane figures, must be inscribed and circumscribed between the spheres of the planets. Thus, mathematics could prove the harmony provided by six planets, as there were only five perfect solids that could fit between their spheres. Years later, in 1619, he admitted that the astronomical observations did not quite match his solution. He attributed the discrepancies to God wishing to create musical harmonies in the eccentricities of the orbits. On Kepler's religious leanings that obscured him from the complete truth, I turn to Laplace's comment: "It is depressing for the human mind to have to see how this great man dwells with delight on his chimerical speculations and regards them as the soul and life of astronomy.[16]"

Despite these discrepancies, after receiving Mästlin's approval, Kepler published his first work, *Mysterium Cosmographicum (The Cosmic Secret)*, in 1596. The publication of this text instigated Kepler to seek more accurate data and brought about his turbulent collaboration with Brahe in 1600. Kepler initially traveled to Prague in search of precise data on planetary distances that he hoped would confirm his speculations in *Mysterium Cosmographicum*, but this data did not exist yet because Brahe did not yet understand the motions of the planets. Thus, Kepler's efforts pivoted to the study of the motion of

[14] Reproduced from *Johannes Kepler Gesammelte Werke*, C.H. Beck'sche Verlagsbuchhandlung, Munich, Vol. 1, p. 12

[15] A regular Platonic solid is a convex, three-dimensional polyhedron with sides composed of identical polygons whose vertices meet at the same three-dimensional angle. The five Platonic solids are the tetrahedron ("pyramid", four sides composed of equilateral triangles), hexahedron ("cube", six sides composed of squares), octahedron, dodecahedron, and icosahedron. Plato hypothesized in *Timaeus* that the classical elements of fire, earth, air, aether, and water, respectively, were composed of these shapes.

[16] Quoted from M. Caspar's introduction to his German translation of the *Mysterium Cosmographicum*, Benno Filser Verlag, Augsburg (1923), p. xxx

Mars, an endeavor that engaged him for the better part of five years and culminated with his first two laws.

His greatest discovery was found in the early months of 1602. After aborting his mission to calculate Mars's motion using area and distance law calculations, Kepler turned to triangulation to find the planet's orbit directly. He first determined the position of three points in helioastral space and found the circle that passed through them. Knowing the center of that circle and the position of the sun, he could then define the orbit's major axis. He found that this was not congruent with his observations and repeated the procedure for further triplets of triangulated positions; each time he found a different proposed orbit. The evidence was damning: Kepler had discovered that the orbit of Mars *could not be a circle*. This discovery, along with the many failures along the way, was written in Kepler's *Astronomia Nova* and published in 1609.

As Kepler investigated astronomical motions, a bit to the south, Galileo (1564-1642) began to apply mathematical approaches to terrestrial movements. Early in his career, Galileo drew upon the work of Aristotle, albeit critically. While watching a storm, Galileo noticed that all of the hailstones landed at the same time, regardless of their size. Assuming they started to fall at approximately the same time, at the onset of the storm, this phenomenon violated the Aristotelian theory that heavier objects fall faster than light ones. Thus in 1595, after Galileo had moved to Padua, he converted to Copernican cosmology. With this shift, Galileo worked in his golden age of kinematics from 1602 to 1608, discovering both the law of freefall and the parabolic path of projectile motion. He made great strides in improving the telescopes of the time and began to observe the moon in 1609. He wrote up his work and his observations in a book titled *Sidereus Nuncius* (The Message of the Stars), published in 1610, and became an acclaimed scientist on the international stage.

This fame ensured widespread readership of his later books, especially those writing about Kepler's work on Copernican astronomy. Galileo's observations supported Kepler's argument that the celestial bodies must orbit other celestial bodies rather than mathematical points. Furthermore, once he discovered the phases of Venus, which confirmed the planets were arranged as Copernicus argued, the Greek distinction between the perfect heavens and imperfect Earth was ultimately destroyed. Galileo had proved that the planets were not as divine as previously thought, beginning the infamous conflict with the Catholic Church that dominated the next twenty years of his life.

Original to the work of Galileo is his focus on describing motion mathematically, rather than trying to explain it. Instead of attempting to give a framework of dynamics, he dedicated himself to gathering precise and correct observations, thus reestablishing mathematics as a key character in the quest to describe nature. He prioritized careful analysis of observations over Aristotle's logical approach, insisting that human intuition could be deceived. This was the secret to his success. Galileo stopped searching for the *why* and focused on the *how*. Rather than explain *why* stones fell to the ground with accelerated motion, Galileo tried to explain the *extent* at which that acceleration occurred.

Another important difference between Aristotle and Galileo was the latter's concept of perfectly mathematical motion. Galileo wrote about the path a body would follow if not disturbed by air resistance and friction, inventing the "ideal physics world" we all depend on. For Aristotle, the agent of a body's motion was the medium — speed came from the "pushing" or "resistance" of the medium. For Galileo, the effect of the medium was only that of a bothersome distraction from perfect, mathematical motion.

Interestingly, though, Galileo describes the acceleration of a ball rolling down a slope in Aristotelian terms. A body moving downward accelerates from a tendency towards its "natural motion" and resists rolling upward because it is carried further from its "natural place."

> A body subject to no external resistance on a plane sloping no matter how little below the horizon will move down [the plane] in natural motion, without the application of any external force. This can be seen in the case of water. And the same body on a plane sloping upward, no matter how little, above the horizon, does not move up [the plane] except by force. And so

> the conclusion remains that on the horizontal plane itself the motion of the body is neither natural nor forced.[17]

His last comment is interesting. Galileo describes such motion as "neutral," rather than mixed, and acknowledges that they may be perpetual: "For if [a body's] motion is not contrary to nature, it seems that it should move perpetually; but if its motion is not according to motion, it seems that it should finally come to rest[18]." While he never explicitly uses the word 'inertia', Galileo clearly recognized the characteristic *persistence of motion* attributed to terrestrial bodies that is encapsulated in Newton's First Law.

Galileo soon realized that by reducing the slope of the plane he was able to reduce the acceleration of a rolling ball. In 1604, he devised a way to measure speed and acceleration. He let a ball roll down a shallow plane (of less than 2°) from rest and marked its change in position (in millimeters) every half-second, as measured by musical beats. He discovered a simple relationship: if the distance traveled in the first interval of time is normalized to be 1, the distance traveled in the second interval is equal to 3, then 5 in the third interval, etc. That is, if *x(t)* measures distance traveled in time *t*:

$$x(1) - x(0) = 1$$
$$x(2) - x(1) = 3$$
$$x(3) - x(2) = 5$$
$$\Rightarrow x(n) - x(n-1) = 2n - 1$$

At last, the application of mathematical precision to the fourth dimension! With simple addition, we see that position increases as a square of the time:

$$x(0) = 0$$
$$x(1) = 1$$
$$x(2) = 4$$
$$\Rightarrow x(n) = n^2$$

Or, in a more recognizable form:

[17] Galileo, *De Motu* p. 66
[18] *Ibid*, p. 73

$$x = \frac{1}{2}at^2$$

This discovery convinced Galileo that indeed, mathematics was the language of nature. He was able to use a formula to predict a body's motion and separate the idealized state from the observed motion. This technique was used by Newton to divide inertial motion and disturbances from forces like gravity and magnetism. Galileo's other major discoveries — such as the parabolic nature of projectile motion, his theory of the tides, and his thoughts on relativity that we refer to as *Galilean invariance* — were huge advancements in the field of natural philosophy. Entire books could be devoted to each of those momentous ideas.

The gist, though, is that these discoveries brought the study of nature to the threshold of greatness and did so governed by mathematics and geometry. Kepler's biggest struggle was his ambition; he was observing motion that was too complex for him to grasp. The complexity in the system of the heavens obscured him from seeing the fundamental laws of dynamics. By focusing on the foundations — one-dimensional motion with constant acceleration — Galileo was able to make far more significant strides than Kepler. The other great advantage that Galileo possessed by studying terrestrial rather than celestial motion is the ability to manipulate initial conditions. The systems in astronomy are unique cases that are unchangeable by humans. Galileo's advancements in the scientific method were possible because he was able to control nearly every variable in his experiments. It is much easier to discover how the angle of a slope affects a kinematic system if it is the only variable to change.

It is not intuitive to believe that going back to the basics furthers our advanced understanding of complex systems, but Galileo is our proof. The paradigm of the immutability and perfection of the heavens was ended by Galileo. The final straws were Descartes' coordinate system and Christiaan Huygens' invention of the pendulum clock, which revolutionized measurement and precision. After Huygens discovered centrifugal force, the first form of the law of conservation of energy, and the law of relative velocities, the study of dynamics was poised for complete transformation. Barbour offers insight into the advancements from this period of science in a wonderful metaphor:

> Huygens is the bird imprinted on his earthbound falconer, Descartes. He had fashioned himself wings with which he might have flown to unimagined heights but was restrained by a string that temperament and circumstances never gave him cause to break. His clarification of the centrifugal phenomenon and the elucidation of the concept of force had an elegance that surpassed Newton's, and anticipated his by several years.
>
> If Huygens was a falcon content to remain on the perch having brought home the sleekest hares ever caught on the Lord's estate, Newton was the soaring eagle with an eye to catch the moon and the very stars.

Indeed, Newton was able to break the Cartesian tether and set forth into uncharted land, which our story will turn to now.

Isaac Newton was born on what was then[19] Christmas Day, 1642, in the hamlet of Woolsthorpe in Lincolnshire. Newton's mother initially wished for him to take over the family's estate, but his academic promise resulted in his being sent to school in Grantham, where he was noted for the brilliance of the mechanical devices he constructed. In the summer of 1661, Newton matriculated at Trinity College, Cambridge, where, like many other universities in Europe, natural philosophy was still largely in the grip of Aristotelianism. Before he graduated, he dreamed up a new system of mathematics that studied how things changed, be it speed, position, velocity, volume, etc. At the age of 22, he embarked on independent study in almost complete isolation from other scientists. Towards the end of this period, in 1665, the university was closed by an outbreak of the Bubonic Plague, and Newton was forced to return to his home. He claimed in later years that in his home garden, he "began to think of gravity extending to the orb of the moon.[20]" The unpublished studies that Newton did in these early years, including pioneering work in mathematics (working out the foundations of calculus), optics (including his experiments on the spectral decomposition of light and the invention of the reflecting telescope named after him), and dynamics were immense and set the stage for greatness.

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[19] It's a long story. Newton's date of birth is contentious. In the 1640's, England switched from the Julian calendar to the Gregorian calendar, which we use today. Depending on which calendar you prefer, Newton was either born on Christmas Day, 1642, the year Galileo died, or January 4th, 1643.

[20] I. Newton, Catalog of Portsmouth Collection, Cambridge (1888), Sec. 1, Division xi, No. 41.

Newton's name entered the international stage primarily through his early work on optics, though knowledge of his work in mathematics was spread through correspondences and actually reached Leibniz. Later, this led to a bitter controversy as to whom the priority in the discovery of calculus should be attributed. It was his reflecting telescope that brought Newton recognition and election to the Royal Society, the world's oldest independent scientific academy. This encouraged him to submit a paper on optics in 1672, which, although well-received, led to a sharp dispute between him and Robert Hooke.

