Fall 2010

1. For the function f whose graph is given, answer the following. If your answer is "does not exist" state this and *explain why*.

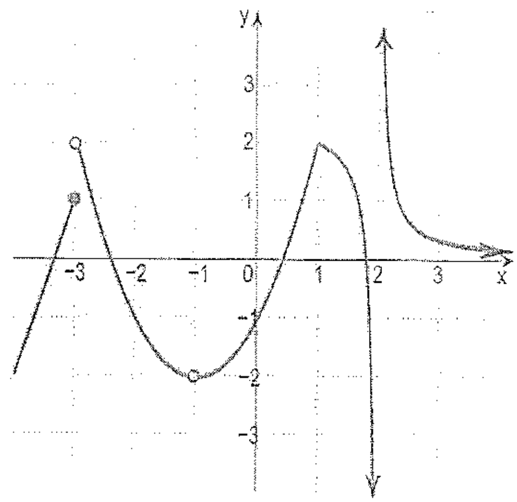
(a) $\lim_{x \to -3^+} f(x)$

(b) $\lim_{x \to -3} f(x)$

(c) $\lim_{x \to -1} f(x)$

(d) $\lim_{x \to 2^-} f(x)$

(e) $\lim_{x \to \infty} f(x)$



2. (a) Fill in the blank to complete the definition of continuity of a function $f(x)$ at a given point $x = a$.

" A function $f(x)$ is continous at a number a if _______________________________________."

(b) *Use the definition of continuity* to find a real number c that makes f continuous on $(-\infty, \infty)$,

$$f(x) = \begin{cases} c^2x^2 - 3c & \text{if } x \le 1 \\ cx - 4 & \text{if } x > 1 \end{cases}$$

3. *Use the limit definition of the derivative* to find $g'(2)$ if $g(x) = \dfrac{1}{x^2}$.

4. Find each limit by using limit laws and analytic methods taught in class. If you use L'Hospital's Rule, indicate this as well as the indeterminate form(s) present where applicable. A table of values or graph will not be accepted.

(a) $\lim_{x \to 0^+} \arctan\left(e^{\frac{1}{x}}\right)$

(b) $\lim_{x \to -\infty} \frac{100 - x^2 + x^7 - 2x^{12}}{1 + 100x + 17x^3 + x^{12}}$

(c) $\lim_{x \to 1^+} \sqrt{\frac{\ln x}{x - 1}}$

(d) $\lim_{x \to \infty} \left(1 + \frac{4}{x}\right)^x$

5. Use the *limit definition(s) of vertical and horizontal asymptotes* to find any vertical and horizontal asymptotes for $f(x)=\dfrac{x-3}{1-2x}$. Your answer(s) should reflect the behavior of the graph of $f(x)$ at the asymptote(s).

6. Use the Intermediate Value Theorem and the Mean Value Theorem to show that $f(x)=2x-1-\sin x$ has exactly one real root.

7. Find the equation of the line that is tangent to the curve $y=7x^2+2x-5$ at $x=1$.

8. Approximate the value $\sqrt{99.9}$ using either linear approximation or differentials. Decimal values are accepted.

9. Given $f(x) = x^4 - 4x - 1$, find the following *showing the proper use of calculus methods*.

(a) Intervals of increase and decrease.

(b) Intervals over which *f* is concave up or concave down.

(c) Any local maximum or minimum values.

10. Given $f(x) = -\ln(x-1)$, find $f^{-1}(x)$ and state the domain and range of this inverse function.

11. Find the derivative of each function. DO NOT SIMPLIFY.

(a) $y = \dfrac{x^2 + x + 7}{x^2 + 1}$

(b) $f(x) = \tan^{-1}\left(e^{2x}\right)$

(c) $y = \sec^3(1-x)$

(d) $h(t) = (t+3)^2(\ln t)^6$

12. Find the derivatives. Simplify.

(a) $y = x^{\sqrt{x}}$

(b) $xy^3 + \cos(x+y) = 1$ (note: y is a function of x)

13. Write TRUE if the statement is always true. Otherwise, write FALSE then explain your answer or give an example.

_______________ (a) If a function $f(x)$ is continuous at $x = a$, then $f(x)$ is differentiable at $x = a$.

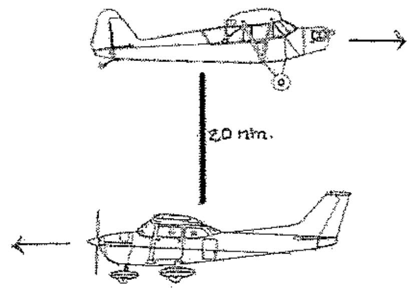
_______________ (b) Every function is guaranteed to have an absolute maximum and an absolute minimum on a closed interval [a, b].

