Departmental Narrative for Math 112(Calculus II)

Every section of Math 112 includes the three GER1 criteria of (1) calculations, done by hand or using a calculator or computer, (2) theoretical explanations of why the procedures used in part (1) actually work, and (3) applications recognizable as such to educated non-mathematicians.

Calculations: Almost every exercise and test problem in Math 112 involves calculations. Here are typical questions:

a) Compute the volume of the solid obtained if the curve $y = x^2$ for $1 \le x \le 4$ is revolved about the y axis.

b) Compute the fifth Taylor polynomial coefficient for the function $y = e^{x^2}$ centered at $a = 0$.

c) Use the trapezoid rule to compute an estimate for $\int_0^1 e^{x^2}\ dx$ that is accurate to 5 decimal places.

d) Many evaluation methods for indefinite integrals (e.g., partial fractions techniques, integration by parts, and clever changes of variable) are essentially computational problems, even though they deal with formulae rather than numbers.

Theoretical issues: Theoretical explanations abound in Math 112. Each of the course's many sections spends a considerable amount of time studying the theory of integration, the theory of infinite series, and the theory of Taylor polynomials.

As in Math 111, one of our goals is to help our students see that, in university mathematics, the question "Why does the method work?" is as important as the question "What method should we use?". For example, we prove the various series convergence testing theorems and we prove that the logistic growth formula is the only possible solution of the logistic differential equation. Typically we do not ask our students to reproduce major proofs on our exams, but we do test students' knowledge of theoretical tools, e.g., to find intervals of convergence for power series and to derive analytic solutions to certain kinds of differential equations.

Applications: Math 112 includes a large section on differential equations, and one of the central goals of Math 112 is for students to develop the ability to set up differential equations models of problems described in words and then to study the models using calculus techniques, and finally translate the mathematical findings back into the problem's real-world context[1]. Here are some examples. We will be pleased if our students realize that examples (e) and (f) are essentially the same.

e) Leaves fall on a square meter of forest floor at the rate of 5 grams per year, and decay processes cause 10% of leaf matter to decompose each year. Give a differential equation that describes this situation, then solve it, and determine what happens to the amount of leaf material in the long run.

[1]At the time of the last GER1 assessment, we argued to the working group and to EPC that developing students' ability to create mathematical models is a more important GER1 skill than working out specific applications problems. We repeat that claim today and would like to discuss our view with EPC in the hope that "mathematical modeling" can be specifically mentioned in the future as a GER 1 expectation.

f) A trust account begins with $15,000 and earns 4% per year continuous interest. The account is required to pay out $1,000 per year to its beneficiaries. Set up a differential equation that describes this situation, solve it, and determine how long the trust will last.

Are these real applications problems? We believe that they are, even though the numbers in them are made up. In addition, these are the kinds of problems that sharpen students' mathematical modeling skills.

But just to make sure that our students see indisputable applications of mathematics, the department has designed the Math 112 laboratory packet (a required part of the course that is worth 15% of each student's final grade) to include more problems of the types described above. Because the department has an NSF grant to explore the feasibility of including a greater emphasis on chemistry in beginning calculus courses, some of the problems have a chemical flavor. Here are three examples that have been used from time to time in the Math 112 lab packet. Once again, even though the numbers are made up, we believe that the problems are true applications and that solving these problems adds to students' ability in mathematical modelling.

g) Ecologists have found that if fishing removes H fish per year from a certain lake, then the population $P(t)$ of fish at time t satisfies the modified logistic differential equation

$$\frac{dP}{dt} = 2P - 0.01P^2 - H.$$

For each of the harvesting values $H = 75, 100, 200$, plot a graph of P against t = time. For which of those three values of H is there an initial population $P(0)$ such that the population P does not eventually die out? For what range of H-values will the population of fish never die out? Make a recommeddation about the policy that should control fishing in the lake.

h) Suppose water is to be raised from a well 40 ft. deep by means of a bucket attached to a rope. In the problems below, the force of interest is the tension in the rope, perhaps as exerted by someone at the well-head. When the bucket is full, it weighs 30 lbs. Assume that that the tension in the rope exactly matches the weight in the bucket (Warning: In reality, the rope tension is greater that the bucket weight.)

(h-1) Neglecting the weight of the rope, find the work done by the tension on the rope as the bucket rises to the top of the well.

(h-2) Now, suppose the bucket has a leak that causes it to lose water at the constant rate of 0.25 lb. for each foot the bucket is raised. If height is measured from the water level in the well, what is the weight of the bucket at x ft above the water?

(h-3) Using the result from part (h-2), what is the work done by the tension in the rope as the bucket moves a short distance x?

(h-4) Using the results from part (h-3) write the integral needed to find the total amount of work done by the tension in the rope as the leaking bucket in raised to the top of the well. You may evaluate the integral by hand or using Matlab.

(i) A reversible adiabatic expansion of an ideal gas satisfies the differential equation

$$C(T)dT + pdv = 0,$$

where $C(T)$ is the molar heat capacity of the gas at constant volume and $p = RT/v$ is the ideal equation of state. (R is the ideal gas constant.) An important case is when $C(T) = C$ is a constant.

(i-1) Solve the differential equation in closed form to obtain a relation between T and v.

(i-2) This result is more typically displayed by eliminating T with the aid of the equation of state, $p = RT/v$. Work out the formula that connects p and v (i.e., no T s allowed) for a reversible adiabatic expansion.

(i-3) Now use the Matlab command *dsolve* to work out the solution to the adiabatic differential equation. You will need to let *dsolve* know that the independent variable is v rather than the default t: $T = dsolve(C * DT + p = 0, v)$. Compare the two versions of solution.