At the time, the Royal Society was "particularly busy investigating and understanding nature and the laws of motion more thoroughly than has been done heretofore… since nature will remain unknown so long as motion remains unknown, diligent examination of it is the more incumbent upon philosophers…[21]". Hooke had especially been developing ideas on the dynamical treatment of the planetary problem. In particular, he proposed that the planets were kept in orbit by a force directed towards the sun. In 1679, Hooke wrote to Newton, asking for an opinion on his theory. This correspondence led Newton to an insight that changed the history of physics forever — Kepler's Laws could demonstrate the planets must be attracted to the sun by a force whose strength decreases the farther the body is from the sun. That is to say, only and inverse square law $F \alpha \frac{1}{r^2}$ would reproduce Kepler's Laws. This critical discovery prompted the synthesis of dynamics.

For years, Hooke, Wren, and Halley had been considering the problem of the planets. They were closing in on the right solution but lacked the mathematical fortitude to get to the finish line. Specifically, Halley and Wren had the idea to partner Kepler's Third Law with Huygens' formula for centrifugal force to show that the gravitational force between objects varies as the inverse square of the distance between them. What they could not derive was the specific path that such a force would drive objects to follow. In August of 1684, Halley traveled to Cambridge and put the problem to Newton: what curve would

[21] *Westfall* (1971), opp. contents page.

be described by a planet subject to a force of attraction from the sun that was inversely proportional to the square of its distance? In response, Newton told Halley that he had solved this problem years ago but could not, at the moment, find his proof. He promised, though, to provide it when he found it. In November, Halley received his answer, an ellipse with the sun at one focus, in a paper titled *De Motu Corporum in Gyrum (Concerning the Motion of Bodies in Orbit)*. Newton's ingenious solution was to invert the problem: assume the orbital path to be an ellipse and find the force required to ensure this.

Upon reading Newton's solution, Halley urged him to enter his work into the register of the Royal Society, as evidenced by a report on the matter to the organization on December 10th, 1684. During this time, Newton's life had utterly transformed. As Barbour so eloquently puts it, "[Newton] started work on one of the most astonishing labors of intellectual man: a comprehensive treatise on motion, the aim of which was to show how the entire gamut of observed motions — both terrestrial and celestial — could be deduced from a mere handful of general principles formulated in a mathematically rigorous framework." His servant at the time noted how intensely this work seized his master: Newton would often completely forget to eat as he was so engrossed in his project.

In the autumn of 1685, Halley returned to Cambridge to see the work done thus far. In the year since his first visit, Newton had transformed an early solution of the planetary problem into a complete theory of universal gravitation. From this point, Halley devoted the whole of his energy to both ensuring Newton's work was published and preparing the scientific community for the dawn of the impending masterpiece. Thus began the series of events that eventually led to the publication of *Philosophiae Naturalis Principia Mathematica* (The Mathematical Principles of Natural Philosophy) in the spring of 1687, a date that defines the scientific age.

The role of Halley in Newton's success is not to be underestimated. Halley referred to himself in later years as 'the Odysseus who produced this Achilles,[22]' and he definitely needed the skills and strength of the Ithacan king. Managing press for temperamental Newton served difficult for his disagreement with Hooke:

> There is one thing more that I ought to inform you of, viz, that Mr. Hooke has some pretensions upon the invention of your rule of the decrease of gravity, being reciprocally as the squares of the distances from the center. He says you had the notion from him, though he owns the demonstration of the curves generated thereby to be wholly your own; how much of this is so, you know best, as likewise what you have to do in this matter, only Mr. Hooke seems to expect you should make some mention of him, in the preface, which, it is possible, you may see reason to prefix. I must beg your pardon that it is I, that send you this account, but I thought it my duty to let you know, that so you may act accordingly: being in myself fully satisfied, that nothing but the greatest candor imaginable, is to be expected from a person, who of all men has at the least need to borrow reputation.[23]

Given that Hooke had no idea of the magnitude of work Newton had accomplished, this was not an unreasonable request. In their correspondences, Hooke had explicitly told Newton that he believed "attraction" was always based on a "duplicate proportion to the distance from the center,[24]" but that he did not know what curve a body subject to a central attraction would follow. Newton's unpublished papers show us that the suggestion of the inverse square law was the least of Hooke's assistance: his real service was giving Newton the idea to consider central attraction in the planetary problem. Once Descartes supplied what later became the law of inertia, all that remained was finding a force responsible for deflecting the path of the planets. Hooke proposed the specifics: an attractive force towards a center. This distinction separates the work of Hooke from that of Descartes, Huygens, and even early Newton. In the cases of the latter, the deflecting force was a *contact force*, given by a string in the case of Descartes and Huygens and a circular rim in the case of Newton.

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[22] *Westfall* (1980), p. 405
[23] *The Correspondence of Isaac Newton,* Vol. 2, ed. H. W. Turnbull, Cambridge University Press (1960), pg. 431
[24] *Ibid,* p. 309

After an initially controlled reply to Halley, Newton wrote an unrestrained letter to Hooke. Apart from the glimpse into Newton's troubled psyche, the main point of the exchange is that it shows us how prominent the planetary motion problem was in the late seventeenth century. Hooke, Halley, and Wren tried and failed. From Descartes, the problem passed through the qualitative stage of Borelli and Hooke and now needed to be solved quantitatively and empirically. The remarkable thing, though, was how suddenly a theory of everything came. Halley asked for a solution to a specific problem and received a theory of universal gravitation with all its consequences fully explored. The relationship between the sun and the planets was established beyond doubt by Newton's work. He demonstrated, in excruciating detail, that every little speck of matter exerts a force on every other bit of matter proportional to its mass and the square of the distance between them. With Newton's tiff tamed, the president of the Royal Society, Samuel Pepys, granted the "imprimatur," and printing was completed on July 5, 1687.

The *Principia* is, in a word, substantial, both in impact and in material: the English translation by Andrew Motte is about 500 pages. It starts with an Ode by Halley and a preface by Newton, after which, the *Principia* proper begins with a set of eight definitions of fundamental concepts. It's interesting to note that Newton writes his science like a mathematician: he offers a theorem, specifies corollaries, and poses problems, all in the formal text utilized since Euclid.

The first fundamental concept that Newton introduces is mass, or, as he puts it, "the quantity of matter." In the history of natural philosophy, little regard had been given to this remarkably important concept since Aristotle. Newton distinguishes mass from weight, and describes the proportionality he has found by "experiments on pendulums, very accurately made." Next, Newton defines what we call *momentum* — "the quality of motion…arising conjointly from the velocity and quantity of matter" — and then *inertia*:

> The *vis insita*, or innate force of matter, is a power of resisting, by which every body, as much as in it lies, continues in its present state, whether it be of rest, or of moving uniformly forwards in a right line.

He identifies that a body only exerts this force when another force acts upon it and utilizes definitions IV-VII to expand upon these "impressed" forces. In definition IV, Newton defines the general case, in which an impressed force is "an action exerted upon a body in order to change its state." Then, he specifies centripetal forces in definition V to be those that "tend toward a center" and of which there are three types, absolute (VI), accelerative (VII), and motive (VIII).

After these definitions, Newton offers his infamous Scholium comparing absolute versus relative time, space, and motion. Up until Galileo, motion had largely been discussed in relation to other motion. The concept of space was an abstraction humans used to compare the different arrangements of a system of objects. Thus, the idea of empty space was a conceptual impossibility, like dividing by zero. The same was true for time. It was believed that there can be no lapse of time without evolution: time is merely a measure of cycles of change in a system. In the *Principia,* Newton declared that space is something real and distinct from objects, and that time is real and passes uniformly without regard to whether anything moves in the world. That is, matter has *no effect* on space or time.[25] This paradigm worked until Einstein, but that is a discussion for another text.

Following this discussion, Newton lists his three fundamental laws of motion:

> *Law I:* Every body continues in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed upon it.
> *Law II:* The change of motion is proportional to the motive force impressed; and is made in the direction of the right line in which the force is impressed.
> *Law III:* To every action there is always an opposed equal reaction: or, the mutual actions of two bodies upon each other are always equal, and directed to contrary parts.

Later in the *Principia*, in the following scholium, Newton discusses the empirical evidence he has collected to prove his Third Law of Motion, but offers no justification for either of the first two. This instance of asserting a claim without offering explicit

[25] From Einstein, of course, we now know that this is not true. Matter, specifically mass, curves spacetime.

justification is not a unique occurrence in *Principia*. While Newton often gives the rationale for his arguments, there are many cases where his reasoning is *not* explicit, or even implied. Just like some textbook authors today, when he doesn't feel like going into it, Newton leaves the proof as an exercise to the reader. A key instance of this will be discussed later, in Chapter Two.

The laws of motion are followed by six corollaries that introduce several core tenets of dynamics. The first two corollaries discuss the composition and decomposition of forces: The former equates the "addition" of two forces into one composite force using parallelograms, and the latter discusses breaking a force into its components, like a triangle. Corollary III states the law of conservation of momentum and Corollary IV claims that the center of gravity of a closed system[26] is either at rest or moves uniformly in a straight line. The fifth corollary gets interesting, so I include the full text and associated proof (italicized):

> The motions of bodies included in a given space are the same among themselves, whether that space is at rest, or moves uniformly forwards in a right line without any circular motion.
>
> *For the differences of the motions tending towards the same parts, and the sums of those that tend towards contrary parts, are, at first (by supposition), in both cases the same; and it is from those sums and differences that the collisions and impulses do arise with which the bodies mutually impinge one upon another. Wherefore (by Law II), the effects of those collisions will be equal in both cases; and therefore the mutual motions of the bodies among themselves in the one case will remain equal to the mutual motions of the bodies among themselves in the other. A clear proof of which we have from the experiment of a ship; where all motions happen after the same manner, whether the ship is at rest, or is carried uniformly forwards in a right line.*

It is peculiar that while Newton *firmly* insists on the reality of absolute space and motion, this discussion of motion appears to be… relative. This corollary, which is supposed to be a direct consequence of his laws of motion, rejects the claim that the speed and direction of absolute motion can be determined from the phenomena, which counters the implications of the Scholium. Finally, in Corollary VI, Newton claims that

[26] In Newton's own words, "*acting upon each other (excluding outward actions and impediments)"*

bodies urged in parallel lines by equal accelerative forces will move in the same way as if they had not been acted upon.

This is shockingly similar to Einstein's Equivalence Principle and his elevator thought experiment. Einstein argued that acceleration in flat spacetime is locally indistinguishable from gravity. That is, if you can only make measurements in a small area around you, you cannot distinguish between acceleration and gravity. Or, if you're stuck in an elevator, you cannot tell whether an object, if you dropped it, was being pulled down by Earth's gravity or pulled up by the elevator accelerating upward. It is *fascinating* how scientists centuries apart can observe the same phenomena and interpret them so differently. In the final scholium of the section, Newton acknowledges the ideas that inspired his *magus opus* and credits Galileo, Sir Christopher Wren, Dr. Wallis, Mr. Huygens, the "greatest geometers of our time", and Edme Mariotte for "laying down such principles" and "confirming by an abundance of experiments". Thus, we move to the body of the *Principia*.

