Dante’s Waterfall: A Hybrid Model of Inflation

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Abstract:

Given recent observational results from the BICEP2 and Planck collaborations regarding the ratio of power in tensor to scalar modes in the cosmic microwave background radiation, and the implications of this ratio for the possible existence of large gravity waves resulting from inflation, it is of current interest to investigate models of inflation which are capable of producing large gravity waves. We examine a hybrid axion monodromy model we call “Dante’s Waterfall”. In this model, the inflaton rolls slowly along a trench in the potential, and inflation ends when this trench becomes unstable and the inflaton rapidly approaches the global minimum of the potential. We find that this model is incapable of producing large gravity waves. We then introduce a vector interpretation of inflaton motion along a potential, and show that this can be useful in considering non-trivial kinetic terms which could give rise to large gravity waves in a hybrid model.

I. Introduction:

From the first half of the twentieth century, the idea of a Big Bang as the origin of the observable universe has been a cornerstone of cosmology. The basic framework of the original Big Bang Theory, however, left many questions unanswered. Among these were the Flatness Problem and the Horizon Problem. The Flatness Problem is concerned with why the matter and energy density of the universe is so close to the critical density necessary for a flat spacetime. This requires such fine tuning that small deviations from the necessary densities would have led to significant differences in the geometry of space-time. The Horizon Problem asks why the entirety of the observable universe is so homogeneous in terms of its energy density. Homogeneity usually arises from different elements in a system having had a chance to interact and rethermalize to an equilibrium, but the observable universe is too large for such interactions to have occurred during its lifetime.

In 1980, Alan Guth offered a solution to these problems which he called Inflation. [1] Guth proposed that shortly after the Big Bang, the universe underwent rapid, exponential expansion for a miniscule fraction of a second. This expansion would help explain the problematic observations noted in the Flatness and Horizon Problems. With regard to the Horizon Problem, if the universe had at one point been contained in a very small space, the rapid expansion of the universe would have magnified small homogeneous regions to beyond the current observable horizon, leading to the homogeneity we see today. Inflation can address the flatness problem as well, since the magnification would cause any curvature in the early universe to approach being flat after the end of inflation.

Guth's theory has been modified since it was initially proposed, and inflation has been a topic of recent discussion as large experimental collaborations such as BICEP2 and Planck have tried to search for empirical evidence for inflation. The evidence these groups seek comes from the Cosmic Microwave Background radiation (CMB), very low temperature (~3 K) thermal radiation which is left over from the Big Bang. This radiation is polarized, and the polarization is sorted into two modes, called E and B modes in analogy to electromagnetism. E modes are scalar, and have no curl, while B modes are tensor modes and have no divergence. One cosmological observable is the ratio of power in tensor to scalar modes, denoted generally by r, but in this paper by $\underline{r}$ to avoid confusion with other variables since we will be using r to refer to a particular scalar field in our model. A sufficiently large ratio $\underline{r}$ is an indication of the presence of large gravity waves after the big bang, which would have been caused by an inflationary period. Therefore, the observation of large $\underline{r}$ values would support the theory of inflation.

In the spring of 2014, the BICEP2 collaboration claimed to have observed $\underline{r} = 0.2$, which is considered large. [2] However, this result was later retracted when it was realized that the data was not sufficiently corrected for the presence of cosmic dust. Later that same year, a collaboration between the BICEP2 and Planck teams released an upper bound on $\underline{r}$, saying $\underline{r} < 0.1$. [3] Although no definite value for $\underline{r}$ has been released yet, the question of just how large these gravity waves should be is a relevant one right now, and any discovery will help constrain the theories that seek to explain inflation.

Given the power of inflationary theory to explain outstanding questions in cosmology and the prospect of finding evidence for the phenomenon, it is of contemporary interest to investigate the theoretical mechanisms by which inflation might happen. In general, inflation is thought to be driven by a scalar field, called the inflaton field, moving towards the minimum of an associated potential. Inflation occurs while the field is in "slow roll". This means that the slope of the potential is shallow, and the field is slowly changing its potential energy. Since the field is staying at a relatively high potential during this period, it continues to drive inflation. Inflation ends with the end of slow roll, when the field rapidly approaches the minimum of the potential. [4]

Different models for inflation can use different potentials, which will lead to different predictions regarding cosmological observables. Various cosmological observables can be written in terms of three slow roll parameters, which depend on derivatives of a canonically normalized potential with respect to the inflaton direction as follows:

$$\epsilon = \frac{{M_P}^2}{16\pi}\left(\frac{V'}{V}\right)^2$$

$$\eta = \frac{{M_P}^2}{8\pi}\frac{V''}{V}$$

$$\gamma = \frac{{M_P}^4}{64\pi^2}\frac{V'V'''}{V^2}$$

Here, V is the potential, and M_P is the Planck mass. V depends on an inflaton scalar field, and all derivatives are taken with respect to that field.

