The College Of WILLIAM & MARY

Math 112
Lab 6

Differential Equations and Slope Fields

A differential equation (DE) is an equation involving a function and its derivatives. Derivatives have many meanings - slopes, rates of change, curvatures, and so on - and these can be used to develop very detailed and dynamic equations capable of explaining detailed and dynamic situations. This is why differential equations are used heavily in science and physics to describe laws of nature. By expressing a law of nature in the form of a differential equation, we have a mathematical story that we can interpret, analyze and maybe even solve!

In this lab we solve one type of differential equation. We also study a picture of the equation, a "slope field", that will give us a "qualitative" graphical feel for what the differential equation is telling us. In many cases, graphically understanding a differential equation is more important than number crunching or rigorous math. Great mathematicians and scientists like Albert Einstein, Richard Feynman, Isaac Newton, and Archimedes solved complicated problems through simple pictures.

I. Solving Separable Differential Equations

A separable differential equation is one that can be written in the form, . It is solved by first cross multiplying and then integrating each side.

Whenever possible, write the final solution as y explicitly in terms of x.

Example 1: (a) Solve the differential equation, $x+3y^2\sqrt{x^2+1}\,\dfrac{dy}{dx}=0$. (b) Find the solution that satisfies the initial condition $y(0)=1$.

II. Application: Logistic Differential Equation:

Example 2: A 1000 acre forest has a carrying capacity of 100 deer. Assume that the deer population grows logistically with $k = 0.2$. Find the deer population $P(t)$ if the initial population is $P_0 = 10$.
(10 deer at day, $t = 0$)

III. Slope Fields:

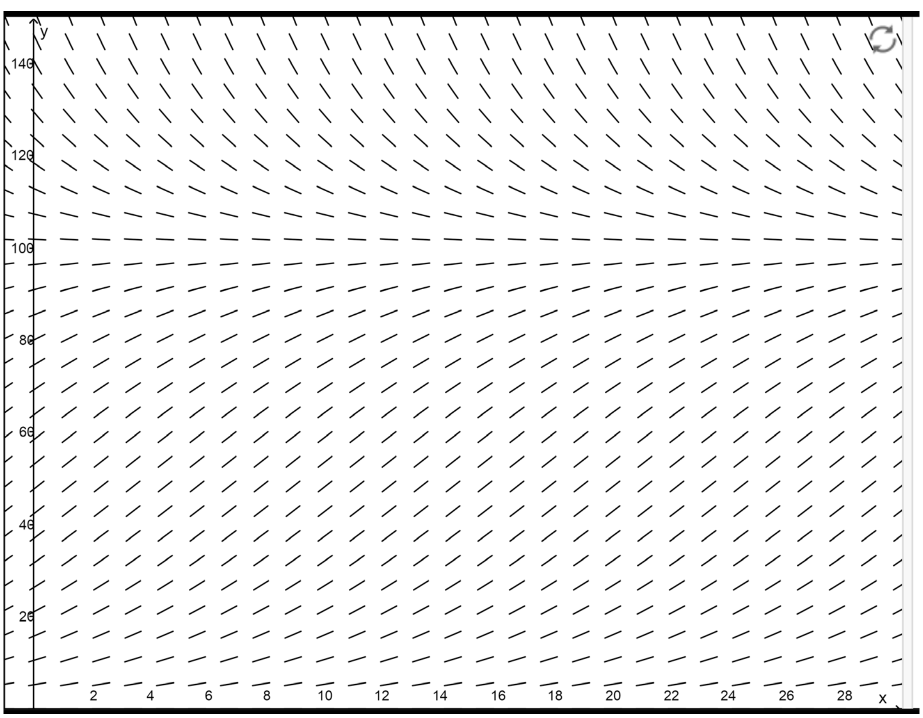
Given a first-order differential equation, $y' = F(x, y)$, we can enter a coordinate pair (x, y) and determine the slope of the solution curve at that point. If we calculate the slopes for multiple coordinate pairs and then sketch small line segments with such slopes, we generate a slope field or direction field for that differential equation. Here we can see what various solution curves look like.

A slope field also shows where the ***equilibrium solution*** to a differential equation. This is where the DE is equal to zero. For our logistic equation this is where the population is neither increasing nor decreasing.

Example 3: Find the equilibrium solutions of the logistic equation, $\dfrac{dP}{dt} = \dfrac{1}{5}P\left(1 - \dfrac{P}{100}\right)$

These equilibrium solutions are easily recognized on a slope field. You can use a slope field generator on a website such as https://www.geogebra.org/m/Pd4Hn4BR. Then, using PrtScr, you can copy and paste to a document.

Example 4: Use a slope field generator to view the logistic equation in Example 3. Mark the nontrivial equilibrium solution and sketch the solution curve for $P(0) = 20$.



Math 112 Lab 6 S24 Exercises Name: _________________________ Section:____ Score:____

You may use your textbook, lab and notes. Students may work cooperatively but must submit their own set of Lab Exercises. Use an online slope field generator where applicable. **No calculators.**

1. Find the general solution to the differential equation, $(\ln x)\dfrac{dx}{dt} - tx = 0$. Your final answer should show x explicitly in terms of t.

2. Solve the initial value problem, $y\dfrac{dy}{dx} = xe^{-y^2}$; $y(0) = -2$. Your final answer should be a single equation in which y is a function of x. (Hint: use the initial condition to find C and this final equation.)

3. A lake is stocked with 500 fish. Assume the fish population grows according to the logistic model with a carrying capacity of 2000 where $k = 0.6$. Find the function, for this fish population.

4. Use the slope field generator on https://www.geogebra.org/m/Pd4Hn4BR or another, if you wish. Generate a slope field for the logistic equation in Exercise 3. Copy and print this slope field. Sketch the solution curve for $P(0) = 1000$. Attach this as a separate page of your submitted Lab 6 Exercises.