The main text of the *Principia* is broken up into three sections. The first book discusses motion with no resistance, the second one motion with resistance, and the third is aptly titled "Concerning the System of the World." To cover all the details contained in *Principia*, I'd have to write a book at least as long as it. Suffice to say, every page is densely packed with an immense range of material. The *Principia* was and is a daunting book — in the seventeenth century, on account of the novelty and intricacies of the subject; in the modern day, on account of the now archaic mathematics and vernacular used. Even when reading an English translation, Newton's methods of derivation are so foreign that there is almost a language barrier to understanding. The way that we approach and complete proofs has *fundamentally changed*.

To analyze this further, we will turn to a derivation from Book II.

Chapter Two: Investigating Newton's Proof (Or Lack Thereof.)

In the *Principia*, Newton performs four experiments to empirically confirm his theories about forces and their interactions. In Book II, which, as stated earlier, covers motion in resisting media, Newton performs two elaborate experiments to measure fluid resistance forces. His ultimate goal with these experiments was to provide conclusive evidence that there is no medium resisting the motions of comets. But, as he was understandably limited to terrestrial experimentation, the investigation of fluid mechanics was of unique interest to him.

Sometime after *De Motu* in 1684, Newton concluded that resisting forces consisted of two components: one owed to the "internal friction" that resulted from the viscosity of the fluid and varied as the velocity of the moving body; and another that resulted from a body's inertia and varied as velocity squared. In the summer of 1685, Newton conducted initial experiments investigating pendulum decay in air and water. He believed that by varying the arc length of the pendulum, he could disaggregate the components of the resistance force. Newton presented the results of this experiment in the first edition of *Principia* in unprecedented detail, thus allowing readers to retrace his steps and confirm his results. He had reason to do this, as he did not come to any precise conclusions. All he was able to deduce was that the resistance force could not involve any power of velocity greater than 2.

Shortly after the first edition of *Principia* was published, Newton discovered the source of error in his pendulum-decay experiment: the oscillating motion of the pendulum caused the medium to also oscillate, resulting in a difference between the velocity of the bob relative to the fluid and to the velocity of the bob relative to the central axis. This was a *fatal* design flaw of his experiment that compelled Newton to scrap it entirely for the second and subsequent editions.

In the redesign, Newton relied on vertical fall to precisely measure the effects of resistance forces in a hope of determining the relationship between the density of a resisting medium and the magnitude of resistance it provided. From the pendulum fiasco, Newton had a good idea that the resistive force depended on the shape of the sphere (specifically, its diameter, d, squared) and the density of the fluid medium, ρ. He deduced from initial vertical-fall experiments that the exact solution would involve the body's velocity, v, squared and a coefficient of proportionality, c, thus creating a resistive force with form $c\rho d^2 v^2$. Taking c as given, Newton could thus predict the time a body would take to fall from a given height, or the height fallen in a given time. After dropping ten spheres in water and six in air from the top of the dome of St. Paul's Cathedral,[27] Newton's plan was to compare the predicted time of descent and the measured time, remarking "if it [the body] encounters another resistance in addition, the descent will be slower, and the quantity of this resistance can be found from the retardation."

The problem is that this remark is nearly all that Newton offers as explanation. Book II, Proposition XL, Problem IX provides the "derivation" — more accurately, the lack thereof — that Newton uses to support his theory of resistive forces. It's just over a page long and horribly incomplete. Newton defines his variables and then offers his formula for height, $h(t)$. He pulls numbers apparently from thin air without including any discussion of where they come from; a method very different from the standard today.

Investigating the differences in experimental design between the investigation into fluid mechanics in the first, second, and third editions of *Principia* gives us a remarkable glimpse into the *development* of classical mechanics. By looking at the changes between editions, we can watch Newton's process. Because of Newton's tendency toward

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[27] Designed by William & Mary's own Sir Christopher Wren!

individual study and isolation, we don't have many papers that show us *how* he thought. We also don't have much record of Newton's failures, but the readaptation of the vertical-fall experiment from the inconclusive pendulum-decay experiment shows us an explicit example of a mistake. Studying the *evolution* of the *Principia* tells us as much about physics, if not more, as the body of the text, itself. The purpose of this chapter is to "update" this derivation and provide an explicit, more thorough explanation of parts of it. I will be including the verbatim text (in English) in `block quotes` and my commentary in normal text. We start by defining our variables:

PROPOSITION XL. PROBLEM IX

> *To find by experiment the resistance of a globe moving through a perfectly fluid compressed medium.*

> Let A be the weight of the globe in a vacuum, B its weight in the resisting medium, D the diameter of the globe, F a space which is to $\frac{4}{3}D$ as the density of the globe is to the density of the medium, that is, as A is to $A - B$.

"A space that is "*to*" $4/3$ D?" This is not a contemporary phrasing, but I think Newton is trying to set up a set of ratios:

$$\frac{F}{\frac{4}{3}D} : \frac{\rho_{globe}}{\rho_{medium}} : \frac{A}{A-B} \qquad (1)$$

Without reference to it, Newton is employing Archimedes' principle: $F_b = -\rho g V$, where F_b is the buoyant force, ρ is the density of the fluid, g is acceleration due to gravity, and V is volume of fluid. If we solve plug $\rho = \frac{mass}{volume}$ into formula (1) and our definitions of A and B into Archimedes' principle, we arrive at a consensus:

$$B = A - \rho_{fluid}V \Rightarrow A - B = \rho_{fluid}V \qquad (2)$$

where $\rho_{fluid}V$ would be the mass of the displaced fluid, which agrees with the second ratio in Eq. (1). Let's continue:

> ...G the time in which the globe falling with the weight B without resistance describes the space F, and H the velocity

> which the body acquires by that fall. Then H will be the greatest velocity with which the globe can possibly descend with the weight B in the resisting medium, by Cor. II, Prop. XXXVIII; and the resistance which the globe meets with, when descending with that velocity, will be equal to its weight B; and the resistance it meets with in any other velocity will be to the weight B as the square of the ratio of that velocity to the greatest velocity, H.

That is to say, we are operating in conditions of freefall: $x = vt \Rightarrow F = HG$, where H is v_{max}, the terminal velocity of the body.

> Let the globe be let fall so that it may descend in the fluid by the weight B; and let P be the time of falling, and let that time be expressed in seconds, if the time G be given in seconds. Find the absolute number N agreeing to the logarithm $0.4342944819\,\frac{2P}{G}$, and let L be the logarithm of the number $\frac{N+1}{N}$;

Note the subtle distinction between "logarithm" and "logarithm of." The difference in the Latin — *logarithmo* versus *logarithmus numeri* — helped us deduce the arguments of each term and figure out that Newton means to say $log(N) = 0.4342944819\frac{2P}{G} \Rightarrow log(N) = log_{10}(e) * \frac{2P}{G} \Rightarrow N = e^{\frac{2P}{G}}$ where the leading coefficient 0.4342944819 is utilized to switch between $log_{10}(x)$ and $ln(x)$, and $L = log_{10}(\frac{N+1}{N})$.

> ...and the velocity acquired in falling will be $\frac{N-1}{N+1}H$, and the height described will be $\frac{2PF}{G} - 1.3862943611F + 4.605170186LF$. If the fluid be of a sufficient depth, we may neglect the term $4.605170186LF$ will be the altitude described; nearly.

This passage embodies why this chapter exists. Newton gives us these equations and offers no evidence. First, let's try to explicate his logic about the last term being minimized. If the fluid is "sufficiently deep," $\frac{2P}{G} >> 1$, so,

$$L = log_{10}(\frac{N+1}{N})$$

$$= log_{10}(1 + \frac{1}{N})$$

$$= ln(1 + \frac{1}{N}) * log_{e}(10)$$

which, if we perform a Taylor expansion, we see that the leading term is proportional to $\frac{1}{N} = e^{\frac{-2P}{G}} << 1$. Okay, great. Any term with L should vanish when $P >> G$. I have no idea, though, where the coefficients 1.386… and 4.605… come from. Upon preliminary research, I have found that $1.3862943611 = ln(4)$ and $4.605170186 = 2ln(10)$, but those numbers hold little significance for me at this time.

BOOK II: THE MOTION OF BODIES 355

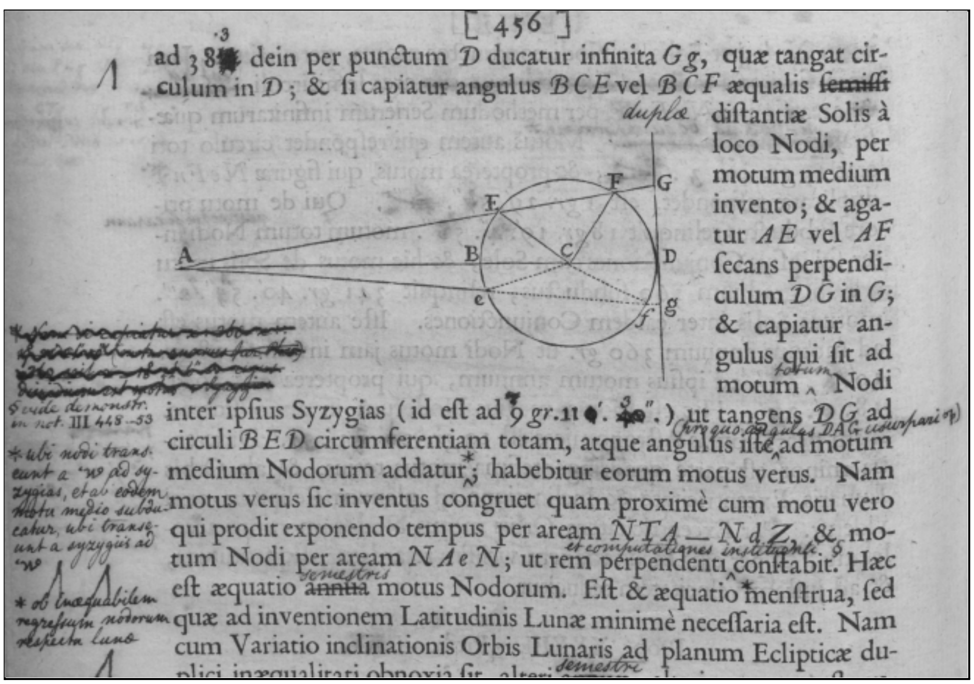
The Times P	Velocities of the body falling in the fluid	The spaces described in falling in the fluid	The spaces described with the greatest motion	The spaces described by falling in a vacuum
0.001G	99999 29/30	0.000001F	0.002F	0.000001F
0.01G	999967	0.0001F	0.02F	0.0001F
0.1G	9966799	0.0099834F	0.2F	0.01F
0.2G	19737532	0.0397361F	0.4F	0.04F
0.3G	29131261	0.0886815F	0.6F	0.09F
0.4G	37994896	0.1559070F	0.8F	0.16F
0.5G	46211716	0.2402290F	1.0F	0.25F
0.6G	53704957	0.3402706F	1.2F	0.36F
0.7G	60436778	0.4545405F	1.4F	0.49F
0.8G	66403677	0.5815071F	1.6F	0.64F
0.9G	71629787	0.7196609F	1.8F	0.81F
1G	76159416	0.8675617F	2F	1F
2G	96402758	2.6500055F	4F	4F
3G	99505475	4.6186570F	6F	9F
4G	99932930	6.6143765F	8F	16F
5G	99990920	8.6137964F	10F	25F
6G	99998771	10.6137179F	12F	36F
7G	99999834	12.6137073F	14F	49F
8G	99999980	14.6137059F	16F	64F
9G	99999997	16.6137057F	18F	81F
10G	99999999 3/5	18.6137056F	20F	100F

This is an opportunity for further research. These numbers contain a lot of significant figures — the precision that Newton displayed in his computations and claimed in his experiments is astounding. In the chart of data and calculations that follows this derivation, pictured above, Newton claims to measure time to the thousandth of a second. This is a remarkable claim, given that oscillations of pendulums constituted the cutting edge of timekeeping.