Given the recent discussion of the possibility of large gravity waves having been produced during inflation, we focus on models which can accommodate large gravity waves. The production of large gravity waves requires a long period of slow roll, a restriction which causes many classes of models to violate what is called the Lyth bound. In order for the early universe to have scalar perturbations which time evolve into the scalar spectral index we observe today, the universe would have needed to expand by between 50 to 60 e-folds (powers of e) during inflation. This would cause inflaton field values to become comparable to the Planck mass, which would cause the effective field theory governing the model to break down. [5] Therefore, we need a potential which allows for a long slow roll period without the inflaton field reaching high values. One class of models which fulfills these requirements is that of axion monodromy models. Axion monodromy models incorporate a shift symmetry when constructing the dependence of their potentials on the fields involved. This periodicity allows the inflaton field to undergo slow roll for an extended period of time without reaching very high field values. In this paper, we examine an axion monodromy model we call “Dante’s Waterfall” and find that it does not allow for very large

$\underline{r}$ values. We then include a brief discussion of how to treat Lagrangians for models with non-trivial kinetic terms, which can potentially help models achieve large $\underline{r}$ values.

II. Dante's Waterfall:

From Dante's Inferno to Dante's Waterfall:

One proposed axion monodromy model for inflation from the existing literature is called Dante's Inferno. [6] This model includes two fields, called r and θ. The potential is a function of both of these fields, and is expressed as follows:

$$V(r,\theta) = \frac{1}{2}m^2r^2 + \Lambda^4\left[1 - \cos\left(\frac{r}{f_r} - \frac{\theta}{f_\theta}\right)\right]$$

Here, Λ, m, f_r, and f_θ are constants. The quadratic term, which depends only on r and is not periodic, explicitly breaks the shift symmetry in r. Thus, for a given θ, the potential has a parabolic cross-section modulated by a cosine function. The periodicity of 2π in θ is preserved.

If we create a plot of this potential in cylindrical coordinates, which highlights the periodicity in θ, we can see that the cosine modulations create a trench which spirals towards the global minimum of the potential, as shown in Fig. 1. The bottom of the trench is a local minimum in the potential with respect to the direction perpendicular to the trench. This means that if the inflaton were to start inside the trench at a point higher on the potential, it would roll along the trench until it reached the potential's global minimum.

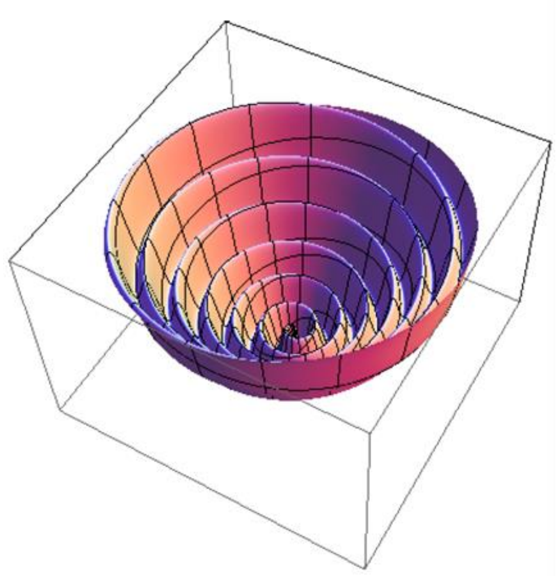

Figure 1: The Dante's Inferno potential, wrapped around 2π to emphasize periodicity. r is in the radial direction, and θ is perpendicular to r.

