Departmental Narrative for Math 111(Calculus I)

Every section of Math 111 includes the three GER1 criteria of (1) calculations, done by hand or using a graphing calculator, (2) theoretical explanations of why the procedures used in part (1) actually work, and (3) applications recognizable as such to educated non-mathematicians.

Calculations: Almost every exercise and test problem in Math 111 involves calculations. Here are typical questions:

a) Compute the slope of the line through $(3, 2)$ and $(5, -1)$

b) Find the slope of the tangent line to the curve $y = e^{2x} + x$ at the point where $x = 0$.

c) Find all intercepts and asymptotes of the curve $y = \frac{x^2-3x+2}{x^2+1}$.

d) Compute the area bounded by the curve $y = 3 - x^2$ and the curve $y = e^x$

e) Use tangent line approximation to estimate $\sqrt{99}$.

Theoretical issues: Theoretical explanations abound in Math 111. Each of the course's many sections will spend a considerable amount of time studying the theory of limits. Each section will include a proof that $\lim_{x\to 0} \frac{\sin(x)}{x} = 1$ when it studies the derivative of $y = \sin(x)$. Each section will discuss the Intermediate Value Theorem, the Mean Value Theorem, L'Hospital's rule and Riemann theory of the definite integral. Each section will discuss a proof that if $f(x) \geq 0$ is a continuous function defined on the interval $[a, b]$ and if $A(x)$ is the area between the x-axis and the graph of $y = f(x)$ between a and x, then $A'(x) = f(x)$.

Most of our students have never paid much attention to proofs before, and one of our goals is to help them see that, in university mathematics, the question "Why does the method work?" is as important as the question "What method should we use?". Typically we do not ask our students to reproduce major proofs on our exams, but we do test students' knowledge of theoretical tools such as limits, L'Hospital's rule, and how data about $f'(x)$ can be used to study extreme points and concavity of $f(x)$. (See, for example, the pre-test problem in which students were given the graph of $f'(x)$ and asked where the function $f(x)$ is increasing, decreasing, concave up, concave down, etc.)

Applications: One of the central goals of Math 111 is for students to develop the ability to set up mathematical models of problems described in words and then study the models using calculus techniques, and finally translate the mathematical findings back into the problem's real-world context. At the time of the last GER1 assessment, we argued to the working group and to EPC that developing students' ability to create mathematical models is a more important GER1 skill than working out specific applications problems. We repeat that claim today and would like to discuss our view with EPC in the hope that "mathematical modeling" can be specifically mentioned in the future as a GER 1 expectation.

However, we are also careful to include in Math 111 a family of problems that we hope will be recognized as applications by the College's review committees. Here are examples from the current textbook.

f) Find the dimensions of the postal tube with the largest possible volume that can be mailed, given the USPS restriction that "length plus girth is at most 84 inches".

g) A painting of height H is hung on a wall so that it is at distance D above eye-level. How far from the wall should an observer stand in order to maximize the angle subtended at the observer's eye by the painting?

h) Two houses are located 1 mile apart along a straight road. The first is set back 0.25 miles from the road, and the second is set back 0.5 miles from the road. We want to connect both houses to a telephone pole located somewhere along the road. Where along the road should we place the pole in order to use the least amount of wire?

i) I stand at the edge of a 300 foot cliff and throw a ball upwards with a velocity of 55 feet per second. How fast is the ball moving when it hits the ground below?

j) A radioactive element is observed to decay by one half of one percent in 10 years. What is the half-life of the element?

Are these real applications problems? We believe that they are, even though the numbers in them are made up. In addition, these are the kinds of problems that sharpen students' mathematical modeling skills.

But just to make sure that our students see indisputable applications of mathematics, the department has designed the Math 111 laboratory packet (a required part of the course that is worth 15% of each student's final grade) to include problems of the following types. Because the department has an NSF grant to explore the feasibility of including a greater emphasis on chemistry in beginning calculus courses, many of the problems have a chemical flavor. Here are three examples from the fall 2007 lab packet.

k) A murder has been committed. At 8am, a forensic scientist discovers that the victim's body temperature is 90 degrees (F) and one hour later a second measurement reveals the victim's body temperature to be 88 degrees. When was the murder committed?

l) Recall that for one mole of an ideal gas at a fixed temperature T, pressure p and volume V are connected by the ideal equation of state $v \times p(v) = RT$ (where R is a constant). The equations of state that we have encountered so far are designed for real not ideal gasses. However, even with a real gas, if we keep T fixed, we may consider a drastic increase in V so that the gas becomes very dilute and behaves more and more like an ideal gas. Chemists capture this idea by writing $\lim_{v\to\infty}(v \times p(v)) = RT$. Is this requirement satisfied by the van der Walls equation of state $p = \frac{RT}{v-b} - \frac{a}{v^2}$?

m) The van der Walls equation for CO_2 is $p(v) = \frac{RT}{v-b} - \frac{a}{v^2}$ where $a = 3.592$ and $b = 0.04267$, with the universal gas constant being $R = 0.0821$. What are the extreme values of $p(v)$ for supercooled CO_2 at 250 Kelvins with $0.6 \leq v \leq 5$?