I'll be transparent — Newton's precision is a bit *too* astounding to sit right with me. It would be interesting to investigate whether we could replicate this level of precision using Newton's methodology today. I am doubtful. Newton's derivations, measurements, and experiments are some of the many enigmas of the *Principia*. To investigate some of the other mysteries buried in the text, I turn now to discuss the annotations written in the margins of William and Mary's copy of the *Principia*.

Chapter Three:
The Mysteries of the Marginalia

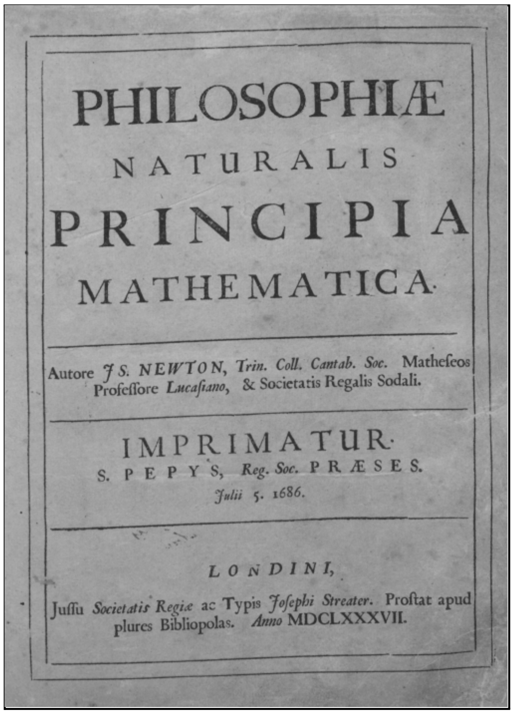
William & Mary's Special Collections owns a first edition copy of *Principia*, donated by Rev. Thomas Staughton Savage in 1869. A recent census by Caltech reports that our copy is one of 386 first editions known to survive to the modern day[28]. Other copies reside in libraries and private collections around the world; in 2016, a first edition sold for over $3.7 million, making it the highest valued scientific book in the world[29]. William & Mary's (W&M) copy, though, is priceless due to the marginalia littering the text. My work for the past three years has been cataloging these notes, deciphering their content, and analyzing what they show us about the creation and development of classical mechanics.


[456]

ad 38 dein per punctum *D* ducatur infinita *Gg*, quæ tangat circulum in *D*; & ſi capiatur angulus *BCE* vel *BCF* æqualis ~~ſemiſſi~~ dupla diſtantiæ Solis à loco Nodi, per motum medium invento; & agatur *AE* vel *AF* ſecans perpendiculum *DG* in *G*; & capiatur angulus qui ſit ad totum motum Nodi inter ipſius Syzygias (id eſt ad 9 *gr*. 11′. ″.) ut tangens *DG* ad circuli *BED* circumferentiam totam, atque angulus iſte ad motum medium Nodorum addatur; habebitur eorum motus verus. Nam motus verus ſic inventus congruet quam proximè cum motu vero qui prodit exponendo tempus per aream *NTA* — *NdZ*, & motum Nodi per aream *NAeN*; ut rem perpendenti et computationes instituenti conſtabit. Hæc eſt æquatio ~~annua~~ semestris motus Nodorum. Eſt & æquatio menſtrua, ſed quæ ad inventionem Latitudinis Lunæ minimè neceſſaria eſt. Nam cum Variatio inclinationis Orbis Lunaris ad planum Eclipticæ duplici inæqualitati obnoxia ſit, alteri semestre

[28] Clavin, Whitney. "Hundreds of Copies of Newton's Principia Found in New Census." *California Institute of Technology*, 10 Nov. 2020, https://www.caltech.edu/about/news/hundreds-copies-newtons-emprincipiaem-found-new-census.
[29] Geggel, Laura. "Isaac Newton's Book Auctioned for Record-Setting $3.7 Million." *LiveScience*, Purch, 15 Dec. 2016, https://www.livescience.com/57229-isaac-newton-book-sold-breaks-record.html.

When I began, my predecessor, Jackson Olsen, left me a few of his notes and a solid starting point. He had noticed that several of the sections in W&M's edition appeared to be crossed out and replaced with passages from Newton's Third Edition. Niccolò Guicciardini, a historian of mathematics at the University of Milan, also pointed out the possibility of a reference to a proof from 1802 in one of the annotations. I was interested in searching for more dates to see if I could further limit the timeline of our alleged annotator(s). I started, as most people do, at the beginning: in my case, on the title page of the *Principia*. Immediately, I noted the absence of the letter *c* in the Latin word *auctore*, "author". In the second and third editions that I have seen, this typo is corrected, but it is still surprising to me that a mistake of this size was not caught before printing. While Samuel Pepys was responsible for typesetting the title page rather than Newton, I note this blunder as a reminder (to both reader and author) to distance yourself from equating genius with perfection. Albeit the embodiment of brilliance, he is not to be blindly trusted: this was the first of many lessons I learned from my time with the *Principia*.


PHILOSOPHIÆ
NATURALIS
PRINCIPIA
MATHEMATICA.

Autore *JS. NEWTON*, *Trin. Coll. Cantab. Soc.* Mathefeos Profeffore *Lucafiano*, & Societatis Regalis Sodali.

IMPRIMATUR.
S. PEPYS, *Reg. Soc.* PRÆSES.
Julii 5. 1686.

LONDINI,
Juffu *Societatis Regiæ* ac Typis *Jofephi Streater*. Proftat apud plures Bibliopolas. *Anno* MDCLXXXVII.

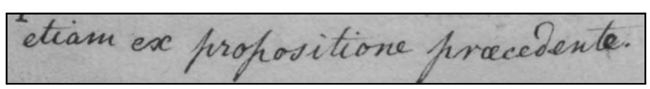
Still shocked, I turned to the epithets attributed to Newton, 'Trin. Coll. Cantab. Soc. Mathefeos Profeffore *Lucafiano*, & Societatis Regalis Sodiali.' and had some difficulty translating them. I reached out to a Linguistics professor, Jack Martin of William & Mary, who taught me about seventeenth-century typesetting practices. The 'f' that I was seeing was an 's' used to conserve space to preserve the width of a line of text. I saw three different types of S's and named them the 'normal,' 'integral,' and 'f' S's; they can be seen in *Soc.*, *Lucasiano*, and *matheseos*, respectively. I thus translated the epithets to be 'Trinity College, Lucasian Professor of Mathematics, & Friend to the Royal Society.[30]'

[30] I was unfamiliar with the Royal Society, founded by Sir Christopher Wren, William Petty, and Robery Boyle, until working on this project. They describe themselves as a "learned society" dedicated to

This gave me my first few leads on possible identities — perhaps our annotator was a work colleague of Newton's, or maybe an editor from the Royal Society.

I continued through the *Principia*, noticing an amalgamation of cross hatching, underlining, bubbles, and dots. I also saw that someone was numbering the sentences and changing the Latin ever so slightly. In Latin, nouns are assigned a certain declension based on their part of speech. While English determines the meaning of a sentence from its word order — that is, ‘the dog chases the cat’ means something very different from ‘the cat chases the dog,’ and ‘dog the chases cat the’ is completely nonsensical — Latin utilizes the last 1-4 letters of a noun to assign parts of speech. Thus, *Canis fugat felem* has the same meaning as *Felem canis fugat*: the dog chases the cat. If I wanted to alter the meaning of the sentence into ‘the cat chases the dog,’ I would need to change it to *Canem fugat feles*. In the *Principia*, I saw several instances of an annotator changing noun endings. The annotator would cross out the original sentence and replace it with another of similar content but containing slight shifts in grammatical structure. This suggested that my annotator was editing the first edition.

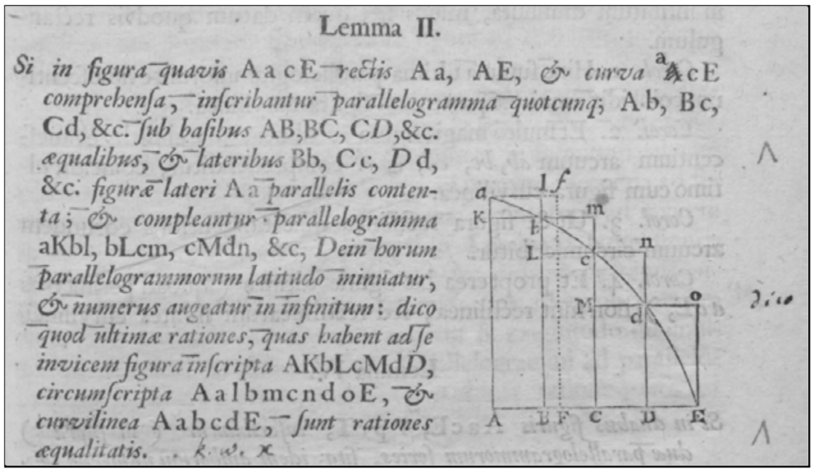
Thus, I started looking into editors associated with Newton and stumbled upon a man by the name of Roger Cotes. After comparing his handwriting with excerpts from the *Principia* and learning that all four copies of the text touched by Cotes are already accounted for[31], I abandoned the thought and turned elsewhere. I fell into a rabbit hole comparing handwriting samples of nearly every name associated with Newton I could find. I scoured every corner of the Internet, thankful for private collectors who had held on to letters, journals, and diaries. After comparing samples from *numerous* connections, I'd developed an efficient system. I noticed that my author had characteristically open ‘p's’ that looked almost like ‘h's’ and his ‘d's’ had a curved stem that hooked over towards the left. Nevertheless, at the end of day one, I found myself stuck in a web of red strings, browser tabs, and personal correspondences,



celebrating excellence in science. Members are on the order of Jocelyn Bell, Stephen Hawking, and Charles Darwin.

[31] From the University of Sydney Library, https://digital.library.sydney.edu.au/nodes/view/7166

trapped like a frantic insect. I must have looked at hundreds of samples, none of which matched the careful script of my annotator. After eating and sleeping, I recentered on the only thing I knew to be certain: my text. I continued on from the first few pages, moving past the Definitions and Axioms, and found something of interest in the second Lemma of Book 1:



Lemma II.