Based on the inflaton's expected trajectory in this model, it seems reasonable to work in coordinates where one field is oriented parallel to the trench, and the other is perpendicular. We can do this by applying the rotation:

$$\begin{pmatrix} \tilde{r} \\ \tilde{\theta} \end{pmatrix} = \begin{bmatrix} \cos\xi & \sin\xi \\ -\sin\xi & \cos\xi \end{bmatrix} \begin{pmatrix} r \\ \theta \end{pmatrix}$$

We can choose the rotation angle ξ such that the argument of the cosine in the potential depends only on $\tilde{r}$. Abbreviating $\sin\xi$ as s and $\cos\xi$ as c, we choose:

$$\mathrm{s} = \sin\xi \equiv \frac{f_r}{\sqrt{{f_r}^2 + {f_\theta}^2}}$$

$$\mathrm{c} = \cos\xi \equiv \frac{f_\theta}{\sqrt{{f_r}^2 + {f_\theta}^2}}$$

$$f \equiv \frac{f_r f_\theta}{\sqrt{{f_r}^2 + {f_\theta}^2}}$$

Then, the Dante's Inferno potential can be rewritten as:

$$V(\tilde{r}, \tilde{\theta}) = \frac{1}{2} m^2 (\tilde{r}c + \tilde{\theta}s)^2 + \Lambda^4 \left[1 - \cos\left(\frac{\tilde{r}}{f}\right)\right]$$

Now, the cosine modulation in the potential is only in the $\tilde{r}$ direction, and the bottom of the trench is a local minimum in $\tilde{r}$. Therefore, the $\tilde{\theta}$ direction is along the trench. In this coordinate system, the inflaton field will move only in the $\tilde{\theta}$ direction. The original Dante's Inferno paper shows that if we assume $f_r \ll f_\theta \ll M_P$ and $\Lambda^4 \gg f m r_{initial}$, this potential can be approximated as an effective single field potential which depends only on $\tilde{\theta}$. The method for reaching this approximation will be shown later, as we use a similar process to obtain an effective single field theory for our Dante's Waterfall potential. In the Dante's Inferno scenario, then, inflation ends when the trench becomes steep enough to prevent a slow rolling of the inflation field.

Although the Dante's Inferno model started out as a hybrid model depending on two fields r and θ, it was ultimately able to be reduced to an effective single-field theory once we rotated the fields to align one field along the trench. One simple change to the Inferno potential, however, can yield a dramatically different model of inflation. By making the quadratic symmetry breaking term negative instead of positive, we obtain a potential which resembles a concave down hill instead of a concave up bowl, still modulated by a cosine function. [7] Let us replace the symmetry breaking term $\frac{1}{2} m^2 r^2$ with the quartic equation $-\frac{1}{2} m^2 r^2 + \frac{\lambda}{4!} r^4 + \frac{3}{2} \frac{m^4}{\lambda}$. This yields the overall potential:

$$V(r, \theta) = -\frac{1}{2} m^2 r^2 + \frac{\lambda}{4!} r^4 + \frac{3}{2} \frac{m^4}{\lambda} + \Lambda^4 \left[1 - \cos\left(\frac{r}{f_r} - \frac{\theta}{f_\theta}\right)\right]$$

Here, m and Λ are constants with dimensions of mass, f_r and f_θ are constants with dimensions of inverse mass, and λ is a dimensionless constant. This potential, like Dante's Inferno, has a trench along which the inflaton field can slowly roll during inflation. Again like Dante's Inferno, we can rotate the coordinate system so that we have one field aligned with the trench and the other perpendicular.

$$V(r,\theta) = -\frac{1}{2}m^2(\tilde{r}c + \tilde{\theta}s)^2 + \frac{\lambda}{4!}(\tilde{r}c + \tilde{\theta}s)^4 + \frac{3}{2}\frac{m^4}{\lambda} + \Lambda^4\left[1 - \cos\left(\frac{\tilde{r}}{f}\right)\right]$$

Unlike Dante's Inferno, however, the fact that the quadratic term is negative causes the trench to eventually cease to be concave up, and become unstable lower on the potential. This is shown in Fig. 2. The mathematical condition for this instability is:

$$\frac{\partial^2 V}{\partial \tilde{r}^2} = 0$$

This point along the trench marks the end of inflation, since the inflaton will no longer follow the trench, but will instead decline rapidly to the global minimum of the potential. The quartic term in r assures that there will be a global minimum, and the added constant term causes the potential to vanish at that minimum. This is a very different end to inflation from the Dante's Inferno model. In the Inferno, the entire process of inflation, from the beginning through the end, depended on effectively a single field. With this modification, we obtain a truly hybrid model. The inflaton field travels along the trench during inflation, but travels perpendicular to the trench after the trench becomes unstable and ends inflation. As with Dante's Inferno, $\tilde{\theta}$ is oriented along the trench and $\tilde{r}$ is oriented perpendicular to the trench. We can refer to $\tilde{\theta}$ as the inflaton field, and $\tilde{r}$ as the waterfall field. Hence, we refer to this model as "Dante's Waterfall".

