1. Evaluate each integral below. You must show all necessary steps including substitutions, integration by parts, partial fractions, etc.

(a) $\int \sin^3(x)\cos^2(x)\,dx$

(b) $\int \frac{x^2}{\sqrt{9-x^2}}\,dx$

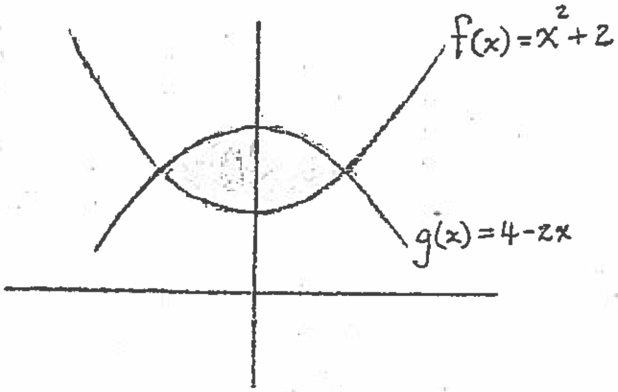
(c) $\int x\cos(3x)\,dx$

(d) $\int \frac{2x^2+x-3}{x^2-x}dx$

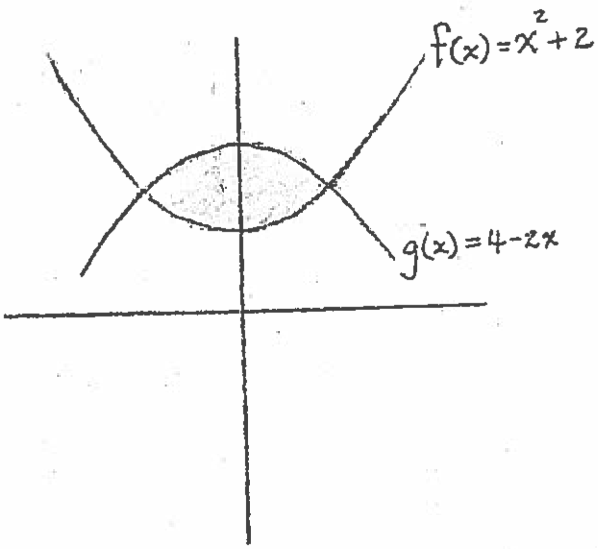
(e) $\int \frac{e^x}{1+e^{2x}}dx$

2. Determine if the integral $\int_3^\infty \frac{\ln x}{x^3}dx$ is convergent or divergent. Evaluate the integral if it is convergent.

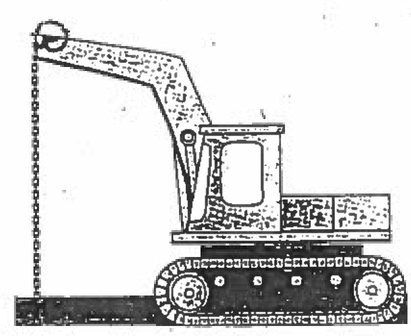
3. Consider the region bound by the curves $f(x)=x^2+2$ and $g(x)=4-x^2$ that is shown below.
(a) Find the area of this region by evaluating the proper integral.



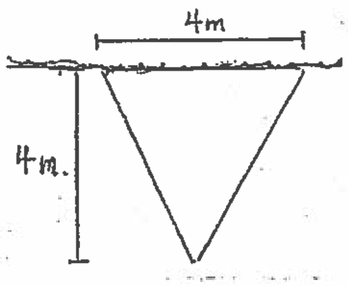
(b) Find the volume of the solid obtained by revolving this region about the x-axis.



4. A uniform chain that *is 30 ft. long and* weighs 60 lbs is hanging vertically from a crane. Find the work required to roll up 20 ft of the chain.



5. An isosceles triangular plate is submerged vertically under the surface of a freshwater lake. The triangular plate has a 4 meter base and a 4 meter height. The base of the plate is on the water surface as shown below. The density of water is $1000\ kg/m^3$, and the gravity constant is assumed to be $10\ m/s^2$. Find the hydrostatic force acting against the plate.



6. Solve the differential equation $\frac{dy}{dx} = (x^2+1)\sqrt{y}, \quad y(1) = 1$.

7. Brine containing $1\,lb/gal$ of salt enters a tank at $2\,gal/\min$ that initially contains $100\ gal$ of fresh water. The stirred mixture leaves the tank at $2\,gal/\min$. Let $A(t)$ represent the amount of salt in the tank at time t.
   (a) Set up the differential equation that models the rate of change of salt in the tank.
   (b) Solve the differential equation for $A(t)$.

8. Express the number $1.\overline{414} = 1.414414414...$ as a ratio of two integers which have no common factors larger than 1.

9. Determine if the series $\sum_{n=1}^{\infty} \frac{e^n}{e^n - 1}$ is convergent or divergent. If convergent, find the sum.

10. Determine if the series $\sum_{n=1}^{\infty} (-1)^n \frac{(\sqrt{2})^n}{3^{2n+1}}$ is convergent or divergent. If convergent, find the sum.

11. Determine if the series $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n + \ln n}$ is absolutely convergent, conditionally convergent or divergent.

12. Find the radius and interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{(2x-3)^n}{\sqrt{n}\, 3^n}$.

13. Evaluate the integral $\int \frac{x}{2+x^7}\, dx$ as a power series.

14. Find the first four terms of the Taylor series for $f(x)=\frac{1}{x^2}$ at $a=1$.

15. (4pts each) If the statement is always true, write TRUE. Otherwise write FALSE and make the proper correction to make the statement true.

________________(a) $\int_2^5 \sqrt{x}\,dx \approx S_8 = \frac{1}{3}\left[\sqrt{2}+4\sqrt{5/2}+2\sqrt{3}+4\sqrt{7/2}+2\sqrt{4}+4\sqrt{9/2}+\sqrt{5}\right]$
Using Simpson's approximation with $n = 6$.

________________(b) If a power series $\sum_{n=1}^{\infty} c_n (x-a)^n$ was found to have a radius of convergence, $R=\infty$, then the ratio (or root) test resulted in a limit equal to ∞.

________________(c) If f is an even function that is continuous on $[-a,a]$, then $\int_{-a}^{a} f(x) = 0$.

________________(d) If $\lim_{n\to\infty} b_n = 0$, then the series $\sum_{n=0}^{\infty} b_n$ is convergent.

________________(e) The distance from $x=0$ to $x=\pi/4$ along the curve $f(x)=\ln(\cos x)$ is equai to the value of $\int_0^{\pi/4} \sec x\,dx$.