Si in figura quavis AacE *rectis* Aa, AE, *& curva* AcE *comprehensa, inscribantur parallelogramma quotcunq;* Ab, Bc, Cd, &c. *sub basibus* AB, BC, CD, &c. *æqualibus, & lateribus* Bb, Cc, Dd, &c. *figuræ lateri* Aa *parallelis contenta; & compleantur parallelogramma* aKbl, bLcm, cMdn, &c. *Dein horum parallelogrammorum latitudo minuatur, & numerus augeatur in infinitum: dico quod ultimæ rationes, quas habent ad se invicem figura inscripta* AKbLcMdD, *circumscripta* AalbmcndoE, *& curvilinea* AabcdE, *sunt rationes æqualitatis.*

My own translation of Lemma II reads: *"If in any figure* AacE, *embraced by the right [lines]* Aa, AE*, and the curve* [a]cE*, any amount of parallelograms can be inscribed;* Ab, Bc, CD, & c.*, having been constrained under equal bases* AB, BC, CD, &c. *& with sides* Bb, Cc, Dd, & c. *parallel to side* Aa *of the figure; & the parallelograms* aKbl, bLcm, cMdn, &c. *are completed. Then, if the width of these parallelograms might be reduced, and [their] number be increased to infinity: I say that the ultimate ratios that the inscribed figure* AKbLcMdD, *the circumscribed figure* AalbmcndoE, *and the curved line* AabcdE *have to each other are equal ratios."*

A knowledgeable reader might recognize the subject of this passage from *Principia* from the diagram alone. In Book 1, Lemma II, Newton argues that a curve can be estimated by a series of rectangles, an idea famously expanded upon by German mathematician Bernhard Riemann 150 years later. Indeed, in Lemma's I-IV, Newton is presenting a geometrical approach to his new flavor of mathematics, infinitesimal calculus. In other sections, when Newton presented an exciting, novel idea such as this, the margins were littered with notes. I found it puzzling, then, that the Latin word *dico*, 'I said,' was the only comment written on this page. Even more peculiar, over the next three pages, *dico* was written and underlined another five times.

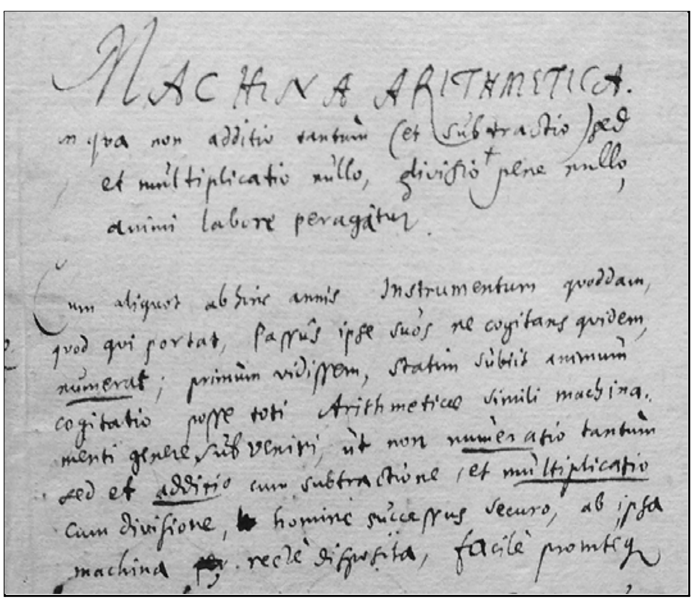
We know what was being said… but who is 'I'?

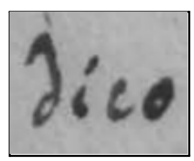
The controversy surrounding Isaac Newton, Gottfried Wilhelm Leibniz, and the discovery of calculus was a bitter, high-profile dispute among scholars. Newton and his fervent supporters slandered Leibniz for the last years of his life. In his own correspondences, Newton's perspective alleging intellectual theft is clear:

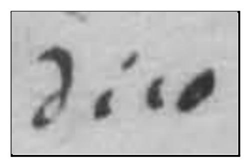
> *"Second inventors have no right. Whether Mr. Leibniz found the Method by himself or not is not the Question… We take the proper question to be,… who was the first inventor of the method. Probity and principle demand a correct answer: To take away the Right of the first inventor, and divide it between him and that other [the second inventor], would be an Act of Injustice.[32]"*

In the face of such criticism, perhaps a miffed Leibniz procured a copy of Newton's text in order to investigate the claims of his opponent. *Dico*, I wondered if Leibniz wrote, *Dico primus*: I said *first*.

I checked online to find a handwriting sample to compare our *dico*'s with. Thankfully, Stephen Wolfram, creator of *Mathematica* and Wolfram|Alpha, had posted pictures of a passage from Leibniz's notes for us. Given the blessing of multiple samples and the expectation that an author's hand will show discrepancies across space and time, I picked two of my favorites to serve as a reference point:





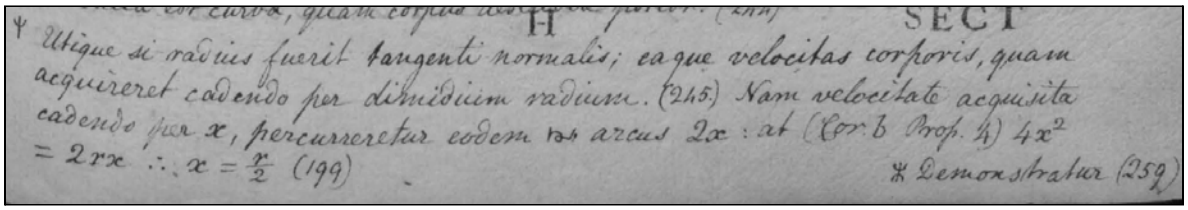


On the left, Leibniz crosses his A's and t's from left to right, evidenced by line thickness and flow of letters, so he is most likely right-handed. Besides that, his scrawl is messy and his hand is heavy. His words, like the *dico*'s from the *Principia* pictured above, are composed of printed,

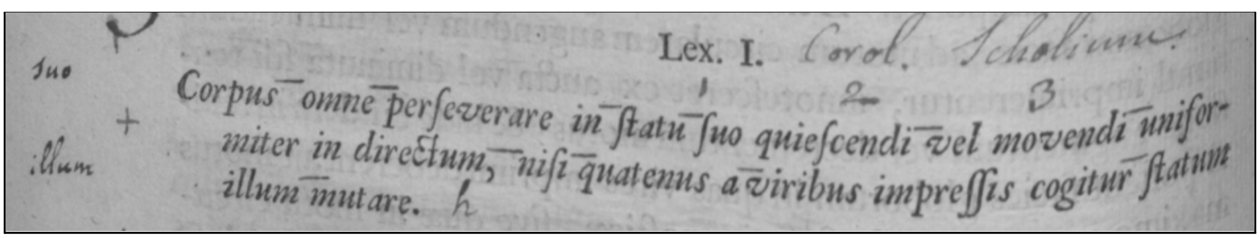
[32]Sir Isaac Newton, *The Correspondence of Isaac Newton*, 7 v., edited by H. W. Turnbull, J. F. Scott, A. Rupert Hall, and Laura Tilling, Cambridge University Press, 1959–1977.

mostly non-connected letters. Looking letter by letter, the d's in both have hooked stems and both o's are completely closed.

At the time, I believed that the annotations in the *Principia* were authored by two individuals. The first, nicknamed "the Doodler," was responsible for the underlining and cross-hatching while a second, more engaged reader wrote the more complex annotations. But what if there was a third? We can see that the majority of the annotations are written in a tidy, flowing cursive, illustrated here:


Ubique si radius fuerit tangenti normalis; eaque velocitas corporis, quam acquireret cadendo per dimidium radium. (245) Nam velocitate acquisita cadendo per x, percurreretur eodem arcus $2x$: at (Cor. 6 Prop. 4) $4x^2 = 2rx \therefore x = \frac{r}{2}$ (199) Demonstratur (259)

I am not convinced that the same person wrote this cursive and those *dico*'s. Like I said before, my annotator has neat, legible handwriting, a strong baseline and rightward slope, and characteristically open p's. While the stems of these lowercase d's are also hooked like Leibniz's and the *dico's*, Leibniz did not write this cursive. And regardless, as my colleague Jack Martin pointed out, all of the *dico*'s in the margins correspond with a *dico* in the same line of text in the *Principia*. We see evidence of this behavior in a few other places, for example, in Newton's first law with *suo* and *illum*. Note the distinctive separation between letters in the margin notes and the 'integral' S's from before in the text:


suo
illum

Lex. I. Corol. Scholium.

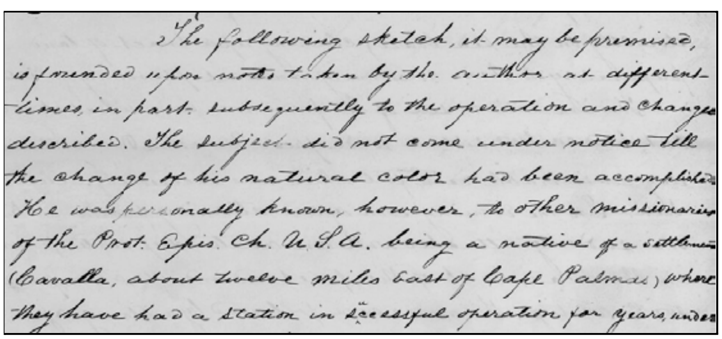
Corpus omne perseverare in statu suo quiescendi vel movendi uniformiter in directum, nisi quatenus a viribus impressis cogitur statum illum mutare.

It seems, then, that our *dico*'s could be attributed to a bored reader, mindlessly writing the words that he had just seen.

Back to the drawing board.

As invigorating as the Leibniz narrative is, the evidence is purely circumstantial. With its heavy pressure and narrow lettering, I admit that the handwriting looks similar. If we assume, that the *dico*, *suo* and *illum* are written by a different person than whoever penned the cursive, the timeline adds up[33]. I have no proof, though, that W&M's copy of the *Principia* was ever in Leibniz's hands. Thus, further investigation into the history and previous whereabouts of our copy is the only way to identify our annotator(s) with certainty. Even if I found a flawless handwriting match, I would need to prove that an individual had access to the text to definitively tie them to our book. Let's turn, then, to the one person we *know* to have had possession of our *Principia*, Thomas Staughton Savage.