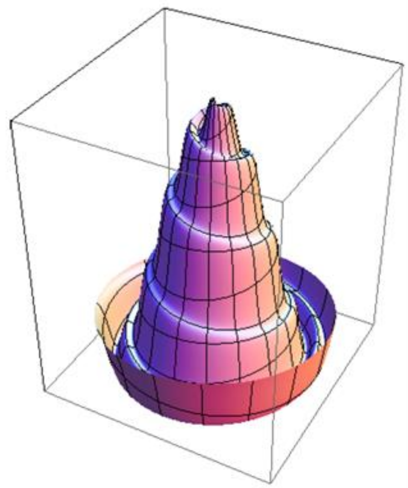

Figure 2: The Dante's Waterfall potential, wrapped around 2π to emphasize periodicity. r is in the radial direction, and θ is perpendicular to r.

Effective Single Field Theory:

Considering the potential we have constructed, we have a qualitative understanding of how inflation should proceed in this model. The inflaton field should roll along the trench in the potential for the duration of the inflationary period, and inflation should end when the trench becomes unstable. From the start of inflation until this instability occurs, we should therefore be able to approximate our model with an effective potential which depends on a single field oriented along the trench. Let us now construct this single-field effective theory, and use it to assess some of the constraints on our model.

The trench is the trajectory along the potential for which $\frac{\partial V}{\partial \tilde{r}} = 0$. Taking this derivative of our potential in terms of the rotated fields $\tilde{r}$ and $\tilde{\theta}$ and setting it equal to 0, we get the trajectory:

$$-m^2 cr + \frac{\lambda}{6} cr^3 + \frac{\Lambda^4}{f} \sin\left(\frac{\tilde{r}}{f}\right) = 0$$

Here, we condense linear combinations of $\tilde{r}$ and $\tilde{\theta}$ and simply write r outside of the sine function in the interest of simpler notation. Now, recall that when we rotated the original fields r and θ to get $\tilde{r}$ and $\tilde{\theta}$, we chose $\tilde{r}$ and $\tilde{\theta}$ such that $\tilde{\theta}$ was roughly parallel to the trench and $\tilde{r}$ was roughly perpendicular. Therefore, we should be able to find a regime along this trajectory where $\tilde{r}$ can be written as a linear function of $\tilde{\theta}$, and the field theory can be written as a function of only $\tilde{\theta}$.

We can rewrite the equation defining the trench to isolate $\sin\left(\frac{\tilde{r}}{f}\right)$. This gives us:

$$\sin\left(\frac{\tilde{r}}{f}\right) = \frac{cf\left(m^2 r - \frac{\lambda}{6} r^3\right)}{\Lambda^4}$$

If we consider the regime where $\frac{\left|cf\left(m^2 r - \frac{\lambda}{6} r^3\right)\right|}{\Lambda^4} \ll 1$, we can approximate $\sin\left(\frac{\tilde{r}}{f}\right) \approx \frac{\tilde{r}}{f}$. Once we expand r back into the appropriate linear combination of $\tilde{r}$ and $\tilde{\theta}$, this allows us to construct a linear relationship between $\tilde{r}$ and $\tilde{\theta}$.

$$\tilde{r} = \left[\frac{f^2 m^2 sc}{\Lambda^4 - m^2 f^2 c^2}\right]\tilde{\theta} \approx \left(\frac{f^2 m^2 sc}{\Lambda^4}\right)\tilde{\theta}$$

We can now reconsider the Lagrangian for our model. Using this linear relationship between $\tilde{r}$ and $\tilde{\theta}$, we can write the kinetic term in the Lagrangian as follows:

$$T \equiv \frac{1}{2}\dot{\tilde{r}}^2 + \frac{1}{2}\dot{\tilde{\theta}}^2 = \left[1 + \left(\frac{f^2 m^2 sc}{\Lambda^4}\right)^2\right]\frac{1}{2}\dot{\tilde{\theta}}^2$$