_______________ (c) If a function f is concave up on the interval $(-\infty, \infty)$ and $f'(2) = 0$, then f has a local minimum at $x = 2$.

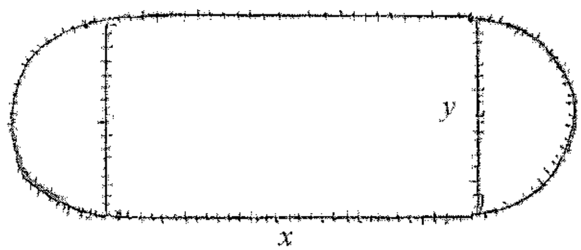
_______________ (d) $\int_{-1}^{2} \frac{1}{x}\, dx = \ln|2| - \ln|-1| = \ln 2$

_______________ (e) If f is continuous on [a, b] then $\int_a^b f(x)\, dx = f(c)(b-a)$, where $f(c)$ is the average value of the function on [a, b].

14. A Piper Cub is traveling east at 100 knots (nautical miles per hour.) A small Cessna is traveling west at 80 knots on a flight path that is 20 nautical miles south of the Piper Cub. At noon the planes are directly abeam one another as shown in the sketch. At what rate is the distance between the two planes changing fifteen minutes later; 12:15 pm? Decimals answers are accepted. Include units in your answer.



15. You have 100 meters of fencing to enclose a rectangle and a semi-circular area at each end; as shown by the sketch. Determine the dimensions x and y that maximize the total area (rectangle plus semi-circles.) Decimal answers are accepted. Include units in your answer.



16. An object is shot vertically upward at an initial velocity of $40\ {}^{ft}\!/_{\sec}$ from the top of a building, 100 feet above the ground. Answer the following, using calculus methods taught in class. Acceleration due to gravity is $-32\ {}^{ft}\!/_{\sec^2}$. Decimal answers are accepted. Include units in your answer.

(a) Find an expression for the height of the object above the ground at any time, *t*.

(b) What is the maximum height reached by the object?

17. For the definite integral $\int_0^1 \sin(\pi x)\, dx$:

(a) Approximate its value as a Riemann sum using $n = 3$ and right endpoints as sample points. (no decimals)

(b) Express the integral as a limit of Riemann sums. Do not evaluate the limit.

18. Evaluate each integral.

(a) $\int_{e}^{e^2} \frac{1-x}{x}\,dx$ (answer in *exact value* not a decimal)

(b) $\int \left(\frac{\sqrt[3]{x}}{2} + \cos x + 7 \right) dx$

19. Let $g(x) = \int_0^x f(t)\,dt$, where f is a continuous function on $[0, 4]$ as shown below.

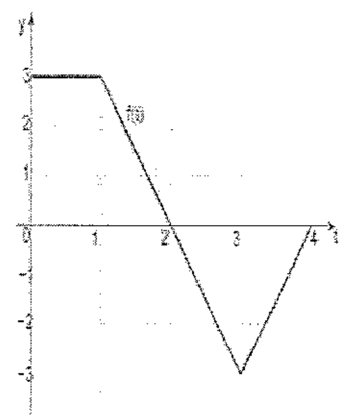
(a) Evaluate $g(0)$.

(b) Evaluate $g(4)$.

(c) On what interval is g increasing?

(d) Where does g have a global maximum value?

(e) Sketch the graph of $g(x)$ on the same coordinate plane.



20. Use the Fundamental Theorem of Calculus to find the derivative of the function $f(x) = \int_{2x}^{3x} e^t\,dt$.