Thomas S. Savage was born in Crowell, Connecticut on June 7, 1804 to a wealthy family. He graduated from Yale in 1825 and received his M.D. from Yale Medical School in 1833. He practiced medicine until 1836, when he was called to practice religion. He graduated from the Virginia Theological Seminary in 1836, after which he was ordained a deacon and priest. In the winter of 1836, Savage traveled to Liberia, becoming the first medical missionary sent by the Episcopal Church. Near the end of his trip in 1847, he discovered a new species of ape while studying chimpanzee bones. He called it *Troglodytes gorilla*, the western gorilla, which catapulted his career in science. Some of his field notes are preserved by the Royal Society[34], left, which is a fair match for our cursive annotator. The rightward slant and tidiness are consistent, along with the open and 'h'-like p's and hooked d's. The lowercase c's don't look the same, but the capital P's are a perfect match. All things considered, I'm willing to overlook a few inconsistencies because of the difference in context between taking notes and a lab


The following sketch, it may be premised, is founded upon notes taken by the author at different times, in part subsequently to the operation and change described. The subject did not come under notice till the change of his natural color had been accomplished. He was personally known, however, to other missionaries of the Prot. Epis. Ch. U.S.A. being a native of a settlement (Cavalla, about twelve miles East of Cape Palmas) where they have had a station in successful operation for years, under

[33] That is, Leibniz lived from July 1646 to November 1716. This includes the release dates of the first and second editions of the *Principia* but predates the Robertson proof of 1802 referenced in an annotation written in cursive.
[34] An account of the desquamation and change of color in a Negro of Upper Guinea, West Africa, by Thomas Staughton Savage, 1846. From The Royal Society, AP/28/21

notebook. It's important to note that handwriting changes over time and place. Savage's handwriting in his letters varies drastically, specifically with regards to tidiness.

Returning to the United States at the end of his mission, Savage served as a rector in Livingston, Alabama at St. James Church from 1848-1849 before transferring to Trinity Church in Pass Christian, Mississippi from 1849-1857. United States census data places Savage in Pass Christian in 1860 and Rhinebeck, New York in 1870. He stayed in Rhinebeck until his death in 1880. Recalling that he donated the *Principia* to W&M's Library in Williamsburg, Virginia in 1869, the whereabouts and affairs of Savage between Mississippi and New York are of interest to me. Of course, there's no guarantee that the annotations were done in this decade, so I wanted all of the content I could get my hands on.

I started my search on the Internet and used Ancestry.com to connect with Savage's living descendants. I met John Cornell, Savage's great-great-grandson, and Dave Rutherford, his great-great-grandnephew. I was thrilled to share details about my project with them. Luckily for me, both gentlemen are passionate about their family's genealogy and history and have thus kept records dating back to Savage's lifetime. From John, I learned that Savage had two sons that graduated from the University of Virginia. This was both pleasing and confusing to hear — I now knew that he spent time in Virginia outside of Seminary, but why wouldn't he have donated the *Principia* to UVA? What was Savage doing in Williamsburg, and why William & Mary? Later, Dave shared with me over email that Savage's son William was a minister in North Carolina and donated his papers to the University of North Carolina at Chapel Hill. I checked out UNC's library to see if they still had the papers.

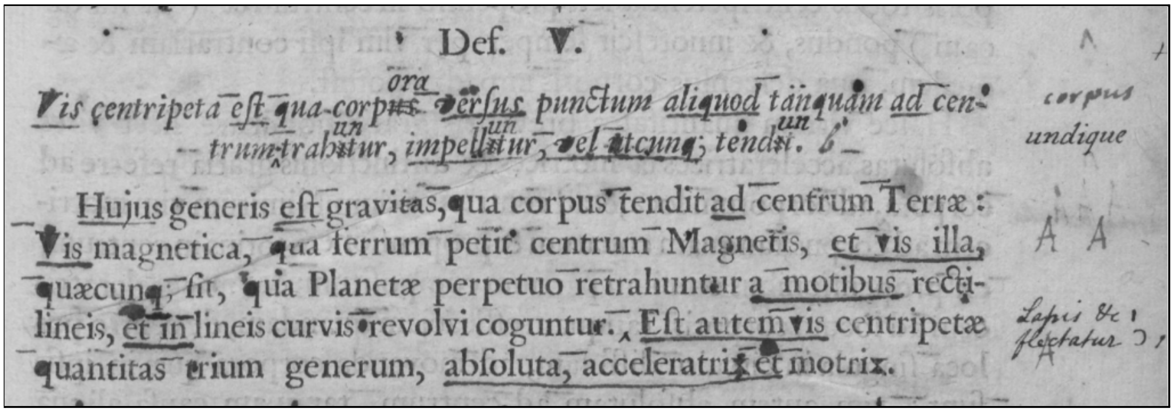
They did, and they had a *lot* of them.

I reached out to my friends in W&M's Special Collections to express interest in taking a peek at this collection. I offered to drive down, but graciously, UNC digitized their documents for us. They quoted us a four-six week timeline, which I was thrilled with. But what was I even looking for? Ideally, I would find a letter that contained an explicit reference to the *Principia*, its annotations, and a detailed history of where the text was before Savage had possession of it. I knew, of course, that this was an unrealistic goal, but even in the worst-case scenario, I would be able to compare handwriting samples.

While I was waiting for the text to be digitized, I pivoted from attempting to identify our annotators to looking at the content of the marginalia itself. Jackson had already had the idea that the annotations were revisions from the third edition of the *Principia* back into the first, so that's where I started. I began to methodically catalog the annotations and concluded with three "categories" of content.

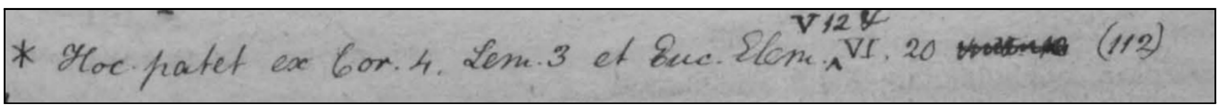
1. **Corrections:** Unoriginal content. Updating our copy with the revisions from the 3rd edition. I also found examples of content paraphrasing the Jesuit edition, a later expansion on Newton's work that offers more details regarding his proofs.
2. **Notes:** Original content of the annotator's made while digesting the *Principia,* explaining or commenting on what he is reading. This category also includes the connections made between the text and external references.
3. **Doodles:** Includes scribbles, cross-hatching, underlines, simple mathematics, absentminded drawings, etc.

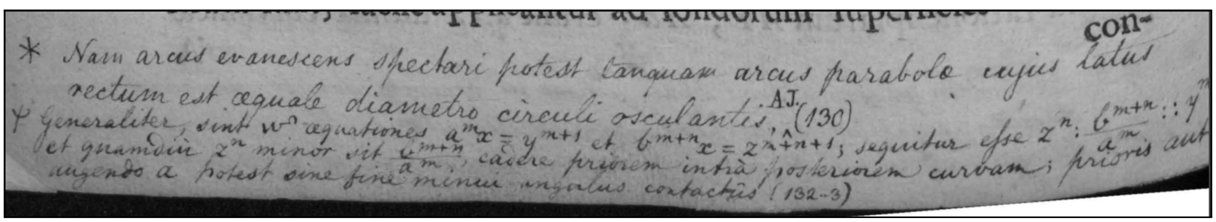
The corrections were easy to spot. I had a copy of the third edition in Latin and was able to directly compare the differences between them. For example, in Definition V., pictured below, *corpus* has been edited into *corpora* and *tendit* to *tendunt*, reflecting the slight changes in sentence structure from the first edition to the third. I found that the majority, but not all, of the changes between the first and third editions were recorded in W&M's copy. Most of the time, they were copied verbatim. Sometimes they were paraphrased, but a few clauses would be missing. Rarely, a passage would be missing that was so long that the annotator did not even attempt to copy it all down. In the first section of Book 1, an extended passage was added to the fifth definition. Instead of writing it word-for-word, the annotator noted the first and last word of the additions, seen in the bottom right corner, *Lapis…flectatur*.


Def. V.
Vis centripeta est qua corpus versus punctum aliquod tanquam ad centrum trahitur, impellitur, vel utcunque tendit.
Hujus generis est gravitas, qua corpus tendit ad centrum Terræ: Vis magnetica, qua ferrum petit centrum Magnetis, et vis illa, quæcunque sit, qua Planetæ perpetuo retrahuntur a motibus rectilineis, et in lineis curvis revolvi coguntur. Est autem vis centripetæ quantitas trium generum, absoluta, acceleratrix et motrix.

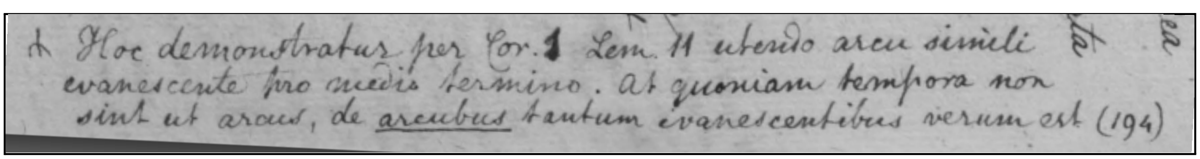
Around Def. V., we also see underlining and several instances of q's and o's being bubbled in. These, along with the ^, +,—, and A's written in the right margin, fall into category 3, evidence of The Doodler.

Category 2 is where things get interesting. Dotted throughout the *Principia* are lines of handwritten text ending with a numerical reference, always in parentheses. There are


* Hoc patet ex Cor. 4. Lem. 3 et Euc. Elem. VI. 20 (112)


* Nam arcus evanescens spectari potest tanquam arcus parabolæ cujus latus rectum est æquale diametro circuli osculantis, A.J. (130)

dozens of annotations just like this, some alluding to what text they are citing from, like A.J.(130) or Euc. *Elem*[35] (112), but some with only a number. Their content seems to be paraphrasing an external source and connecting it with the text of *Principia*. For


Hoc demonstratur per Cor. 1 Lem. 11 utendo arcu simili evanescente pro mediis termino. At quoniam tempora non sint ut arcus, de arcubus tantum evanescentibus verum est (194)

example, this annotation reads "This was demonstrated by Cor. 1. Lem. 11 used with a similar arc having disappeared by the middle boundary. But because there are no conditions as an arc, it is certainly so much concerning disappearing arcs (194)." Most of the references start with something along the lines of *hoc demonstratur per*, "this is

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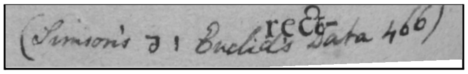
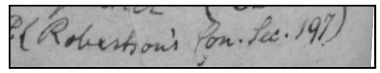
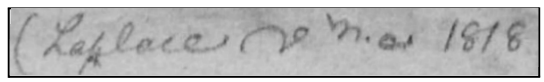
[35] Almost certainly referring to Euclid's *Elements*.

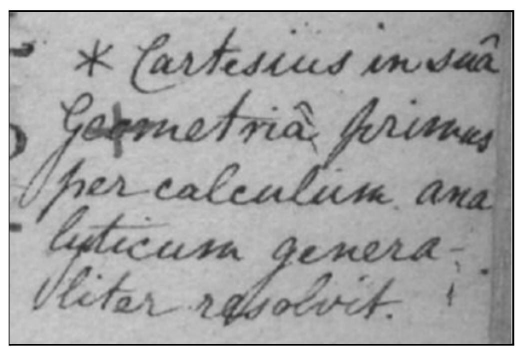
demonstrated by ___”. This content is important because we can date our author based on when the referenced text was written. While the original note to Euclid’s *Elements* isn’t much help as it came out in 300 BCE, this reference to Simson, Euclid, and *Data* point to Scottish mathematician Robert Simson. Simson published the second edition of his critique of *Elements,* in which he tacked on a couple of books from Euclid’s *Data* in 1762. Furthermore, (Robertson’s Con. Sec. 197) is almost certainly in reference to Abram Robertson’s *A Geometrical Treatise of Conic Sections*, written in 1802. On page 197 of this text, we find a demonstration of Proposition 10 from Book 1, where the conclusion is (as it should be): “the centripetal force is reciprocally as $\frac{1}{CB}$, or directly as the distance CB.” 1802 is two years before Thomas Staughton Savage was born, so our timeline is still intact. The most contemporary reference is to Laplace, 1818, written in pencil here.