If we choose $\frac{f^2 m^2 sc}{\Lambda^4} \ll 1$, we can neglect one term on the right hand side and are left with a canonically normalized kinetic term in $\tilde{\theta}$. Now we can perform the same substitution in our expression for the potential, further simplifying terms by assuming $s \ll 1$ and $\frac{f^2 m^2}{\Lambda^4} \ll 1$. This gives us:

$$V_{eff} = -\frac{1}{2} {m_{eff}}^2 \tilde{\theta}^2 + V_0$$

Here,

$$m_{eff} = ms, \;\; V_0 = \frac{3}{2\lambda} m^4$$

We now have a canonically normalized Lagrangian which describes the inflaton field's motion along the trench during slow roll, and have identified $\tilde{\theta}$ as our inflaton field. This should allow us to calculate slow roll parameters, and show that there exist parameters for our model which yield the correct values for cosmic observables such as the scalar tilt (n_s), the scalar amplitude (${\Delta_R}^2$), and the number of e-folds by which the universe expanded over the course of inflation (N). The slow roll parameters, as mentioned earlier, depend on combinations of the canonically normalized potential and its derivatives with respect to the inflaton field. Substituting our effective potential into the equations for the three slow roll parameters ϵ, η, and γ, we get:

$$\epsilon = \frac{{M_P}^2}{4\pi} \frac{\tilde{\theta}^2}{\left(\frac{2V_0}{{m_{eff}}^2} - \tilde{\theta}^2\right)^2}$$

$$\eta = -\frac{{M_P}^2}{4\pi} \frac{1}{\left(\frac{2V_0}{{m_{eff}}^2} - \tilde{\theta}^2\right)}$$

$$\gamma = 0$$

We can express cosmological observables in terms of the slow roll parameters, evaluated at $\tilde{\theta}_i$, an initial value of $\tilde{\theta}$ which is spaced far enough from the instability in the potential which ends inflation to allow for 50 to 60 e-folds during inflation. These expressions for the scalar tilt and the scalar amplitude are the following:

$$n_s = [1 - 6\epsilon + 2\eta]_{\tilde{\theta}=\tilde{\theta}_i}$$

$$\Delta_R{}^2 = \left[\frac{8}{3{M_P}^4}\frac{V}{\epsilon}\right]_{\tilde{\theta}=\tilde{\theta}_i}$$

Substituting our expressions for the slow roll parameters and the effective potential, as well as known values for the cosmological observables ($n_s = 0.9603$, $\Delta_R{}^2 = 2.2 \times 10^{-9}$) [8], we find that these two equations can be simultaneously satisfied only if $m_{eff} < 8.31 \times 10^{-7}$. We express all observable and parameter values in natural units, where $M_P = 1$.

Once we constrain the equations for these observables by plugging in their known values, we have two equations with three variables. If we choose a valid m_{eff}, we determine V_0 and $\tilde{\theta}_i$. Recalling that we defined m_{eff} and V_0 for our effective theory in terms of parameters of the full theory, fixing values of these parameters allows us to start constraining parameter values in the full theory. We are now equipped to show that there exists a set of parameters of the full theory which is consistent with the effective single-field theory and the known values for cosmological observables.

Let us choose $m_{eff} = 1.2 \times 10^{-7}$. This yields $V_0 = 2.885 \times 10^{-14}$ and $\tilde{\theta}_i = 0.0838$. We can also choose $s = 0.0010$, thereby fixing m and λ. We then need to search for Λ and f which will allow us to attain between 50 and 60 e-folds before the trench becomes unstable. The following set of parameters for the full theory is consistent with the restrictions that result from the choices we made with regard to the effective theory.

$$s = 0.0010$$

$$\lambda = 1.078 \times 10^{-2}$$

$$\Lambda = 0.0001$$

$$m = 0.00012$$

$$f = 2.453 \times 10^{-5}$$

With these parameters, there is a trajectory along the trench in the potential which connects an initial point $(\tilde{r}, \tilde{\theta})_i = (8.099 \times 10^{-6}, 8.377 \times 10^{-2})$ to a final point $(\tilde{r}, \tilde{\theta})_f = (3.647 \times 10^{-5}, 2.485 \times 10^{-1})$ which satisfies the trench instability condition $\frac{\partial^2 V}{\partial \tilde{r}^2} = 0$. Recall that $\tilde{\theta}_i$ was fixed by our choice of m_{eff} in the effective theory to yield the correct values for n_s and ${\Delta_R}^2$, and that we found $\tilde{\theta}_f$ by following the trench up to the point where the instability occurred. We still need to check that this trajectory provides between 50 to 60 e-folds before the end of inflation. The number of e-folds, N, can also be expressed in terms of the potential and slow roll parameters.

$$N = \frac{2\sqrt{\pi}}{M_P} \int_{\tilde{\theta}_i}^{\tilde{\theta}_f} \frac{1}{\sqrt{\epsilon}} d\tilde{\theta}$$

Using our effective potential, we get the following expression for N:

$$N = \frac{4\pi}{{M_P}^2} \left[\frac{2V_0}{{m_{eff}}^2} \ln\left(\frac{\tilde{\theta}_f}{\tilde{\theta}_i}\right) - \frac{1}{2}\left({\tilde{\theta}_f}^2 - {\tilde{\theta}_i}^2\right) \right]$$

Our values of $\tilde{\theta}_i$ and $\tilde{\theta}_f$ give us $N = 54.4$, which is within the required bounds of 50 to 60 e-folds. Thus we show that, using approximations based on an effective single-field theory, we can find parameters that yield accepted values for cosmological observables. Later, we will use numerical methods to check that the approximations used here are fair ones to make, and to find parameters that better match known observable values.

A Bound on $\underline{r}$:

Before we leave the effective single-field theory, however, let us use it to find some bounds on $\underline{r}$, the ratio of power in tensor to scalar modes in the CMB. In terms of the slow roll parameters, we can express this as:

$$\underline{r} = [16\epsilon]_{\tilde{\theta}=\tilde{\theta}_i}$$

Based on the constraints we placed on the effective theory in order to ensure the correct values for n_s and ${\Delta_R}^2$, we can write $\underline{r}$ as a function of m_{eff}.

$$\underline{r}(m_{eff}) = \frac{2}{9\pi {C_0}^2}\left[C_1 \pm \sqrt{{C_1}^2 - 4C_0 {m_{eff}}^2}\right]$$

Here,

$$C_0 = \left[\frac{{\Delta_R}^2}{144\pi}\right]^{\frac{1}{3}}$$

$$C_1 = 6\pi {C_0}^2(1 - n_s)$$

If we plug in known values for n_s and ${\Delta_R}^2$, we can obtain a maximum value for $\underline{r}$ for each sign on the square root.

$$\underline{r}_{max+} = 0.107$$

$$\underline{r}_{max-} = 0.053$$

We can continue to narrow this down by requiring between 50 and 60 e-folds during inflation. Maximizing our expression for N with respect to $\tilde{\theta}_f$, we find that N_{max} corresponds to ${\tilde{\theta}_f}^2 = \frac{2V_0}{{m_{eff}}^2}$. This means that N_{max} is determined by a choice of m_{eff}, with the sign of m_{eff}

corresponding to the sign on the square root in the expression for $\underline{r}$. If we take the positive square root, N_{max} never exceeds 42.4. This is below our required minimum of 50, so solutions from the positive square root can be immediately eliminated. With the negative square root, we find that we cannot choose values of m_{eff} which simultaneously are large enough to yield $\underline{r} = 0.053$ and small enough to keep $N_{max} > 50$. Searching numerically for the threshold, we find that the imposition that $N_{max} > 50$ places the following bound on $\underline{r}$:

$$\underline{r} < 0.03$$

Of course, the validity of this bound depends on the effective theory being a reasonable approximation of the full theory. As mentioned before, we can test this numerically.

Numerical Analysis:

After having made several approximations and assumptions in order to arrive at an effective single field theory, it is worth using numerical methods to perform calculations using the full theory and see just how accurate our effective theory was. Substituting the same parameter set we presented when discussing the effective theory, we find the following observable values, where n_r is the tensor spectral index.

$$n_s = 0.956$$

$${\Delta_R}^2 = 1.833 \times 10^{-9}$$

$$\underline{r} = 6.70 \times 10^{-4}$$

$$n_r = -1.47 \times 10^{-5}$$

$$N = 49.44$$

Qualitatively, this agrees fairly well with the values these same parameters gave us using the effective theory, which means that the effective theory is a reasonable approximation of the full theory. From comparing the observable values obtained numerically using the full theory, and analytically using the effective theory, we see that the full theory yields slightly smaller values of N and $\underline{r}$ than the effective theory for the same parameters. This means that the upper bound of $\underline{r} < 0.03$ found using the effective theory should be valid for the full theory as well.

Using numerical tools to find solutions based on the full theory, we are able find parameters for the potential which yield more accurate values for known cosmological observables. One possible set of parameters is:

$$\frac{f}{s} = 0.1043$$

$$\frac{f}{c} = 3.127 \times 10^{-4}$$

$$m = 1.367 \times 10^{-4}$$

$$\lambda = 1.314 \times 10^{-3}$$

$$\Lambda = 3.654 \times 10^{-4}$$

$$\left(\tilde{r}_i, \tilde{\theta}_i\right) = (1.112 \times 10^{-4}, 0.322)$$

$$\left(\tilde{r}_f, \tilde{\theta}_f\right) = (4.738 \times 10^{-4}, 1.039)$$

This set of parameters gives us the following values for known cosmological observables:

$$n_s = 0.960$$

$$\Delta_R{}^2 = 2.23 \times 10^{-9}$$

$$\underline{r} = 7.45 \times 10^{-3}$$

$$n_r = -1.42 \times 10^{-4}$$

$$N = 59.7$$

Note that there is no particular motivation for choosing this specific set of parameters. We have merely shown that there exists a set of parameters which can yield the correct value for known cosmological observables.