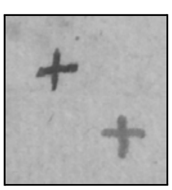






Besides referencing other sources, the reader shows his engagement with the text through little notes. Here, our annotator writes “Descarte in his *Geometria* first solves generally through analytic calculation.” This is in reference to a passage in the first edition that Newton cuts out of the following ones: “and so we have in this Corollary a solution of that famous Problem of the ancients concerning four lines, begun by *Euclid*, and carried on by *Appollonius*; and this not an analytical calculus but a geometrical composition, such as the ancients required.” This edit shows us that our annotator is well educated in mathematics and is familiar with the classic texts of the time. You might also note the presence of a second pen on the ‘o’ of *Geometria*. Plus signs are a favorite scribble of The Doodler, but it seems our annotator held little regard for avoiding them. Further study might reveal which pen wrote first or if there even was a different pen. John Loud did not patent the first ballpoint pen until 1888, decades after the *Principia* was safely housed in



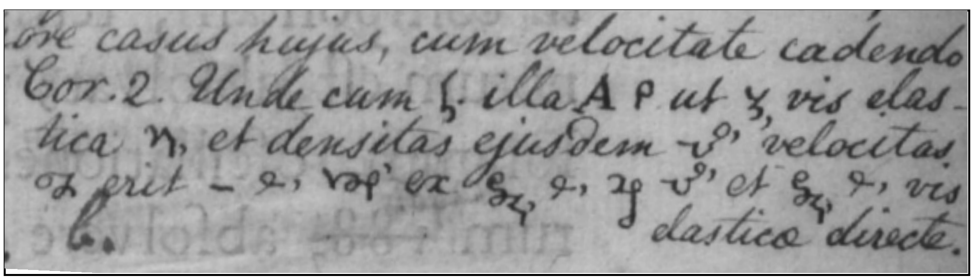


W&M's Library. Thus, both our annotator and The Doodler were likely writing with refillable ink — this might explain the differences in opacity between some of the annotations.

The most identifying characteristic of our annotator, though, is not his pen, nor his handwriting, nor his notes. *It's his secret code.*

Disguised among the Latin, our annotator concealed his most secret thoughts in complex symbols. At first, they might appear to be stray lines, a comma, or maybe even a Greek letter. They are discrete, designed for you to skip over them. In fact, they've already appeared in three different images in this chapter — did you catch them[36]?

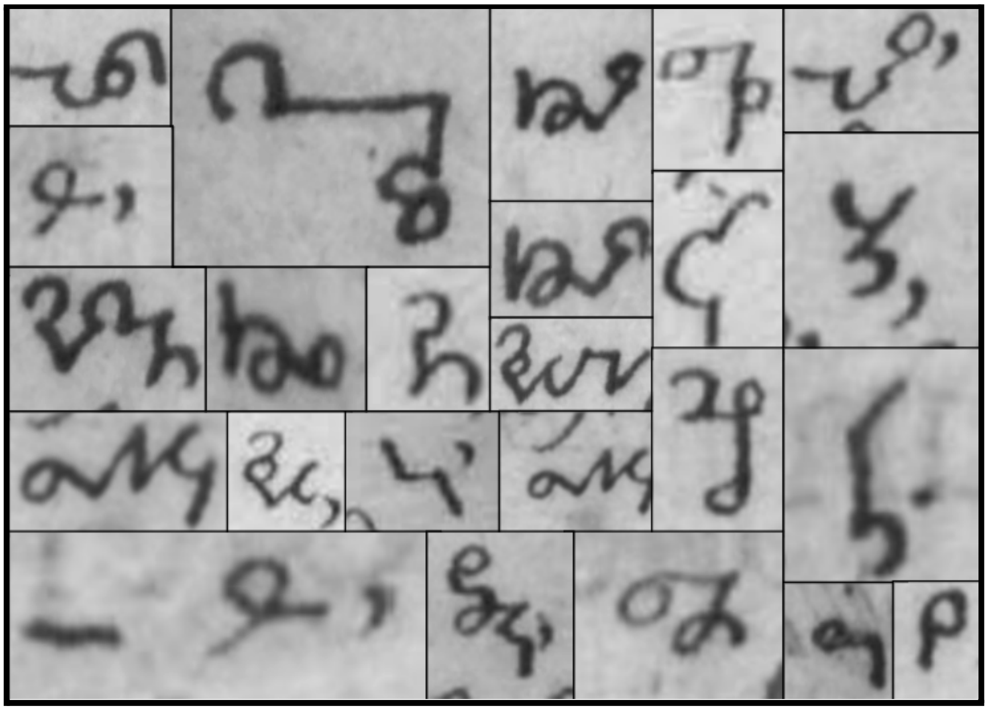
If not, don't worry. I dismissed them as planetary symbols for months while I worked on identifying my annotator. But once I saw them, I saw them *everywhere*.



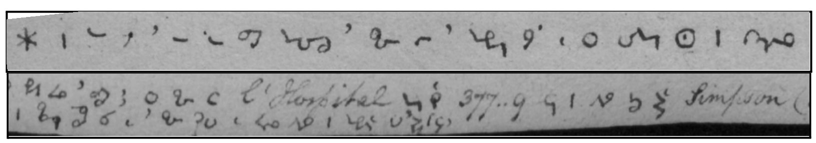
Take, for example, the code in this passage. If there weren't eleven symbols nearly in a row, I might not have noticed they were even there. They are inconspicuous and unassuming, the perfect recipe for clandestine affairs.

I started my hunt. Once again, I returned to the text, poring over the annotations in search of clues. I found hundreds of samples and began to catalog them based on size, orientation, and surrounding text.

[36] See pages 40 and 41.



There were even instances where entire annotations were written in code. With time and analysis, I started to see patterns. I noticed some repeating elements like circles, horizontal lines, vertical lines, hooks, and 'C' shapes. To rule out any existing languages, I utilized Detexify[37], a free online symbol recognition tool. If you draw a symbol, Detexify will identify it using shape matching and spit out the command to type it into LaTeX. I got no results. I tried the same thing for Unicode using ShapeCatcher[38] & didn't get anything there, either, despite the database of nearly twelve *thousand* unique glyphs. I thus felt confident that I was working with a new alphabet, a shorthand of sorts, rather than an existing one. Stuck, my advisor and I reached back out to Jack Martin to pick his brain about paths forward and see if he was familiar with any of our symbols.



While he didn't recognize our symbols, Jack thought we were on the right track with the shorthand idea. He believed that each arc, circle, line, and dash was either a letter or a

[37] Created by Philipp Kühl and Daniel Kirsch; can be found at https://detexify.kirelabs.org/classify.html
[38] Created by Benjamin Milde; can be found at http://shapecatcher.com/

letter sound, also known as a *phoneme*. We knew that a portion of the annotations were perfect reflections of the third edition back into the first. Jack recommended that I identify that category of entries and chart each symbol along with the corresponding Latin in the *Principia*. He also pointed us to some sources on seventeenth and eighteenth-century British shorthand. With this in mind, I compiled a table of nearly a hundred instances of code.

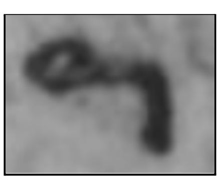
The game plan was to create a neural network, feed it our shorthand-Latin pairings and an analysis of the average frequency of each letter in the entire Latin written language, and have the code pop out our solved alphabet. Easier said than done, but presumably doable. W&M students can request access to the supercomputer and I figured that someone in the Computer Science department could teach me how to code AI.

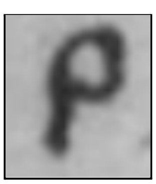
Luckily for me, I accidentally cracked the secret code by hand and was able to avoid that can of worms entirely.

I have found myself on more than one occasion knee-deep in binders of *Principia*, sticky notes, Latin textbooks, and tables of secret code, sitting cross-armed on my couch, glaring at the pages of a book that will not reveal its secrets to me. "I am more stubborn than you," I tell it, "and we're going to have to work together *whether you like it or not*."

She finally yielded at the end of September 2021.

In my table of symbols, I found two entries corresponding with different forms of the same Latin word, *sum*, "to be". In English, our "to be" — is, am, are, was — is used the



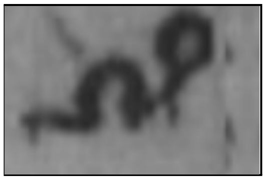


most common verb and the second, behind *the*, most frequently used word in the language[39]. *Sunt* (left), "they are" and *sit* (right), "he would be," look very similar. Our secret code is an *abjad* alphabet: a writing system in which only the consonants are notated, while the vowels are left for the reader to infer. Thus, the only difference between *s[u]nt* and *s[i]t* is the letter "n". The only difference in the

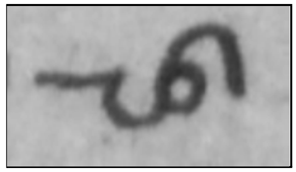
[39] *The Oxford English Corpus*

symbols is the horizontal line in *s[u]nt*. If “n” was represented by “—”, that would leave “s” to be “o” and “t” to be “|”.