III. Discussion:

We have shown that one can choose parameters such that the Dante's Waterfall model is consistent with established cosmological data. However, the model cannot fully accommodate one less-established observational result, namely the large upper bound for $\underline{r}$ ($\underline{r} < 0.1$) released by the recent collaboration between Planck and BICEP2. As discussed earlier, the Dante's Waterfall model requires that $\underline{r} < 0.03$. On one hand, there is no claimed measurement for $\underline{r}$ in the Planck and BICEP2 results, and therefore it is still possible that the actual value of $\underline{r}$ could fall below the maximum possible value we predict. On the other hand, the failure of this model to accommodate larger $\underline{r}$ values is striking, since the main motivation for turning to axion monodromy models is that they allow fields to reach higher energies (which can produce higher $\underline{r}$ values) without reaching field values that cause the effective field theory to break down. This begs the question, what factors determine whether or not an axion monodromy model is capable of producing large gravity waves, and therefore large $\underline{r}$?

One possibility is the introduction of non-trivial kinetic terms in the Lagrangian. One example of this is the Spiral Inflation model explored in [9]. The Spiral Inflation potential is identical in form to the potential in the Dante's Waterfall model. The difference between the two models lies in the interpretations of the two fields involved. The Spiral Inflation potential is written in terms of the fields ϕ and θ, which both originate from a single complex field Φ, defined as:

$$\Phi = \frac{\phi e^{i\theta}}{\sqrt{2}}$$

The kinetic term in the Lagrangian for Φ will be proportional to $\dot{\Phi}^2$, which is:

$$\dot{\Phi}^2 = \frac{1}{2}\left[\phi^2\dot{\theta}^2 + \dot{\phi}^2\right]$$

The factor of ϕ^2 multiplying the $\dot{\theta}^2$ shows that this potential is not canonically normalized in terms of ϕ and θ. The expressions for the slow roll parameters that we have been using depend on derivatives of a canonically normalized potential with respect to the inflaton direction. Since this potential is not canonically normalized, we cannot simply differentiate with respect to the direction of the trench as we could with the Dante's Waterfall model.

It would be convenient, both in replicating the results in this model and in generalizing to a broader class of models, to be able to take the necessary derivatives without needing to manually find an expression for the inflaton direction. We can make headway on this front by recognizing that if we treat ϕ and θ as polar coordinates, the kinetic term in the Lagrangian could be rewritten in Cartesian coordinates with the x and y terms uncoupled. In Cartesian coordinates, the potential would be canonically normalized. We can define the following position and velocity vectors:

$$\vec{X} = x\hat{x} + y\hat{y}$$

$$\vec{v} = \dot{x}\hat{x} + \dot{y}\hat{y}$$

The inflaton field I will be a linear combination of the x and y fields which is parallel to the resultant direction of motion of the two fields. We can express this as the projection of the position vector in the direction of the velocity vector. The waterfall field R would then be the linear combination of the x and y fields which is perpendicular to the direction of the velocity vector. These can be expressed as dot and cross products respectively as follows.

$$I = \vec{X} \cdot \hat{v}$$

$$R = \left|\vec{X} \times \hat{v}\right|$$

Here we introduce a velocity unit vector $\hat{v}$, where:

$$\hat{v} = \frac{1}{\sqrt{\dot{x}^2 + \dot{y}^2}} (\dot{x}\hat{x} + \dot{y}\hat{y})$$

Taking derivatives of the potential with respect to the inflaton direction is likewise the projection of the gradient of the potential in the direction of the velocity vector. We can define the operator:

$$\frac{\partial}{\partial I} = \hat{v} \cdot \nabla$$

Given a potential V, the first and second derivatives are:

$$\frac{\partial V}{\partial I} = \hat{v} \cdot \nabla \mathrm{V}$$

$$\frac{\partial^2 V}{\partial I^2} = \hat{v} \cdot \nabla(\hat{v} \cdot \nabla \mathrm{V})$$

We have arrived at these expressions for the derivative by considering Cartesian coordinates, but we could use them to work in any system of coordinates, as long as we know how to take the gradient and express the velocity unit vector in those coordinates. This helps us more easily tackle the Spiral Inflation potential, since we can take the gradient and express the velocity unit vector in polar coordinates.