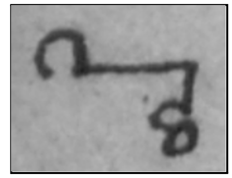
numeris



invicem



manentibus

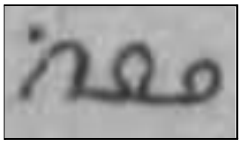


Using “s”, “n”, and “t”, I plugged these letters into other symbols and deduced that “m” is “⌒”, “r” is “/”, “b” is “(”, etc. From there, I found more words with these letters and was able to fill out the complete alphabet, which is as follows:

a	’ *above*	**h**	r	**o**	· *above*	**v**	◡
b	(	**i**	, *below*	**p**	σ	**w**	‘ *below*
c	\, o *(s sound)*	**j**	q *(g sound)*	**q**	‘ *above*	**x**	\o
d	)	**k**	\	**r**	/	**y**	q (j sound)
e	’ *in line*	**l**	/	**s**	o	**z**	o
f	◡	**m**	⌒	**t**	|		
g	q	**n**	—	**u**	· *in line*		

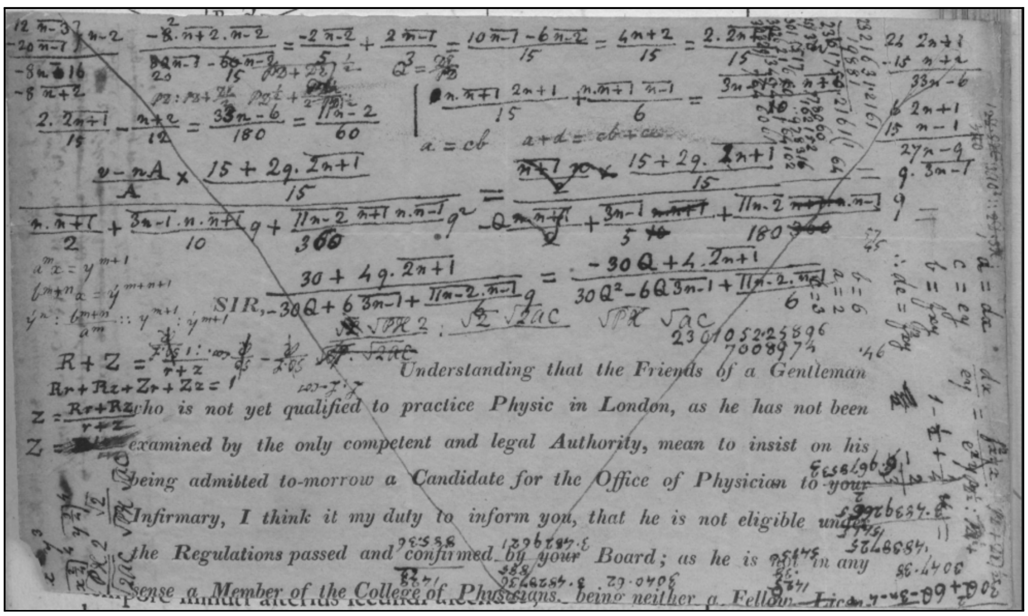
Some brief oddities to note: the vowels are only included if they are the first or last letter of a word and the Latin vowel has a macron[40]. A few of the letters — namely c/s, c/k, y/j, s/z — are conflated based on sound. That is, the first “c” in *circle* would be spelled with a “O” in shorthand and the “ph” in *philosophy* would be spelled with a “◡”. Additionally, the “ch” sound is denoted with a ᒣ. The same symbol is used for “r” / “l” and “f” / “v” — this can make translation difficult sometimes. Common Latin prefixes like *circum, magna, inter*, *hyper*, and *super* are abbreviated by the corresponding shorthand for the prefix’s first letter. The letter “w” is not present in Latin; “w” was written in shorthand only once in the marginalia.

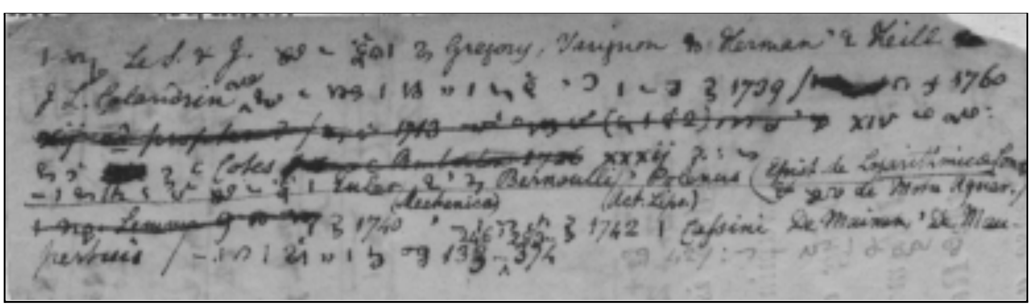
[40] Namely the ablative case.



Finally, I found a handful of times where the symbols for “s” and “t” were swapped incorrectly. For example, this glyph should read *ordinationis,* but actually spells *ord[i]n[a]s[io]n[i]s*.

With the code cracked, I could finally turn back to identifying my annotator. In Book 2, I discovered a slip of paper tucked into the text, pictured below. The front, pictured first, is covered with polynomials and general mathematical notes, likely related to some proof in the *Principia* while the back, pictured second, contains some English (!) writing and some secret code, undoubtably written by our annotator as the handwriting is identical.





The first line of text on the back of the insert references LeS[eur] and J[acquier], authors of the Jesuit Edition. In the *Monitum* of the first book (1739), the two authors acknowledge David*[41] Gregory, Varignon, Jakob Hermann, and John* Krill and in the

[41] The names David and John are written in code.

monitum of the second book (1740), they refer to Euler's *Mechanica*.[42] These names and dates are stated on the paper, making our secret code a bilingual alphabet.

I haven't been able to glean much meaning from the scribbled math on the front, but the printed content of the letter, which reads as follows, is of interest to me:

> *SIR,*
> *Understanding that the Friends of a Gentleman who is not yet qualified to practice Physic in London, as he has not been examined by the only competent and legal Authority, mean to insist on his being admitted to-morrow a Candidate for the Office of Physician to your Infirmary, I think it is my duty to inform you, that he is not eligible under the Regulations passed and confirmed by your Board; as he is not in any sense a Member of the College of Physicians, being neither a Fellow, Lice[ntiate]*[43]*—*

The Royal College of Physicians (RCP) is a British organization, headquartered in London, dedicated to improving the practice of medicine. It is the oldest medical college in England, founded by royal charter in 1518. The main role of the RCP in the seventeenth and eighteenth centuries was accrediting physicians via an oral examination. In order to be admitted, applicants must prove they were "classically educated", i.e. graduated from the Universities of Cambridge or Oxford, and "groundedly learned" in a myriad of subjects.[44] In 1767, members of the RCP began to raise complaints concerning the fellowship's refusal to admit candidates from non-Oxbridge institutions. The internal dispute lasted until 1835, when physicians educated from all universities finally became eligible for fellowship.

Thomas Savage worked in medicine for years and writes in 1833 to his friend Dr. Charles Osgood about his practice:

> *...Dr Holmes has gone to Hartford — 8th place would have been left vacant had I not succeeded him — he was [unintelligible] that I should come — and in company with another gentlemen of the*

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[42] Guicciardini Niccolò. *Reading the Principia the Debate on Newton's Mathematical Methods for Natural Philosophy from 1687 to 1736*, Cambridge Univ. Press, Cambridge, 2003, p. 248.
[43] That is, a doctor who is licensed to practice and pays a fee to the RCP.
[44] "History of the Royal College of Physicians." *RCP London*, Royal College of Physicians, 24 Feb. 2022, http://www.rcplondon.ac.uk/about-us/who-we-are/history-royal-college-physicians.

> *place, waited upon me at Middletown for that purpose — my intention is to go to the [unintelligible] southwest section of U.S. — my wish is to get in with some old practitioner who is about retiring and wishes to act only as a consulting physician. Dr Miner has written to that effect to his friends in that direction...*[45]

and in 1834:

> *...and if you refer to my much esteemed friend James Johnson (1777-1845), M.D. of London, you will find that he agrees with me on this head...I will give you* ***in shorthand*** *what I have done within the last six months...* [46]

The front of the rejection letter in Book 2 of the *Principia* definitively links our annotator with the Royal College of Physicians. Savage's letter to Osgood in 1833, along with dozens of other sources, show that Thomas maintained a respectable career in medicine and was motivated to leave Middletown, CT to work for an older doctor. In Savage's later correspondence in 1834, he explicitly references writing to Osgood in shorthand and mentions his colleague James Johnson. Johnson, a doctor 27 years older than Savage, was admitted a Licentiate of the RCP in 1821 and practiced in London until his death in 1845.[47]

Savage is clearly a well-educated physician with demonstrated interest in fields ranging from zoology to chemistry. We know that the oral examination required for admittance into the RCP covered various scientific fields and that the annotator of the *Principia* was clearly an engaged student. All of this evidence points to Thomas Savage. I believe that Savage was studying for the test to be admitted to the RCP, but as a graduate from Yale Medical School, he was not eligible for fellowship. He might have been writing in shorthand to conceal his notes, as entrance into the RCP was incredibly competitive.

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[45] Dr. Charles Osgood papers, RHC-185. Grand Valley State University Special Collections and University Archives.
[46] *ibid*
[47] Munk, William. "James Johnson." *James Johnson | RCP Museum*, Royal College of Physicians, https://history.rcplondon.ac.uk/inspiring-physicians/james-johnson.

But, of course, all of this evidence is circumstantial. There's a lot of it, and it paints a convincing picture, but I cannot definitively prove anything. The explicit link that I would love to find — a document written by Thomas Staughton Savage that references the *Principia* and the notes he made within it — almost certainly does not exist. If it did, I would have found it. I have investigated every letter penned by Thomas Staughton Savage's hand that exists in modern record. I have read his vacation letters, his accounts on neighborhood gossip, his treatment plans for the Cholera epidemic, and a ranking of his favorite cities in the southern United States[48]. If it was penned by his hand, you will find it in my files. I have combed the branches of his family tree and shaken every apple loose. I have investigated the Dioceses he worked for, his property records, the schools he founded, and the ministries he led. I have scoured both sides of the Atlantic, nine states, and D.C., accumulating newspaper clippings, correspondences, and hundreds of writing samples. I am confident in saying that the circumstantial evidence on which I base my theory is complete, well researched, and so far, the only theory which fits the facts of the *Principia*. In the future, should new evidence come to light, as all science does, I will reevaluate and possibly adjust my hypothesis.

And yet, even after all of this, the *Principia* contains more secrets — there are still further areas of study that might turn up new clues. I have not yet identified The Doodler, nor researched all of the references mentioned in the annotations. I am not done cataloging the annotations, and I haven't investigated all of the markings. For example, at the end of a few dozen definitions, one of my annotators writes a single letter. I have seen references to the attributed letters in some annotations, but they don't make sense to me. Finally, if there is a way to do so non-destructively, I would love to explore the ink in the annotations. The differences in opacity and shade might be from different pens, which might give us more information about how many annotators we are working with. Of course, one annotator might use several pens, but I think the line of investigation could be worthwhile.

[48] Just in case you were wondering, Mobile, AL is his favorite and New Orleans, LA is his least favorite.

In this thesis, I placed the *Principia* in its proper historical context so that we could analyze its contents separated from the reverence associated with its author. I expanded upon Newton's proof discussing his experiment on resistive motion. I was able to offer more detail and make Newton's lines of logic more explicit. I cataloged the vast majority of the annotations in William & Mary's first edition copy of the text and sorted them by content: corrections, notes, and doodles. I discovered and cracked the shorthand in the margins and attributed Thomas Staughton Savage as their author. I investigated the donor of William & Mary's copy by contacting his descendants, collecting his records, finding new letters, corresponding with museums and libraries around the world, and pilfered through the archival documents of churches he founded. I narrowed down the possible time range of annotation to a 51 year period and compiled new information about the history of the document.

At the highest level, this thesis discovered new information about the discovery and evolution of classical mechanics. I found new knowledge by looking *backwards* in time rather than forwards. I am unspeakably grateful to have participated in this project and I hope that this work will be foundational for future scholars to explore the inspiring intersection of Physics and Classical Studies.

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21. Vergil, *The Aeneid*

22. Westfall (1971), opp. contents page.

23. Westfall (1980), p. 40.