Interestingly, when we use this method to take derivatives for the Spiral Inflation model and calculate values for cosmological observables, we are unable to find a parameter set which is entirely consistent with known observable values. Regardless of parameter choice, we were unable to obtain a scenario which simultaneously preserves large $\underline{r}$, the correct value for n_s, and an acceptable number of e-folds $50 < N < 60$. Even if the Spiral Inflation model itself doesn't work under our constraints, however, the vector notation we use still allows us to extend our analysis to many scenarios with non-canonical kinetic terms.

IV. Conclusion:

The Dante's Inferno model is a hybrid one in which the two different fields on which the potential depends play different roles throughout the process of inflation. The $\tilde{\theta}$ field is the inflaton field, which rolls slowly along a spiral trench. Once this trench becomes unstable, the inflaton moves in the $\tilde{r}$ direction to quickly reach the global minimum of the potential. This trajectory is shown in Fig. 3 through a contour plot.

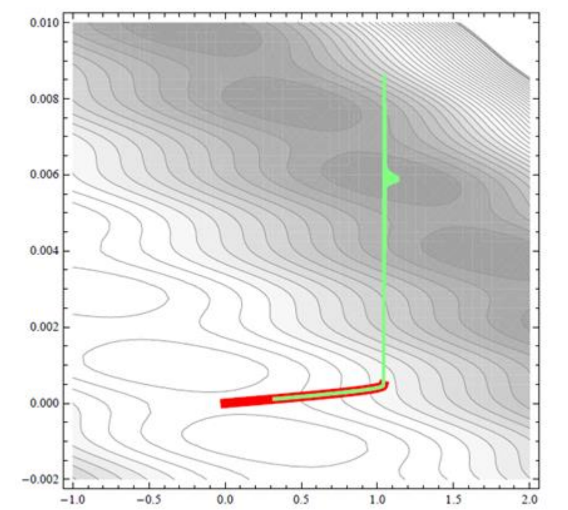

Figure 3: Contour plot of Dante's Waterafall potential, with the trench (red) and inflaton trajectory (green) highlighted. $\tilde{\theta}$ is along the horizontal axis, and $\tilde{r}$ is along the vertical axis.

We find that when we constrain this model with the values of known cosmological observables, we obtain an upper bound on $\underline{r}$ of $\underline{r} < 0.03$. This model is not capable of producing large $\underline{r}$ and therefore large gravity waves, and therefore would be ruled out if $\underline{r} \sim 0.1$, the current observational upper bound, is observed.

We also interpret inflaton motion using a vector notation, a method which could be used to investigate the role of non-trivial kinetic terms in generating large values for $\underline{r}$. This notation opens up the possibility of studying a potential which depends on three fields, and looking at inflaton motion in spherical or cylindrical coordinates. Between investigating two-field potentials with various natural coordinate systems and expanding the scope of the investigation to potentials depending on more than two fields, future extensions of this research could help provide a more comprehensive understanding of when axion monodromy models of inflation can yield large values for $\underline{r}$, an understanding which will be helpful in explaining the observations of groups like Planck and BICEP2.

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VI. References:

[1]: A. H. Guth, Phys. Rev. D 23, 347456 (1981)

[2]: P. A. R. Ade et al. [BICEP2 Collaboration], Phys. Rev. Lett. 112, 241101 (2014) [arXiv:1403.3985 [astro-ph.CO]]

[3]: P. A. R. Ade et al. [BICEP2 and Planck Collaborations], Phys. Rev. Lett. 114, 101301 (2015) [arXiv:150200612[astro-ph.CO]]

[4]: A. R. Liddle and D. H. Lyth, *Cosmological Inflation and Large Scale Structure* (Cambridge University Press, Cambridge, 2000)

[5]: D. H. Lyth, Phys. Rev. Lett. 78, 1861 (1997) [hep-ph/9606387]

[6]: M. Berg, E. Pajer and S Sjors, Phys. Rev. D 81, 103535 (2010) [arXiv:0912.1341 [hep-th]]

[7]: C. D. Carone et al, Phys. Rev. D 91, 043512 (2015) [arXiv :1410.2593[hep-ph]]

[8]: P. A. R. Ade et al. [Planck Collaboration], arXiv:1502.01589 [astro-ph.CO].

[9]: G. Barenboim and W. Park, Phys. Lett. B741, 252255 (2015) [arXiv:1412.2724 [hep-ph]]